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EFFECT OF PARTITIVE VARIATION TEACHING STRATEGY ON PUPIL'S MOTIVATION AND PERFORMANCE IN BASIC SCHOOL ALGEBRA IN BENUE STATE, NIGERIA

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ABSTRACT

This study investigated the effect of partitive variation teaching strategy on pupils' motivation and academic performance in basic five algebra in Benue state, Nigeria. The quasi experimental, pretest-posttest design was adopted for the research. Four schools were systematically selected in Zone B education zone of the state. There were 113 pupils (54 male, 59 female) in the experimental group and 141 pupils (77 male, 64 female) in the control group. Two research questions and two hypotheses guided the study. There were two instruments used for the study: the algebra performance test (APT) which was an objective test and the Algebra Motivation Questionnaire (AMQ). These were administered on the respondents before and after teaching using carefully prepared lesson plans for both groups of pupils. The experimental procedure took 6 weeks from training to completion. The data collected were analysed using SPSS for Windows analytical software. Means and standard deviations were used to answer the research questions. The multivariate analysis of covariance (MANCOVA) was used to compare the means of motivation ratings, while analysis of covariance (ANCOVA) was used to test the hypotheses at 0.05 level of significance. Findings revealed that there was statistically significant difference $F(6,490) = 44.64$; $P = 0.00 < 0.05$ in the mean motivation ratings and mean performance $F(3,245) = 43.12$; $P = 0.00 < 0.05$ between the experimental and control groups. The study concluded that the partitive variation teaching strategy motivates pupils and improves algebra performance among Basic 5 pupils.

Keywords:partitive variation, motivation, performance, algebra, Basic school

INTRODUCTION

Mathematics education undoubtedly takes center stage in a world of unending scientific and technological advancements, as human kind continually seeks answers to questions about its needs and those of the society. Countries continue to place premium on the teaching and learning of school mathematics. Students need to acquire mathematical knowledge and skills to compete and survive in life as well contribute their own quota to their immediate society and the world at large. These skills include logical reasoning, problem solving, and the ability to think in abstract ways. The challenge in education today however, is to effectively teach students of diverse ability and different pace of learning so they are able to learn mathematics by developing positive motivation and improved performancesin mathematics learning.

Motivational theories seek to investigate what gets individuals to move towards what activities and to describe the characteristics of these activities (Pantziara&Philipou, 2015). Motivation is defined as an internal state that arouses, directs, and maintains behavior, but simply stated motivation is a reason of students' thinking in a given situation (Garut, 2011). Motivation may also be seen as a theoretical concept utilized to clarify human behaviour. It provides the motive for human beings to react and fulfill their needs (Gopalan, Bakar, Zulkifli, & Mat, 2017). The question of how to motivate students in the classroom has become a leading concern for teachers of all disciplines.

Addressing the issue of content, Iji and Omenka (2015) found that the responses of the subjects involved in their study showed poor agreement in the classifications of the mathematics concepts in algebra, number and numeration, geometry, trigonometry and statistics. The high rate of failure in public examination in Nigeria remains a huge problem to researchers, teachers and indeed all stakeholders in education. Studies have identified factors responsible for the high rates of failure to include among others, students' negative attitude to the subject, lack of qualified teachers, inadequacy of teachers, lack of necessary learning skills, specialized language of the subject and inadequate and unsuitable textbooks. Also, it has been established that there is a significant relationship between teachers' method of teaching, teachers' attitude and students' achievement in mathematics (Avong, 2013; Daso, 2013).

The variation teaching strategy which is based on the variation theory is one of those novel teaching strategies employed by mathematics teachers. Variation theory is a theory of learning and experience that explains how a learner might come to see, understand, or experience a given phenomenon in a certain way and why two students sitting in the same class might come to understand a concept differently. It is a necessary component in teaching in order for students to notice what is to be learned (Bussey, Orgill&Crippen, 2012; Learning Project Team of HKU, 2011; Kullberg, Kempe&Marton, 2017).

Lai and Murray (2013) opine that procedural variation is derived from three forms of problem solving in algebra: Varying a problem; extending the original problem by varying the conditions, changing the results and generalization.Partitive Variation Teaching Strategy (PVTS) is when a part or parts of a particular problem are held constant, while other parts are changed. For example, given y = 2, evaluate:

a. $5y$
b. $5y - 3$
c. $5y^2 - 3$
d. $5y^2)^2 - 3$

It can be used to address individual differences in the classroom by allowing students to draw upon their personal experiences and apply them in their learning (Cheng, 2016).Random Variation Teaching Strategy (RVTS) on the other hand refers to the teaching of algebra, in which cases, entire set of problems come with a varied structures.For example, given that y = 2, evaluate:

a. $x + 5$
b. $4x - 9$
c. $3x^2 + 7$
d. $9 (x - 2)$

The object of learning for both examples is to evaluate algebraic expressions by substituting letters for numbers. A mathematics pedagogy that is rooted in variation is one that purposefully provides learners with the means to experience variation through strategically designed activities in order to create a mathematically rich learning environment that allows learners to discern the object of learning. It is defined by its critical features that must be discerned in order to constitute the meaning aimed for. So as a pedagogic approach, a pattern of variation is a useful tool for structuring teaching to make the learning of the object of learning possible (Mhlolo, 2013). This ultimately leads to improved performances as evidenced by a number of studies.

The study by Ifelunni, Ugwu, Aneke, Ibiam, Ngwoke, Ezema, Charles, Oraelosi, & Ede (2019) investigated motivation as a determinant of academic achievement of primary school pupils in Mathematics in South-East, Nigeria. A population of 357,115 primary 5 pupils in all the 5,378 public primary schools in South-East, Nigeria was used for the study. The sample for the study comprised 400 primary 5 pupils. The findings of the study revealed, among others that there is a significant correlation between intrinsic motivation and primary school pupil's academic achievement. Liu (2018) carried out a study on "Potential reciprocal relationship between motivation and achievement: A longitudinal study". It included students from 1,052 high schools across the United States. Motivation had a greater influence on follow-up mathematics achievement (0.079, $p<0.001$). Jing, Tarmizi, Bakar and Aralas (2017) investigated the effect of utilizing Variation Theory Based Strategy on students' algebraic achievement and motivation in learning algebra. The study used quasi-experimental non-equivalent control group research design and involved 56 Form Two (Secondary Two) students in two classes (28 in experimental group, 28 in control group) in Malaysia. Result from analysis of covariance (ANCOVA) indicated that the experimental group students achieved significantly better test scores than the control group. In addition, result of Multivariate Analysis of Variance (MANOVA) also showed evidences of significant effect of VTBS on experimental students' overall motivation. These results suggested the utilization of VTBS would improve students' learning in algebra. Effect of integrated curriculum delivery strategy on secondary school students' achievement and retention in Algebra in Benue state was experimented by Anyor and Iji (2010). The population comprised 1,368 Senior Secondary 1 students out of which 149 were purposively sampled. The study found among other things that Integrated Curriculum Delivery Strategy (ICDS) enhanced students' achievement and retention in algebra taught during the course of the study. The ICDS highlighted the importance of creativity which is akin to the partitive variation teaching strategy, in the teaching and learning of algebra in schools. These empirical studies provided gaps and conclusions which were further investigated in the present study.

The following research questions guided the study:

i. What is the effect of the PVTS on the mean motivation ratings of Basic 5 pupils as compared to those taught using the RVTS?
ii. What is the effect of the PVTS on the mean algebraic performance of Basic 5pupils as compared to those taught using RVTS?

Two corresponding null hypotheses were formulated and tested at 0.05 level of significance:

i. There is no significant effect of the PVTS on the mean motivation ratings of Basic 5 pupils as compared to those taught using the RVTS.
ii. There is no significant effect of the PVTS on the mean algebraic performance of Basic 5pupils as compared to those taught using RVTS.

MATERIAL AND METHODS

The quasi experimental, pretest-posttest, control group design is adopted for the study. This design is seen as suitable due to the inability to manipulate and randomize the respondents who were primary school pupils studying in Basic Five. Denga (2017) posited that quasi-experimental studies are conducted under conditions that do not permit control, manipulation of variables or random selection. Random assignment can be achieved but the intact groups coupled with administrative constraints (random selection may lead to a disruption of school organization and classes) do not allow randomization, control or manipulation. It is an investigation of the effectiveness of a teaching method where random selection of subjects is not possible being a good example, in this study, the Partitive Variation Teaching Strategy.

The area of study is Benue Education Zone B, also called Benue North West Senatorial Zone. The population comprised20,895 pupils from 1,804 primary schools in Zone B education zone of Benue state (SUBEB, Makurdi, 2019). The sample for this study was 254 pupils. There was a total of 113 pupils (54 male and 59 female) in the experimental group, while the control group had 141 pupils (77 male and 64 female).

Procedure Methodology

The study made use of 2 instruments: Algebra Motivation Questionnaire (AMQ) and the Algebra Performance Test (APT). The AMQ was a 30-item motivation questionnaire containing both pleasant (positively skewed) and unpleasant (negatively skewed) items, designed by the researcher. The Algebra Performance Test (APT) was a 20-item test set by the researcher. There were 2 sets of lesson plans for the research in each sub-topic treated, one for the Partitive Variation Teaching Strategy (experimental group) and the other for the Random Variation Teaching Strategy (control group). Some lesson plans were for a period of 1 hour, while others were for 30 minutes.

Statistical Analysis

The means and standard deviations of the variables studied were used to answer the research questions. The Analysis of Covariance (ANCOVA) was used to test the hypotheses relating to students' algebraic performance. Motivation on the other hand, was analysed using Multivariate Analysis of Covariance (MANCOVA) with data obtained from the AMQ. Both hypotheses were tested at 0.05 level of significance.

RESULT

The data is presented according to the objectives of the study. This is done by placing data for a research questions first and followed immediately by the corresponding hypothesis.

Research question one: What is the effect of PVTS on the mean ratings of Basic 5 pupils' motivation as compared to those taught using the RVTS?

Table 1:Descriptive statistics for mean motivation ratings in the experimental and control groups

		Experimental (N = 113)		Control (N = 141)	
	Motivation	Mean	Std. Dev.	Mean	Std. Dev.
Posttest	Attention	3.62	0.25	2.26	0.38
	Relevance	3.29	0.34	2.73	0.38
	Confidence	3.04	0.42	2.85	0.43
	Satisfaction	3.10	0.44	2.86	0.37
	Interest	3.22	0.35	2.71	0.41
	Total	**3.26**	**0.42**	**2.68**	**0.45**
Pretest	Attention	2.49	0.37	2.36	0.32
	Relevance	2.84	0.35	2.56	0.30
	Confidence	2.94	0.43	2.90	0.42
	Satisfaction	2.91	0.42	2.83	0.43
	Interest	2.94	0.41	2.67	0.37
	Total	**2.83**	**0.43**	**2.66**	**0.42**

Results in *Table 1* display the means and standard deviations of the responses with respect to motivation of pupils in the experimental and control groups. It reveals that in the experimental group, the pupils consistently improved individually and in the total means (2.83 to 3.26) while the standard deviation got smaller from 0.43 to 0.42 which shows an improvement in the data distribution across the sub-scales of attention, relevance, confidence, satisfaction and interest. Results in the table also illustrate the means and standard deviations of the responses with respect to motivation of pupils in the control group. It reveals that the pupils appear to have responded in a similar fashion individually and in the total means (2.66 and 2.68). The standard deviations were 0.42 and 0.45 across the sub-scales of attention, relevance, confidence, satisfaction and interest. The standard deviations in both groups showed elements of homogeneity in their responses.

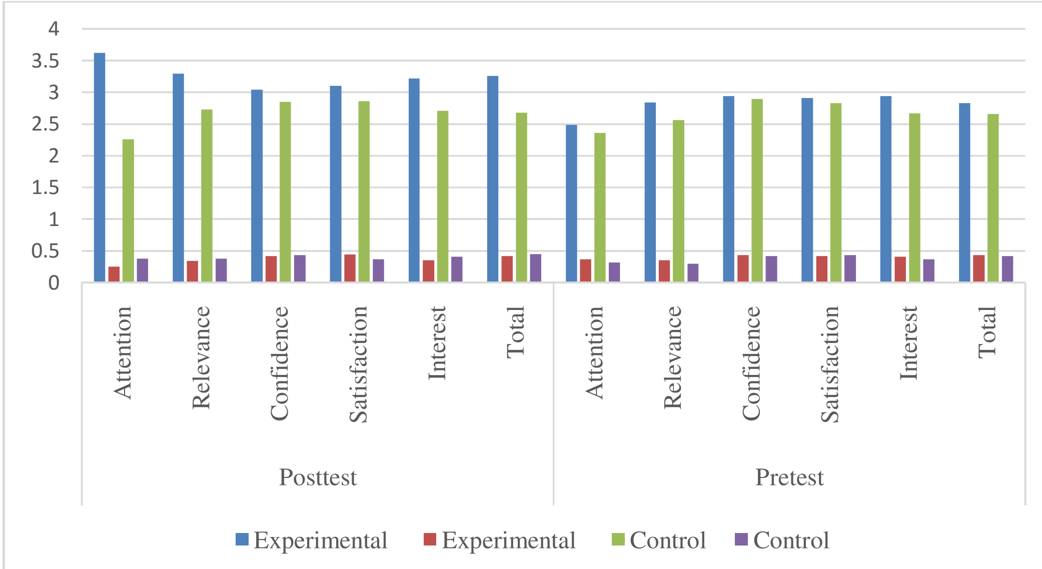

Figure 1: Bar chart for mean motivation ratings in the experimental and control groups

The figure is a pictorial representation of Table 1. It displays the means and standard deviations of the experimental group in blue and red respectively, while the means and standard deviations of the control group are displayed in green and purple respectively.

Table 2: Descriptive statistics for the motivation responses of pupils in the four sub-groups

	Group	Mean	Std. Deviation	N
Posttest	Experimental1	3.26	0.22	71
	Experimental2	3.25	0.19	42
	Control1	2.68	0.27	34
	Control2	2.68	0.27	107
Pretest	Experimental1	2.81	0.28	71
	Experimental2	2.86	0.24	42
	Control1	2.67	0.25	34
	Control2	2.66	0.24	107

Results in Table 2 show that the mean responses of the pupils in Experimental Group One increased from 2.81 in the pretest to 3.26 in the posttest, with standard deviations of 0.28 and 0.22 respectively. In the Experimental Group Two, the pupils had a mean response on 2.86 in the pretest and increased to 3.25 in the posttest, with standard deviations of 0.24 and 0.19 respectively. In the Control Group One, the

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pupils had mean response of 2.67 in the pretest and 2.68 in the posttest with standard deviations of 0.25 and 0.27 respectively. The mean response of the pupils in Control Group Two was 2.66 with a standard deviation of 0.24, in the pretest and mean of 2.68 and standard deviation of 0.27 in the posttest, this demonstrated an almost static response in the control group.

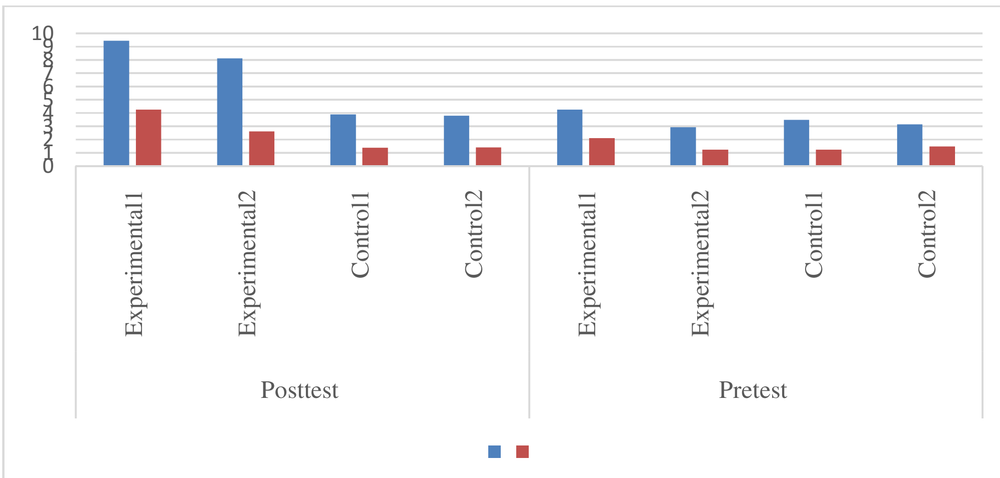

Figure 2: Bar chart for the motivation responses of pupils in the four sub-groups

The bar chart in *Figure 2* illustrates the mean and standard deviations of the mean motivation ratings of the pupils in the experimental and control groups as individual groups.

Table 3:Summary descriptive statistics for mean motivation ratings according to the experimental and control groups

Group	N	Pretest Mean	S.D.	Posttest Mean	S.D.	Mean Gain
Experimental	113	2.83	0.43	3.26	0.42	0.43
Control	141	2.66	0.42	2.68	0.45	0.02
Mean difference		**0.17**		**0.58**		**0.41**

The experimental and control groups had a mean motivation rating of 2.83 and 2.66 in the pretest respectively with a mean difference of 0.17 in favour of the experimental group. The data also showed that the pupils in the two groups had corresponding means of 3.26 and 2.68 in the posttest, this gave a mean difference of 0.58, also in favour of the experimental group. There was a mean gain of 0.43 for the experimental group to the control group's mean gain of 0.02, which gave rise to a mean gain difference between the groups of 0.41. The standard deviation for the experimental group (0.42) was smaller than those of the control group (0.45) in the posttest; this indicates that the responses of the pupils in the experimental group were more homogenous than those in the control group. This depicts a positive effect of the PVTS motivation of pupils in learning algebra among the sample of pupils, to answer the research question.

Hypothesis one: There is no significant effect of the PVTS on the mean motivation ratings of Basic 5 pupils as compared to those taught using the RVTS

Table 4:Summary of multivariate tests for mean motivation ratings between the experimental and control groups

Effect		Value	F	Hypothesis df	Error df	Sig.
Groups	Pillai's Trace	0.59	33.94	6	492	0.00
	Wilks' Lambda	**0.42**	**44.64**	**6**	**490**	**0.00**
	Hotelling's Trace	1.38	56.26	6	488	0.00
	Roy's Largest Root	1.38	112.93	3	246	0.00

The data is interpreted using Wilks' Lambda, the recommended measure (Lund & Lund, 2020) for a multivariate statistic (MANCOVA) for the study. Data in *Table 4* shows that there is a statistically significant difference $F(6, 490) = 44.64$ and $p = 0.00 < 0.05$ in the mean motivation rating of the respondents. The research hypothesis one is therefore rejected, which implies that the mean motivation ratings of pupils taught using the PVTS differ significantly from those taught using the RVTS. This suggests that the pupils showed greater attention, relevance, confidence, satisfaction and interest in the learning of algebra when taught using the intervention strategy.

Research question two: What is the effect of the PVTS on the mean score of Basic 5 pupils' algebraic performance as compared to those exposed to RVTS?

Table 5:Descriptive statistics showing the mean scores at APT in the experimental and control groups in terms of the four sub-groups

	Group	Mean	Std. Deviation	N
Posttest	Experimental1	18.99	7.74	71
	Experimental2	21.05	9.95	42
	Control1	9.82	3.79	34
	Control2	9.81	3.69	107
Pretest	Experimental1	11.10	5.70	71
	Experimental2	8.86	2.58	42
	Control1	8.38	3.04	34
	Control2	7.48	3.54	107

Results in Table 5 reveal the means and standard deviations of the 4 sub-groups in terms of the algebra performance test. The Experimental Group One had a mean of 11.10 in the pretest with a standard deviation of 5.70 and a mean of 18.99 and standard deviation of 7.74. The Experimental Group Two had a mean of 8.86 and 21.05 in the pretest and posttest respectively, with standard deviations of 2.58 and 9.45. The Control Group One had a mean of 8.38 in the pretest and 9.82 in the posttest, with standard deviations of 3.04 and 3.79 respectively. The Control Group Two had a mean of 7.48 in the pretest and 9.81 in the posttest, with corresponding standard deviations of 3.54 and 3.69 respectively. On the whole, the two groups demonstrated similar data characteristics going by their standard deviations. However, there appears to be a disproportionate data behaviour in Experimental Group Two, where the standard deviation in the posttest appears abnormal to the other results in relation to the mean and total number of respondents.

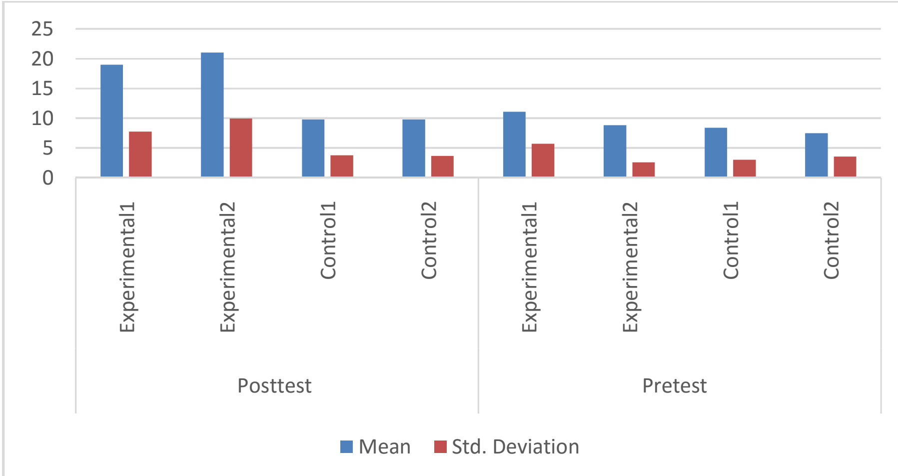

Figure 3: Bar chart showing the mean scores at APT in the experimental and control groups in terms of the four sub-groups

The means in blue and standard deviations in red illustrate the scores of pupils in the groups with respect to the pretest and posttest performance.

Table 6: Summary descriptive statistics showing the mean difference between the experimental and control group at APT

Group	N	Pretest Mean	S.D.	Posttest Mean	S.D.	Mean Gain
Experimental	113	9.98	4.14	20.02	8.85	10.04
Control	141	7.93	3.29	9.82	3.74	1.89
Mean difference		**2.05**		**10.20**		**8.15**

Results in *Table 6* shows that pupils in the experimental group had a mean of 9.98 and 20.02 and standard deviation of 4.14 and 8.85 in the pretest and posttest respectively; this gives a mean gain of 10.04 for the experimental group. The control group on the other hand had a mean performance score of 7.93 and 9.82 with standard deviations of 3.29 and 3.74 in the pretest and posttest respectively with a mean gain of 1.89. There was a mean difference of 2.05 and 10.20 in the pretest and posttest, and 8.15 in the mean gain. The standard deviations of the two groups are an indication that the data sets share similar traits, because they fall within the same side and partition of the normal curve. To answer the research question, the PVTS appears to have caused an increased algebraic performance in the APT by pupils in the experimental group as compared to pupils that were taught using the RVTS.

Hypothesis two: The PVTS has no significant effect on Basic 5 pupils' algebra performance mean score as compared to those taught using the RVTS.

Table 7: Summary tests of between-subjects (experimental and control groups) effects with respect to the APT using posttest as the dependent variable

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	7433.87	8	929.23	25.18	0.00
Intercept	4701.44	1	4701.44	127.42	0.00
APTpretest	939.46	1	939.46	25.46	0.00
Gender	54.74	1	54.74	1.48	0.22
APT	4773.41	3	1591.14	43.12	0.00
Gender * APT	157.16	3	52.39	1.42	0.24
Error	9039.95	245	36.90		
Total	67952.00	254			
Corrected Total	16473.83	253			

The analysis shown in *Table 7* reveal that there was a statistically significant effect, where $F(3, 245) = 43.12$ and $P = 0.00 < 0.05$ of the PVTS when compared to the RVTS in the mean performance scores of pupils. The null hypothesis is rejected which implies that the PVTS has a significant effect on Basic 5 pupils' algebra performance mean score as compared to those taught using the RVTS.

DISCUSSION

This study found a significant difference in the mean motivation ratings of pupils in the experimental group and the control group in favour of the experimental group which agrees with Jing, Tarmizi, Bakar and Aralas (2017).The findings also agree withYakubu (2017) who found that pupils with mathematics learning challenge in the treatment group showed higher motivation than pupils in the control group. There were evidences of significant effect of the partitive variation theory-based strategy on experimental students' overall motivation. This indicated that the partitive variation teaching strategy may be capable of triggering both the intrinsic and extrinsic motivation of pupils in the middle basic level of education in Benue state, Nigeria. This was made more consistent by the more improved mean motivation ratings in the 5 subscales of attention, relevance, comprehension, satisfaction and interest, which the pupils in the experimental group consistent had higher mean responses.

There was statistically significant difference in the performance of students between the experimental group and the control group, which agrees with Anyor and Iji (2010) as well as Jing, Tarmizi, Bakar and Aralas (2017). The positive interaction of organizational culture, education in this case, and human resource management (teaching) would result in self-esteem and self-actualization. Generally, it is agreed that dispositions such as motivation, curiosity and perseverance can be recognised when students persist at difficult tasks, take risks and exhibit open mindedness (Al-Shara, 2015). The findings of this study also agree withthose of Liu (2018) as well as Ifelunni, Ugwu, Aneke, Ibiam, Ngwoke, Ezema, Charles, Oraelosi, and Ede (2019) that there is a significant influence of motivation or that there is a potential reciprocal relationship between motivation and academic performance of students. García, Rodríguez, Betts, Areces and González-Castro (2016) stated that Mathematics enjoyment or satisfaction positively predicted mathematics achievement as has been confirmed in this study. Higher motivation ratings which may be due to the partitive variation teaching strategy predicatedbetter algebraic performances among Basic school learners.

CONCLUSION

This study concluded that pupils in the experimental group significantly showed higher motivation ratings and consistently outperformed those in the control group in the algebra performance test. The study further concluded that the partitive variation teaching strategy may be useful for the improvement of algebra teaching and learning in Middle Basic (Basic 5) level of education in Benue state, which may be replicated in other locations.

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ANALYSIS OF ERRORS IN COMPLETING SQUARE WHEN SECONDARY SCHOOL STUDENTS SOLVE QUADRATIC EQUATION BY NEWMAN ERROR ANALYSIS PROCEDURE IN NASARAWA STATE

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ABSTRACT

The purpose of this study was to diagnose, using the Newman error analysis model. The error committed when students solve quadratic equation using completing square method. The target population was all SS3 students in Nasarawa State public schools. Three research questions and three hypotheses were answered and tested respectively. Survey design was used for the study. Data collected was analysed using student t-test. The result of the findings showed that all the students committed errors at various stages of Newman model and there is no significance difference in the means of errors committed by SS3 students that solved quadratic equation using completing square method.

INTRODUCTION

The importance given to mathematics in the curriculum from primary to the secondary level reflects the role played by mathematics in contemporary society. It is in understanding of this that many countries now resort to making comprehensive and well programmed efforts towards effective teaching and learning of science and mathematics at all levels of the educational system through the development and implementation of improved programmes and projects (Azuka, 2001). In most schools in Nigeria for example, the study of mathematics is made mandatory for all students. In order to secure admission for most courses at higher levels of education, a credit pass in mathematics is a pre-requisite. In spite of importance of mathematics in human existence, students still perform poorly in the subject. Musa (2014) stressed that West African Examination Council (WAEC) results in mathematics between 2004 and 2013 attest to the candidates' poor performance. In this period, the percentage credit pass and above in mathematics at the Senior Secondary Certificate Examination (SSCE) level ranges about 40% pass. The West Africa Examination Council

(WAEC) Chief Examiners Report consistently reported lack of skill in answering almost all the questions asked in general mathematics. The most affected areas include, geometry of 3-dimensional problems, algebraic expression (quadratic equation) and words problems in equations, statistics and percentage errors to mention a few. In specific terms the examiners reported students' weaknesses in quadratic equation. Some areas of the syllabus that were also reported to be poorly attempted by candidates were the reading and drawing of quadratic graph among others. The weakness is also evident in the Chief Examiners' reports of WASSCE 2012, 2013, 2015, 2016 and 2018.In view of these general weak performances of students in mathematics, efforts are being made every now and then by mathematics educators at various quarter to bring mathematics teaching and learning meaningful. This is observed in the trends of workshops for teachers of mathematics and different research works in areas of teaching, student achievement in mathematics, attitude of students towards the subject and strategies to improve on the teaching and learning of mathematics. Learners are evaluated thereof on the way they conceive the concepts in mathematics and the quality of teaching. This will help to reveal learner errors and misconceptions (Riccomini, 2005). According to Riccomini, Mathematics teachers do not treat learner errors committed when solving mathematical problems seriously. He also said teachers do not have courage and patience to investigate learners' errors and problems experienced in solving mathematical problems.

Error can be defined here as a way of doing a thing wrongly. Reviewing the research method on errors, specifically, classifying students' errors based on the step of solving problems or the sources of difficulties in solving problems. Students correctly follow wrong algorithms, which is contrary to many teachers views that students wrongly follow an algorithm. This study focuses on where the error originated, which is linked between conceptual and procedural knowledge. One of the main methods used to analyze student's errors is to classify them into certain categorization based on analysis of students' mistakes. The errors are classified in terms of Language difficulties, difficulties in the processing iconic and visual representation of mathematical knowledge, deficiency in the requisite skill, facts, and concepts; for example, student may forget or be unable to recall related information in solving problems, incorrect association rigidity; that is negative transfer caused by decoding and encoding information and application of irrelevant rules or strategies. Titus (2016) also used the classifying method but based his own on the model of problem solving (Polyer). He thought that Students errors may be due to deficiency in one or more of the above steps.

Analysis of error is the ability to establish the existence of errors in a computation. There are many factors that help students to turn up at a correct result while solving mathematical problems. The method is based on the fact that in the process of problem solving there are two major types of hurdles that hinder students from arriving at correct answers. They are; troubles in reading fluency and abstract understanding that helps reading and understanding meaning of problems. The other one is trouble of processing mathematical problems that consist of transformation, process skill, and encoding results. It is important to find where students commit such errors and continue to repeat them.
Studies have been conducted over years to determine the predictor of mathematics achievement among various groups of individuals. Some of the predictions are, socio-economic status of students, teaching methods teachers use, gender and environmental factors of students among others (Owolabi and Adejoke, 2014; Adeyinka and Kaino, 2014). One variable that has over the years, received considerable attention in many studies on science achievement in general and mathematics achievement in particular is gender. According to Owolabi and Adejoke (2014) studies conducted among middle and high school students show a significant gender effect favoring males in overall sciences and mathematic achievement. They also said in another study that boys outperform girls in science but in reading and writing, girls had the advantage. Abubakar and Oguguo (2011) study show no significant difference in gender achievement in relation to Number and

Numeration, Algebraic process and statistics. However, in some studies, both male and female perform at par having applied certain strategies (Adeleke, 2007).

Another variable which is of interest in this study is school location. There are different school locations in Nasarawa state which include urban, semi-urban rural or remote villages. The results of some studies outside the shores of Nigeria have shown that location is a variable to consider on students' achievement, example is Indonesia, Jambi province. Effandi and Siti (2010) attest that students in rural schools perform poorly in mathematics at the point of comprehension, given that the effect of mother tongue or bilingual conflict is prominent. The urban schools more errors are committed at the process skill and transformation stage.

One other factor the researcher considered is the student academic orientation (Science/arts). The orientation of the students may or may not narrow their understanding of specific concepts and will not be able to engage them in problem solving. Students who do not have background knowledge in mathematics usually display numerous errors in solving mathematical problems and this therefore results in most students grappling with quadratic equations (Sello, 2014). If a research could characterize students learning difficulties, it would be possible to design effective instruments to enhance students learning. The research on student's errors is a way to provide such support for both teachers and students.Li (2006), pointed out one way of trying to find out what makes algebra difficult is to identify the kind of errors students commit. There was also need to classify the errors based on the steps of solving problems. He reported that Radatz first classified students' errors in terms of language difficulties, mathematics is like a foreign language for students who needs to know and understand mathematical concepts, symbol, and vocabulary. Misunderstanding the logic for mathematics language could cause students error at the beginning of problem solving, difficulties in processing the representation of mathematical knowledge, like of requisite skills, knowledge, and concepts, poor associations or rigidity; that is, wrong transfer caused by decoding and encoding information and application of irrelevant rules or strategies.

Anne Newman (1980) than used the classifying method but based her model on problem solving.

The model of the sequence of steps in problem solving: reading and comprehension, transformation, process skill, and encoding to identify students' possible errors. She thought that student's error may be due to deficiency in one or several of the above steps. This is what is today referred to as Newman error analysis procedure (Newman Model) which the researcher will adopt to diagnose errors students commit by senior secondary school three when solving quadratic equation by completing square method.

STATEMENT OF THE PROBLEM

In SSCE mathematics examination, students have presented difficulties in solving equations. The Chief Examiner's Report of 2011 to 2017 all emphasized that students are weak in algebraic process; Quadratic equation is one topic that every year WASSCE features to test the concept among students and also to test their procedural understanding of quadratic equations. These mistakes lower students' achievement in SSCE Mathematics achievement it is therefore important to identify the types of errors students commit and where they commit them.The Newman Model; Newman Error Analysis Procedure has been found useful for analyzing students' errors when solving quadratic equations since it takes them through the steps needed to reach the solution. The focus of this study therefore was to analysis the errors committed when senior secondary school students solve quadratic equations by completing square method using Newman error analysis procedure.

OBJECTIVES OF THE STUDY

The objectives of this study were using Newman Error Analysis procedure;

1. compare the means and standard deviations of errors committed by male and female senior secondary school (SS3) students' when solving quadratic equation by completing square method

2. established the means and standard deviations of errors committed by urban and rural senior secondary school (SS3) students committed when solving quadratic equation using method of completing square
3. compare the means and standard deviation of errors committed by Science and Art senior secondary school (SS3) students' when solving quadratic equations using the method of completing square.

RESEARCH QUESTIONS

The following research questions guided the study.

Using Newman error analysis Procedure;

1. what are the means and standard deviations of errors committed by male and female senior secondary school three (SS3) students' when solving quadratic equations by completing square method?
2. what are the mean and standard deviations of errors committed by senior secondary school three (SS3) students' in the urban and rural areas when solving quadratic equation using completing square?
3. what are the mean and standard deviations of errors committed by science and arts senior secondary school three (SS3) students' when solving quadratic equation by completing square mwthod?

STATEMENT OF HYPOTHESES

The following null hypotheses were tested at 0.05 level of significance.

Using Newman error analysis procedures;

Ho_1: There is no significant difference in the mean scores of errors committed by male and female senior secondary school students' when solving quadratic equations by completing square method

Ho_2: There is no significant difference in the mean scores of errors committed by urban and rural senior secondary school students' when solving quadratic equation using completing square method

Ho_3: There is no significant difference in the mean scores of errors committed by Science and Art senior secondary school students' when solving quadratic equations using completing square method

RESEARCH METHODOLOGY

The research designs used for the study was survey research designs.This design involved the collection of data with a short span of time from randomly selected sample of the target population. The cross-sectional survey design also called parallel-sample design was used for the study.

Population

The population of the study comprised all senior secondary school three (SS3) students in public senior secondary schools in Nasarawa State.

Sample and Sampling Procedure

One senatorial district was randomly selected for the survey. Multi-stratified random sampling procedure was used to select the schools for the study in terms of gender, school location and background

Method of Data Collection; Instrumentation

Data were collected using, Quadratic Equation Diagnostic Test.

Quadratic Equation Diagnostic Test (QEDT):

The researcher selected the items of this test from past SSCE questions papers and work examples from text books that are recommended by WAEC in the syllabus. This was attempted by all the students in the sampled schools. Test was administered and scripts were returned and marked. The scores recorded with respect to the Newman error analysis procedure stages and was tested.

Techniques for Data Analysis

The research questions were answered using means and standard deviation of scores for errors committed by SS3 students while the Hypotheses were tested using student independent t-test at $\alpha \leq 0.05$.

Results and Data Analysis

Research Question 2; What are the means and standard deviations of error committed by male and female SS3 students that solve quadratic equations using completing square method?

Table 1: Means and Standard Deviations of Error Committed by Male and Female Students that Solve Quadratic Equation Using Completing Square Method

Variables	Means	Standard Deviation	t-test
Male	1.612	0.209	0.095
Female	1.780	0.227	

Table 1 shows the means and standard deviations of male and female SS3 students that solve quadratic equation using completing square method. The mean scores for male students was 1.612and standard deviation was 0.209. The mean scores for the female students was 1.780 and standard deviation was 0.227

Research Question 3; What are the means and standard deviations of error committed by urban and rural SS3 students that solve quadratic equations using completing square method?

Table2: Means and Standard Deviations of Error Committed by Urban and Rural Students that Solve Quadratic Equation Using Completing Square Method

Variables	Means	Standard Deviation	t-test
Urban	1.600	1.191	0.020
Rural	1.704	0.174	

Table 2 shows the means and standard deviations of urban and rural SS3 students that solve quadratic equation using completing square method. The mean scores for urban students was 1.600and standard deviation was 1191. The mean scores for the rural students was 1.704and standard deviation was 0.174.

Research Question 3; What are the means and standard deviations of error committed by science and art SS3 students that solve quadratic equations using completing square method?

Table3: Means and Standard Deviations of Error Committed by Science and Art Students that Solve Quadratic Equation Using Completing Square Method

Variables	Means	Standard Deviation	t-test

Science	1.640	0.244	0.898
Art	1.648	0.182	

Table3: shows the means and standard deviations of science and art SS3 students that solve quadratic equation using completing square method. The mean scores for urban students was 1.640and standard deviation was 0.244. The mean scores for the rural students was 1.648and standard deviation was 0.482.

Hypothesis1: There is no significant difference in the means score of errors committed by male and female SS3 students that solved quadratic equations by completing square.
Table1, presents the t-test analysis of errors committed by male and female SS3 students that solved quadratic equation by factorization methods. The t-test for difference of two means was 0.095. Therefore, as P = 0.095 is greater than α =0.05 (P=0.095>α =0.05). The difference was not significant at α<0.05 therefore we do not reject thenull hypothesis.

Hypothesis 2 There is no significant difference in the mean scores of errors committed by urban and rural SS3 students when solving quadratic equation by completing square.
Table2 presents the t-test result of errors committed by urban and rural SS3 students that solve quadratic equation by completing square method. The t-test difference of two mean was 0.020. It is observed that P = 0.020 is greater than α = 0.05 (since P = 0.020>α = 0.05). The difference between the means of error that urban and rural SS3 students committed when solving quadratic, equation by completing square is not significantly different at α<0.05 therefore we reject the hypothesis.

Hypothesis 3
There is no significant difference in the mean scores of errors committed by science and arts SS3 students when solving quadratic equation by completing square,Table 3 presents t-test analysis of errors committed by science and arts SS3 students that solve quadratic

equation by completing square method. The t-test for difference of two means was 0.898. It is observed that P = 0.898 is greater than $\alpha = 0.05$ ($P = 0.898 > \alpha$ 0.05). The difference between the means error that science and arts SS3 students committed when solving quadratic equation by completing square is not significantly different $\alpha < 0.05$. we therefore accept the hypothesis so stated.

DISCUSSION OF FINDINGS

The discussion was made on the bases of the research questions and the corresponding hypotheses stated and tested.The types of errors committed by students when solving quadratic equation using completing square by Newman Model reflects all the error types enumerated in the model. The most error type that the SS3 students committed was in the translation stage followed by the process skill, decoding, encoding and the comprehension stages. The difference between the errors type committed are however closely related. The presentation is in agreement with the findings of Effidini&siti (2010), Teoh (2010), and Nande (2013) that shows significant errors appearing at the transformation stage and the process skill the most.The result in the case of male and female table 1 shows that there were errors committed by both male and female students this agrees with previous researches (Bosire, Mondon&Barmoa 2008), who reported that irrespective of the schools, male respondent perform better than female. In regards to table 2, the mean score of errors committed by urban students when solving quadratic equation using completing square method was less than that of the rural students. This means the urban students committed less error than the rural students. The t-test was 0.020 which is greater than $\propto \leq \mathbf{0.05}$, since P = 0.020> 0.05, the hypothesis was rejected for the method of completing square. The findings here supported the finding in (Teoh,2010 and Shio, 2012). Even though the errors were more in different locations that is the rural pupils' errors were found mostly at comprehension and transformation stages while the urban students' errors were more at the process skill. However, in all, the rural students committed more errors than the urban students. Table3 shows that the mean score of errors committed by science students when solving quadratic equation by completing square was less than that of the Arts students. This shows that science students committed less errors when solving quadratic equation by completing square. The t-test was 0.898, this is greater than $\propto \leq 0.05$. Therefore, since P = 0.898, $\geq \propto = 0.05$, the hypothesis was not accepted, indicating that the mean are not significantly different in completing squaremethod. This did not support Trance (2013), that science and engineering students achieve better in mathematics than other discipline

CONCLUSION

The results of thefindings showed that all the SS3 students presented difficulties at all stages of the Newman Model of Decoding, Comprehension, Transformation, Process kill and Encoding. That there is no significant difference in the means of SS3 students that solved quadratic equation by completing square gender, location and background. However, the means of urban and rural SS3 students that solve quadratic equation by completing square method are significantly different.

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REGULATORY FRAMEWORK AND POLITICAL PARTY FINANCING IN NIGERIA; ANALYZING 2011-2019 APC AND PDP PARTICIPATION IN GENERAL ELECTIONS

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ABSTRACT

This paper examine the regulatory framework on political party financing in Nigeria. There is an ongoing debate on the theme that excessive illicit financing of political parties and massively corrupt financial support of individual candidates on electoral competitions constitute serious threat to the process, and negatively influences the development of politics and consolidation of democracy in developing countries. The study is restricted to political party financing and consolidation of democracy in Nigeria within the period of 2011 to 2019 in two major political parties in Nigeria- All Progressive Congress (APC) and Peoples Democratic Party (PDP). The primary sources of financing political parties in any democracy across the globe is through payment of membership dues, but none the less, Nigeria presents an exception because such dues are hardly paid, another source of funding for the political parties are government grants to political parties which is a universal practice. Party financing is the livewire of political parties that further enables political parties to create awareness about party manifestoes as a means for mobilization and recruitment of more members, this is key to winning elections and execution of party programs through government policies.

INTRODUCTION

Democracy is a capital-intensive venture, and political parties cannot thrive effectively in the arena without sufficient funds. Cole (2016) notes that Nigeria practices one of the most expensive political systems in the world, Finance has been the major bane in the development of the country's political system, as there is no effective laws, or culture, or strong institutional regulatory framework about how to finance political parties or the limit of expenses on elections, and sources of such finances. There is an ongoing debate on the theme that excessive illicit financing of political parties and massively corrupt financial support of individual candidates on electoral competitions constitute serious threat to the process, and negatively influences the development of politics and consolidation of democracy in developing countries. Financial involvement in obtaining political nomination forms for elective public offices is astronomically higher and beyond the reach of average political aspirant. This high bid system of financing is a systemic instrument being used to ward-off financially weak-based politicians. Sourcing money for political party business beyond the approved limit has derailed politics and weakened the course of democracy (Stephen, 2008).

This paper, as a primary objective, intends to examine the regulatory framework on political party financing in Nigeria hence the need for a corresponding research question thus: Is there regulatory framework on political party financing in Nigeria? In an attempt to do justice to the foregoing objective and research question, this study is restricted to political party financing and consolidation of democracy in Nigeria within the period of 2011 to 2019 in two major political parties in Nigeria-

All Progressive Congress (APC) and Peoples Democratic Party (PDP). On the same basis, relevant literature will be reviewed to identify the knowledge gap with a view to filling in same.

LITERATURE REVIEW

This segment reviews relevant literature drawing its intellectual strength from the work of others with the aim of critically commenting on them in the light of conventional academic parameters.

Concept of Party Financing

The term "Political Finance" has been defined by Ojo (2000), Obiorah (2004), Pinto-Duschinsky (2001 and 2004), Emelonye (2004) and (Ayoade) as the use of money or the use of other material resources for political activities. It also embodies the sources or means through which political activities are sponsored in a given polity. The concept of political finance has two broad connotations viz money used for electioneering (campaign funds) and money used for political party expenses (party funds). Though there are other forms of political finance but these two will form the basis of our discussion because they constitute the foundation of every political activity. This broad definition of political finance while capturing the essence of the term does not acknowledge the multiplicity of forms and ways in which the monetization of politics may be used to influence political outcomes. The definition offered by Pinto – Duschinky (2001) also fails to capture the centrality of "political". That is, it shies away from explicating the ambits of the term "political". According to Emelonye (2004:34), what the present author advocates here is not a semantic description but rather a clarification as to construction of the term when it comes to foreign contributions. For example, in issued, the definition of "political" is narrowed soon that foreign payments for technical assistance" and training are permitted. But such terms may be guises for more partisan contributions with political undertones or motives such as support for private governments' business forms and convert propaganda). Pinto-Duschinsky (2004) modified his earlier thesis by positing that political finance is "money for electioneering". Since political parties play a critical part in election campaigns in many parts of the globe, and since it is difficult to draw a distinct line between campaign costs of party organizations and their routine expenses, party funds may reasonably be considered "political finance", too. He goes on to argue that party funding includes not only campaign expenses but also the costs of maintaining permanent offices, carrying out policy research, and engaging in political education, voter's registration, and other regular functions of parties.

Therefore, a definition of political finance should include the under listed aspect which Pinto-Duschinsky (2004) subsequently identified in his contribution:

1. That political finance is a feature of non-democratic, as well as democratic regimes
2. The expenditure on elections and parties is only a part of a more far reaching issue. Political funding can be for activities ranging from lobbying, propaganda, support of interest groups to blatant bribery and
3. That the regulation of political finance is hindered by a plurality of avenues of obtaining and using money for political ends.

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The Electoral Acts (2002, 2006 and 2010) contain numerous provisions in relation to political party and election finance. The Electoral Act (2002) defines election expenses in section 84 (1) as follows: "expenses incurred by a political party within the period from the date notice is given by the commission to conduct election up to and including the polling day in respect of the particular election. This definition is flawed totally because experience has shown that in Nigeria most election expenses are incurred by the candidates themselves and not the political parties. This definition is restrictive automatically excludes the election expenses incurred by candidates from whatever limitations on election expenses.

The Electoral Act (2006) has introduced ceilings on contributions by individuals to political parties and on the campaign expenses by political parties and candidates alike. Section 93 stipulates that election expenses by every candidate shall not exceed:

1. N500 million for presidential candidates
2. N 100 million for Governorship
3. N 20 million for Senate
4. N 10 million for House of representatives
5. N 5 million for State Assembly
6. N 5 million for Chairmanship of Local Government council and
7. N 500, 000 for Councillorship.

In addition, no individual shall donate more than N1 million to any candidate. Notation of this provision attracts fines ranging from N100,000 or one (1) month imprisonment or both for councillorship candidates, to N1 million or 12 months imprisonment or both for presidential candidates, while any individual who donates more than N1 million to any candidate is liable to a fine of N500, 000 or 9 months imprisonment or both. The electoral commission is left to fix the maximum donation any person can make to a political party (section 92) as opposed to a candidate who is stipulated in section 93. No party can accept or keep anonymous contributions of more than N100, 000 unless it can identify the source of the money and must keep records of all donations over N1 million.

The under listed puzzles come up for the Independent National Electoral Commission for consideration:

1. Has INEC undertaken an examination and audit of the accounts of the political parties?
2. Did the Commission place any limit on the amount of contribution which individuals or cooperate agencies made to political parties in the course of fund raising for the 2003, 2007 and 2011 elections?
3. Do all political parties have records of all contributions to their campaign funds?
4. Does INEC have a record, which shows the total expenses of all the political parties for the purposes of invoking the provisions of section 84, 92 and 93 sub sections (2), (3) and (6) of the 2004, 2006 and 2010 Electoral Acts?
5. What steps have been taken to sanction corporate bodies that contributed to the campaign funds of political parties in total disregard of the provisions of section 38 (2) of the company and

Allied matters Act (1990), which prohibits donations or gifts of any of its property or funds to a political party or association.

At present, only INEC can attempt the above questions. For the purpose of this paper the term 'political finance" refers to the deployment of financial and material resources by both political parties and politicians as prescribed by law of the polity to cover political expenses. The Draft Campaign Financing Bill, 2011 ("the Draft Bill") of Kenya, which is currently undergoing a stakeholder review process by the Constitutional Implementation Committee of Kenya is a welcome initiative that will foster greater transparency and accountability in the financing of election and referendum campaigns. However, a number of shortcomings in the Draft Bill like those of Nigeria jeopardize these objectives, and a series of amendments are required before the Draft Bill complies with international standards on freedom of expression and information. The Campaign Financing Bill, 2011 sets out major reforms for funding of election campaigns, use of campaign funds in the nomination process, election campaign and elections. It will provide for the management, spending and accountability of funds during election and referendum campaign. It is important that the draft bill is clear on the concept of campaign financing to prevent any political and administrative frustration and even litigation in Court. CMD-Kenya believes that campaign financing refers to the manner in which political parties and individual candidates who seek to get elected to political office gather, utilize, and recover funds for electoral campaigns and in the case of political parties seek to maintain themselves as organizations. In this context the scope of the legislation should cover all aspects of campaign financing. We believe that the conceptual framework needs to be reflected in the interpretation to give the legislation effective statutory interpretation (CMD-Kenya, 2011).

In the analysis, ARTICLE 19 (2012) emphasizes that transparency in campaign financing is indispensable for embedding accountability and integral to the promotion of good governance and democracy. Only with full access to information can the media scrutinize the conduct of election candidates and inform public debate on the dynamics and distribution of political and economic power in Kenya. The engagement that transparency fosters between candidates for public office and the electorate also maximizes enjoyment of the right to political participation. The analysis finds that positive measures in the Draft Bill include the establishment of limits on political campaign expenditures, caps on the amount individuals can donate to candidates, and the imposition of a ban on anonymous donations. The establishment of a framework for the collection and reporting of data to a new Oversight Committee is a significant step towards furthering a culture of accountability in the financing of political campaigns.
However, ARTICLE 19 (2012) also finds that various elements of the Draft Bill fall short of international standards on freedom of expression and access to information. The Draft Bill designates as confidential all campaign financing information submitted to the oversight Committee, with only limited disclosure exceptions for information that is the subject of a complaint or investigation. This runs counter to the principles of proactive and maximum disclosure that are central to the right of access to information. The selection criteria for the Oversight Committee are also left ambiguous, and there are inadequate safeguards to ensure the accountability of this committee to the public. In

conclusion, ARTICLE 19 (2012) urges the Kenyan legislature to revise the Draft Bill and adopt it only after it is brought into compliance with international standards on freedom of expression and information. The need for greater transparency in all aspects of public life in Kenya further demonstrates the urgent need for a comprehensive access to information framework to be implemented in the country.

Political Party Financing and Consolidation of Democracy in Nigeria

Financing political party for its functions and sponsoring election campaigns is vital in a vibrant democracy. Ballington et.al (2014) note that regular elections organized between competing political parties is the dominant method of selecting democratic governments. For political parties and their nominees to reach out to the teaming electorates to sell the parties manifestoes, it becomes imperative to have access to enough money so as to off-set election expenses. Election expenses, according to Electoral Act (2010) means "expenses incurred by a political party within the period from the date notice is given by the Commission to conduct an election up to and including the polling day in respect of the particular election". Magolowondo et al (2012) note that parties may win or lose elections well before they are held simply on account of their resource endowment or lack thereof. In other instances, how parties' practice or fail to practice intra-party democracy has to some extent been influenced by the way they are financed and how these resources are allocated within the different parties. The concern for possible negative impact of money on politics and governance warranted the incorporation of the regulatory clause in the 1999 Constitution and the 2010 Electoral Act of the Federal Republic of Nigeria to curb the excesses and unregulated donations to political parties and individual candidates which breeds corruption. Money exercises undue influence on politics, and undermines the integrity of elections, credibility and legitimacy of government. In the first republic (1960-1966), there was indefinite electoral law on campaign finance.

Funding election activities and other political parties" functions were the responsibility of the individual parties and their candidates. From knowledge of hindsight, there were political parties who were accused to have used the state funds and investments to sponsor party activities and campaigns, such as the National Council of Nigeria Citizens (NCNC) and Action Group (AG). The 1979 Constitution of the second republic (1979-1983) provided regulatory law on campaign finance which prohibited associations, other than political parties, from campaigning on behalf of a candidate or contributing funds to parties and election expenses of candidates. There was the budgetary provision for annual grants to political parties; and political parties were empowered to receive donations from individuals and corporate bodies, but it prohibited donations from external bodies. The unquantifiable party-donations from individuals and corporate bodies, and unprecedented expenses incurred on party activities and campaigns due to the fact that the limit of funds political parties and aspirants could raise from individuals and corporate bodies was not specified by law. The prevalent corrupt practices, electoral irregularities, and uncordial interparty relations in the second republic amongst the National Party of Nigeria, Nigerian Peoples Party, Unity party of Nigeria, Great Nigerian Peoples Party, People Redemption Party, etc. breached the provisions of subsisting Constitution and exacerbated party expenses. Political party antagonistic clashes, south-western zone post-election crisis, and increasing tension in the heated polity abruptly brought the republic to tragic end through military coup d'état.

Government in the fourth republic is not oblivious of the fact that politicians would abuse party financing if their activities are unregulated and unsupervised by electoral umpire. On the strength of the perceived dangerous threat to democracy, Section 221 of the 1999 Constitution of the Federal Republic of Nigeria provides that "no association, other than a political party, shall canvass for votes for any candidate at any election or contribute to the funds of any political party or to the election expenses of any candidate at any election". Section 225(2) stipulates that "every political party shall submit to the Independent National Electoral Commission (INEC) a detailed annual statement and analysis of its sources of funds and other assets together with a similar statement of its expenditure in such form as the Commission may require". 225(3) states that "no political party shall hold or possess any funds or other assets outside Nigeria; or be entitled to retain any funds or assets remitted or sent to it from outside Nigeria". Similarly, Section 225(4) provides that "any funds or other assets remitted or sent to a political party from outside Nigeria shall be paid over or transferred to the Commission within twenty-one days of its receipt with such information as the Commission may require"; while 225(5) stipulates that "the Commission shall have power to give directions to political parties regarding the books or records of financial transactions which they shall keep and, to examine all such books and records" (FRN, 1999).

METHODOLOGY

The study utilizes both primary and secondary methods of data collection. The secondary were gathered through library research and the internet. The materials consisted of books, journals, articles, reports, periodicals, monographs, newspapers and magazines. Information gotten shall be used to evaluate how political party financing affects consolidation of democracy and what needs to be done to regulate political party financing. The primary method includes questionnaire which shall be administered purposively to respondents.

Analysis and presentation of data followed the path of statistical package for social science (SPSS) method of data analysis. This summarizes data, creates appropriate tables and examines relationships among variables. Thus, data for this investigation will be analyzed using simple percentile, descriptive statistical technique, and the results are further described using tables.

The simple percentage formula to be used is:

$$\frac{NR}{TNR} \times 100$$

Where NR= the number of responses to each questionnaire,

TNR= is the total number of responses. This tool of analysis enables the researcher assess weights of opinion to a questionnaire and the percentage allocated to that weight of opinion. Ethically, in order to conform to the standards of conduct involved in the research, a permission to carry out the study was granted in the department, starting with the approval of the topic. Written consent was obtained from the respondents and were assured of confidentiality. (See Appendix 1)

The Sources and Legal Frameworks for Funding of Political Parties in Nigeria.

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The sources of funding political parties in Nigeria between the Second, Third and the Fourth Republics were:

1. Statutory allocation
2. Fees and subscription and
3. Lawful donations and public collection respectively.

A number of constitutional provisions and legislative enactments relate to political finance. The Constitution of Nigeria provides the basic framework for the implementation and enactment of other laws in the polity. The supremacy of the constitution is further emphasized in section 1 (3), which provides "if any other law is inconsistent with the provisions of this constitution, this constitution shall prevail, and that other law shall to the extent of the inconsistency be void" (The Constitution, 1999:1).

In other words, every other law in the country must be in line with the provisions of the constitution. It also follows that any inadequacy in the constitution will automatically taint the provisions of subsequent laws in the same subject matter. The 1999 constitution in section 221 prohibits any association other than political parties from making political donations. The constitution in section 225 provides as follows:

1. Every political party shall, at such times and in such manner as the Independent National Electoral Commission may require, submit to the Independent National Electoral Commission a statement of its assets and liabilities.
2. Every political party shall submit to the Independent National Electoral Commission a detailed annual statement and analysis of its sources of funds and other assets together with similar statements of its expenditure in such form as the Commission may require.
3. No political party shall – (a) hold or possess any funds or other assets outside Nigeria; or (b) be entitled to retain any funds or other assets outside Nigeria
4. Any funds or other assets remitted or sent to a political party from outside Nigeria shall be paid over or transferred to the Commission within twenty-one days of its receipt with such information as the Commission may require.
5. The Commission shall have power to give directions to political parties regarding the books or records of financial transactions which they shall keep and, to examine the all such books and records.

The Commission was also empowered in subsection 6 of the above section to audit the account of political parties through its staff or professional auditors. The commission is further empowered by section 226 of the constitution to prepare and submit a report on the financial account of the political parties to the National Assembly and are authorized to have unlimited access to the records of the political parties. The National Assembly is empowered in section 228 of the 1999 constitution to make laws for the punishment of any individual or party who fails to observe the above provisions and the disbursement of annual grants to political parties.

The Electoral Act 2002

The provision of this law covers virtually every process of electoral activities in the country. Section 76 provides for the oversight function of the Electoral Commission over the activities of the political parties and also provides for a fine of N500, 000 for non-conformity by any individual to lawful directions by the Commission in carrying out its supervisory functions. Section 77 makes provision for a fine of N500, 000 for the contravention of section 225 (3) (a) and (b) of the 1999 Constitution relating to ownership of foreign asset by any political party and any donation from outside the country. Section 78 provides for period of time, which the annual account of a political party should cover. It also empowered the Commission to audit the account of political parties periodically. Section 79 makes provision for a separate finance statement for election expenses as prescribed in section 100 of the act not latter than 90 days after the election. Surprisingly section 100 of the Electoral Act has no provision whatsoever that relates to party finances it rather talks about qualification of a person who can contest elections. Any political party that fails to submit the audited return of election expenses is guilty of an offense punishable on conviction with a fine of N100, 000. Section 80 makes provision for the disbursement of grants to political parties that are contesting elections. It provides that 30% of the grant shall be distributed equally among the political parties before the election and the remaining 70% shall be shared among the political parties after the result of the election has been known, in proportion to the number of seats won by each party in the National Assembly. Section 81 provides that the National Assembly may make an annual grant to political parties and 30% of such grants should be shared among the political parties in proportion to number of seats won by each party in the National Assembly. Section 82 provides as follows: No political party shall be eligible to receive a grant under section 93 unless it wins a minimum of 10 percent of the total votes cast in the local government election in at least two-thirds of the states of the federation. Section 93 which is referred to in the above provision has no such provisions. Section 83 empowers the Commission to place the limitation on the amount of money or other assets, which an individual or corporate body can contribute to a political party. Also, it stipulates for a record of all contributions.

Electoral Act 2006

Under the 2006 Electoral Act which was used in the conduct of the 2007 elections while the recommendations of the Uwais Panel were being debated, the National Assembly was empowered to approve a grant to be disbursed to political parties. The 2006 law also stipulates how the grant should be divided, 10 percent going to be shared equally among the registered political parties and the remaining 90 percent disbursed in proportion to the number of National Assembly seats won by each party. The law also gives INEC the power to place a limit on the amount of money or other assets an individual or group can contribute to a political party. For a presidential candidate the sum is N500 million, governor N100 million, senator N20 million and a representative N10 million. A state assembly candidate, or chairman N5 million and a local councillorship, N500,000. It is an open question whether this aspect of the electoral law has ever been paid attention to not to talk of being enforced. Some of the then 50 parties have not in any way justified the money they receive from government. It has been discovered that some of the parties only exist on the pages of newspapers and magazines. They only function when elections are coming or when funding is released by government. They collect the funds, share and go home to rest till another round of funding is available. A few of the parties are even run by close-knit family members. So what does a party exist for if it is only to share government funds?

As the nation moved towards 2011 elections, it became imperative to revisit the issue of political financing in Nigeria. The Uwais Panel report recommended the continued funding of parties by government through INEC, but suggests a ceiling for individual donations for each category of office. These figures run from a limit of N20 million for individual donations for a presidential candidate to N15 million for a governor, N10 million for a senator, N3 million for a local government chairmanship candidate. It makes eminent sense for party members to fund their own organization. The Uwais panel recommends that only parties that score 2.5 percent of the votes in the 2011 elections should be eligible to receive funds from public grants, but this like many other issues where expunged in the 2010 Electoral Act.

Electoral Act 2010

The 2011 General Elections are over with local and international acclamation to the electoral commission. The elections were not flawless; however, Nigerians and foreign witnesses are unanimous that the just concluded polls were held in substantial compliance with the nation's electoral laws. It is too early to pre-empt the political parties on the veracity of the election expenses they will submit to INEC in the next 6 months. But then, is six months not too long? I should think three months after the polls is okay, more so as candidates, who spend the bulk of the campaign money, are not yet under obligation to submit election expenses report.

This post-election period, two major things must happen. The first is for the Independent National Electoral Commission (INEC) to rise up to its constitutional duty to enforce political finance provisions as contained in the statutes viz. the 1999 Constitution (as amended), the Electoral Act 2010 (as amended) as well as the Political Party Finance Manual and Handbook. The second matter of urgent national importance is the amendment of these laws to make them more enforceable. The current legal framework requested three reports from the political parties. The first, according to section 89 of the Electoral Act 2010, is the annual statement of assets and liabilities, analysis of their sources or funds and other assets as well as their statements of expenditure. INEC is mandated to publish the report in three national newspapers. The other report which is of greater interest to campaign finance experts is stated in section 92 of the current electoral act. Sub-section 3 of the clause says "Election expenses of a political party shall be submitted to the Commission in a separate audited return within 6 months after an election and such return shall be signed by the party's auditors and counter signed by the chairman of the party and be supported by a sworn affidavit by the signatories as to the correctness of its contents". Sub-section 5 states that the return shall show the amount of money expended by or on behalf of the party on election expenses, the items of expenditure and the commercial value of goods and services received for election purpose. Sub section 6 mandated the political parties to publish this report in at least two national newspapers. The third report is requested of political parties in section 93 (4) and it states that "A political party sponsoring the election of a candidate shall within 3 months after the announcement of the results of the election, file a report of the contributions made by individuals and entities to the Commission". Hitherto, these provisions have been violated with impunity. If the truth will be told, the last general election in Nigeria was the most expensive in the annals of our electoral democracy. Given the resources deployed by some of the wealthy candidates during the elections, there is no gainsaying the fact that the contestants showed scant regards for the provision of section 91 subsections 2 – 5 of the Electoral Act 2010 which placed a cap on the amount of money they are to spend on their campaigns. Predominant among the issues at stake is that of godfatherism which poses a great threat to democratic consolidation in Nigeria.

Limitations on Nigeria's Election Expenses

The fourth republic has witnessed enactment of Electoral Acts and subsequent amendments by the National Assembly to guide conduct of elections since the return of democracy in 1999. The Electoral Act of 2002 guided 2003 elections; 2006 Act (as amended) was used for 2007 elections; and 2010 Act (as amended) guided 2011, 2015 and 2019 general elections. The 2006 Act categorically provided funds limitations on campaign expenses to curtail the unhealthy influence of money on party activities and electioneering campaigns. Hereunder illustrates the stipulations. Section 90(1) of the 2010 Electoral Act provides that "the Commission shall have power to place limitation on the amount of money or other assets, which individual or group of persons can contribute to a political party". In the same manner, Section 91(2)-(7) states that: the maximum election expenses to be incurred by a candidate at a Presidential election shall be one billion naira (N1,000,000,000.00); the maximum election expenses to be incurred by a candidate at a Governorship election shall be two hundred million naira (N200,000,000.00); the maximum amount of election expenses to be incurred in respect of Senatorial seat by a candidate at an election to the National Assembly shall be forty million naira (N40,000,000.00), while the seat for House of Representatives shall be ten million naira (N10,000,000.00); in the case of State Assembly election, the maximum amount of election expenses to be incurred shall be ten million naira (N10,000,000.00); in the case of Chairmanship election to an Area Council, the maximum amount of election expenses to be incurred shall be ten million naira (N10,000,000.00); in the case of Councillorship election to an Area Council, the maximum amount of election expenses to be incurred shall be one million naira (N1,000,000.00) (Electoral Act, 2010). Subsection 91(9) provides that "no individual or other entity shall donate more than one million (N1,000,000.00)" to either a political party or to a candidate. Subsection 91(10) states that a candidate who knowingly acts in contravention of this section commits an offence and on conviction shall be liable, in case of presidential election, to a maximum fine of N1,000,000.00 or imprisonment of 12 months or both; in the case of a governorship election, to a fine of N800,000.00 or imprisonment for 9 months or both; in the case of senatorial seat election in the National Assembly to a fine of N600,000.00 or imprisonment for 6 months or both; in the case of House of Representatives election in the National Assembly to a fine of N500,000.00 or imprisonment for 5 months or both; in the case of a State House of Assembly election to a fine of N300,000.00 or 3 months imprisonment or both; in the case of Chairmanship election to a fine of N300,000.00 or 3 months imprisonment or both; in the case of Councillorship election to a fine of N100,000.00 or 1 month imprisonment or both (Electoral Act, 2010). The 2010 Act also made a provision in Section 93(1) that "no political party shall accept or keep in its possession any anonymous monetary or other contributions, gifts, properties, etc from any source whatsoever. For the purpose of this research, political party financing represents the means through which the political party receive funding to execute their day to day activities before, during and after elections. In our view, financing, a major component in the development of political parties and by extension democracy itself this is largely due to the fact that democracy can only flourish where there are strong and vibrant political parties and therefore vibrant and strong political parties can only be possible when they are well funded to carry out their primary mandate of education, mobilization, information dissemination, and recruitment.

In the same vein it is pertinent to note that every political association must fulfill certain conditions before it can be registered and recognized as a political party, one of which is the source of funding. The primary sources of financing political parties in any democracy across the globe is through payment of membership dues, but none the less, Nigeria presents an exception because such dues are hardly paid, another source of funding for the political parties are government grants to political parties which is a universal practice. Party financing is the livewire of political parties that further enables political parties to create awareness about party manifestoes as a means for mobilization and recruitment of more members, this is key to winning elections and execution of party programs through government policies.

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OTHEORETICAL EVALUATION OF STEPSAPPROACHING ZERO EMISSION ON A DOUBLE THICK BARRIER OF A GAMMA PARTICLE

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ABSTRACT

The goal of this work is to obtain tunneling probability of a gamma particle. The application of SchrÖdinger's equation in barrier penetration has been applied to gamma particle decay for light, medium and heavy nuclei. Gamma particle tunneling probability has been calculated analytically. Decay probability computed for each gamma particle emitting nucleus shows interesting variations. Log plot of calculated Decay constant plotted against atomic number (Z), mass number (A) and Energy for gamma particle emitting nucleus shows the variations interesting. Half-life which is a function of decay probability plotted against gamma particle energy or against atomic number of gamma particle emitting nucleus shows the variations of decay probabilities. Log plot of Calculated Half-life plotted against atomic number (Z), mass number (A) and Energy for gamma particle emitting nucleus shows interesting variations of decay probabilities. Calculated half-lives compared with experimental half-lives for each gamma particle emitting nucleus shows results which are in good agreement.

Key word:SchrÖdinger's equation, Emission, Half-life,Gamma and Decay constant.

INTRODUCTION

Gamma decay is a type of radioactive decay in which gamma rays are emitted.

Gamma decay occurs when a nuclide is produced in an excited state, gamma emission occurring by transition to a lower energy state. It can occur in association with alpha decay and beta decay (Raju et al., 2006). A gamma ray or gamma radiation (symbol γ), is a penetrating electromagnetic radiation arising from the radioactive decay of atomic nuclei. It consists of the shortest wavelength electromagnetic waves and so imparts the highest photon energy. Paul Villard, a French chemist and physicist, discovered gamma radiation in 1900 while studying radiation emitted by radium (Villard, 1900a). In 1903, Ernest Rutherford named this radiation gamma rays based on their relatively strong penetration of matter; he had previously discovered two less penetrating types of decay radiation, which he named alpha rays and beta rays in ascending order of penetrating power (Rutherford, 1903). Gamma rays from radioactive decay are in the energy range from a few kilo electron volts (keV) to approximately 8 Mega electron volts (~8 MeV), corresponding to the typical energy levels in nuclei with reasonably long lifetimes. The energy spectrum of gamma rays can be used to identify the decaying radionuclides using gamma spectroscopy. Very-high-energy gamma rays in the 100–1000 tera electron volt (TeV) range have been observed from sources such as the Cygnus X-3 micro quasar. Natural sources of gamma rays originating on Earth are mostly as a result of radioactive decay and secondary radiation from atmospheric interactions with cosmic ray particles (Villard, 1900b. However, there are other rare natural sources, such as terrestrial gamma-ray flashes, which produce gamma rays from electron action upon the nucleus. Notable artificial sources of gamma rays include fission, such as that which occurs in nuclear reactors, and high energy physics experiments, such as neutral pion decay and nuclear fusion. Gamma rays and X-rays are both electromagnetic radiation, and since they overlap in the electromagnetic spectrum, the terminology varies between scientific disciplines. In some fields of physics, they are distinguished by their origin: Gamma rays are created by nuclear decay, while in the case of X-rays; the origin is outside the nucleus. In astrophysics, gamma rays are conventionally defined as having photon energies above 100 keV and are the subject of gamma ray astronomy, while radiation below 100 keV is classified as X-rays and is the subject of X- ray astronomy. This convention stems from the early man-made X-rays, which had energies only up to 100 keV, whereas many gamma rays could go to higher energies. A large fraction of astronomical gamma rays are screened by Earth's atmosphere.

MATERIALS AND METHOD

Materials

The materials usedare the$Schrödinger's$ equation.

Method

We now consider the beam of a particle incident upon a square potential barrier of height V_O presumed positive for now and width a. As mentioned above, this geometry is particularly important as it includes the simplest example of scattering phenomenon in which a beam of particles is 'deflected' by a local potential. Moreover, this one-dimensional geometry also provides a plat form to explore a phenomenon peculiar to quantum mechanics quantum tunneling (Dyson, 1951).

The potential energy variation in the case of a rectangular potential barrier shown in figure 1 is given by

$$V(x) = \begin{matrix} 0, & x < 0 \\ V_o, & 0 < x < L \end{matrix} \Big\}$$

$$V(x) = \begin{matrix} 0, & x < 0 \\ V_o, & 0 < x < L \end{matrix} \Big\}$$

(1)

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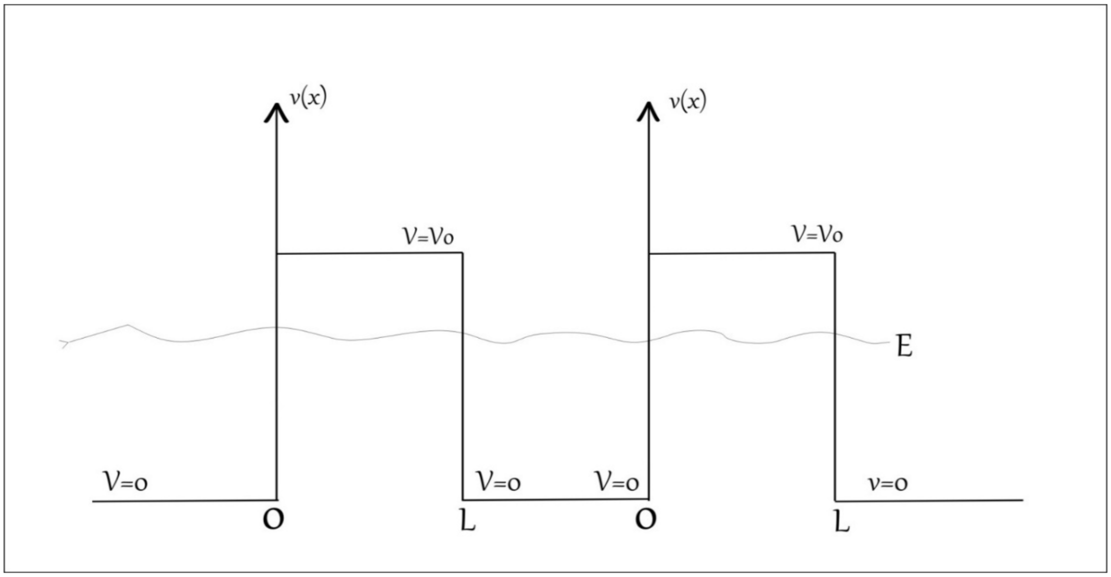

Fig.1: a rectangular double thick potential barrier of width L and height V_0.

Let us consider two cases

(i) $0 < E < V_0$ Classically a particle of energy E if incident from the left would be reflected at the double thick barriers as it cannot enter $(0 < x < L)$ in which its K.E is negative. To describe the behavior of particle quantum mechanically, we will have to solve the Schrödinger equation,

$$\left(\frac{d^2\varphi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\varphi(x)\right)\left(\frac{d^2\varphi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\varphi(x)\right) = 0$$

Or

$$\left(\frac{d^2\varphi(x)}{dx^2} + k^2\varphi(x)\right)\left(\frac{d^2\varphi(x)}{dx^2} + k^2\varphi(x)\right) = 0, k^2 = \frac{2mE}{\hbar^2}, x < 0 \text{ and } x > L \qquad (2)$$

And

$$\left(\frac{d^2\varphi(x)}{dx^2} + \gamma^2\varphi(x)\right)\left(\frac{d^2\varphi(x)}{dx^2} + \gamma^2\varphi(x)\right) = 0, \gamma^2 = \frac{2m(V_0 - E)}{\hbar^2}, 0 < x < L \qquad (3)$$

The general solutions of these equations are given by

$$\varphi^2(x) = \left(A e^{ikx} + Be^{-ikx}\right)\left(A e^{ikx} + Be^{-ikx}\right), x < 0 \qquad (4)$$

$$\varphi^2(x) = \left(Ce^{\alpha x} + De^{-\alpha x}\right)\left(Ce^{\alpha x} + De^{-\alpha x}\right), 0 < x < L \qquad (5)$$

$$\varphi^2(x) = \left(F e^{ikx} + Ge^{-ikx}\right)\left(F e^{ikx} + Ge^{-ikx}\right), x < L \qquad (6)$$

Notice that we allow for waves traveling in both the directions for $x < 0$ representing the incident and reflected waves. We must also allow for $e^{\gamma x}$ and $e^{-\gamma x}$term in the region $0 < x < L$ because x is finite and there is no danger of φ becoming infinite. We have only a wave traveling from left to right of $x > L$ as there cannot be any wave travelling from right to left (reflected wave) since there is no discontinuity in the potential. Hence we must set G=0. The solution, therefore would be

$$\varphi^2(x) = \left(F\, e^{ikx}\right)\left(F\, e^{ikx}\right), x > L \tag{7}$$

The continuity conditions (that is, φ and $d\varphi/dx$ be continuous) at $\mathrm{x} = 0$ and at $\mathrm{x} = L$ yield

$$\text{At } \mathrm{x} = 0, A + B = C + D \text{ and } ik(A - B) = \alpha(C + D) \tag{8}$$

At $x > L$,

$$(Ce^{\gamma L} + De^{-\gamma L})(Ce^{\gamma L} + De^{-\gamma L}) = \left(F\, e^{ikL}\right)^2 \text{ and } \gamma(Ce^{\gamma L} + De^{-\gamma L})\gamma(Ce^{\gamma L} + De^{-\gamma L}) = \left(ikF\, e^{ikL}\right)^2 \tag{9}$$

There are number of ways of solving these equations. If solution leads to

$$\left.\begin{aligned} C^2 &= \left(\frac{[(\gamma+ik)A+(\gamma-ik)B]}{2\gamma}\right)\left(\frac{[(\gamma+ik)A+(\gamma-ik)B]}{2\gamma}\right) \\ D^2 &= \left(\frac{[(\gamma-ik)A+(\gamma+ik)B]}{2\gamma}\right)\left(\frac{[(\gamma-ik)A+(\gamma+ik)B]}{2\gamma}\right) \end{aligned}\right\} x = 0$$

(10)

Similarly

$$\left.\begin{aligned} C^2 &= \left(\frac{[(\gamma+ik)Ae^{-(\gamma-ik)L}F]}{2\gamma}\right)\left(\frac{[(\gamma+ik)Ae^{-(\gamma-ik)L}F]}{2\gamma}\right) \\ D^2 &= \left(\frac{[(\gamma-ik)Ae^{(\gamma+ik)L}F]}{2\gamma}\right)\left(\frac{[(\gamma-ik)Ae^{(\gamma+ik)L}F]}{2\gamma}\right) \end{aligned}\right\} x = L$$

(11)

Equating the values of C^2 and D^2 to each other yield

$$\left((\gamma + ik)A + (\gamma - ik)B\right)^2 = \left((\gamma + ik)Ae^{-(\gamma-ik)L}F\right)\left((\gamma + ik)Ae^{-(\gamma-ik)L}F\right)$$

(12)

And

$$\left((\gamma - ik)A + (\gamma + ik)B\right)^2 = \left((\gamma - ik)Ae^{(\gamma+ik)L}F\right)\left((\gamma - ik)Ae^{(\gamma+ik)L}F\right)$$

(13)

And so

$$(B/A)^2 = \left(\frac{(\gamma-ik)}{(\gamma+ik)}\left[e^{(\gamma+ik)L}\, F/A - 1\right]\right)\left(\frac{(\gamma-ik)}{(\gamma+ik)}\left[e^{(\gamma+ik)L}\, F/A - 1\right]\right) \tag{14}$$

Putting the above value of $(B/A)^2$ in to (3.14) yields

$$\left(\frac{(\gamma + ik)+(\gamma - ik)^2}{(\gamma + ik)}\left[e^{(\gamma+ik)L}\frac{F}{A} - 1\right]\right)^2 = \left((\gamma + ik)e^{(\gamma+ik)}\frac{F}{A}\right)\left((\gamma + ik)e^{(\gamma+ik)}\frac{F}{A}\right)$$

Or

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$$\left(\frac{(\gamma+ik)^2+(\gamma-ik)^2}{(\gamma+ik)}\left[e^{(\gamma+ik)L}\frac{F}{A}-1\right]\right)^2 = \left((\gamma+ik)^2e^{(\gamma+ik)}\frac{F}{A}\right)\left((\gamma+ik)^2e^{(\gamma+ik)}\frac{F}{A}\right)$$

Or

$$((\gamma+ik)^2-(\gamma-ik)^2)^2 = \left(\frac{F}{A}\left[(\gamma+ik)^2e^{(\gamma+ik)L} - (\gamma-ik)^2e^{(\gamma+ik)L}\right]\right)\left(\frac{F}{A}\left[(\gamma+ik)^2e^{(\gamma+ik)L} - (\gamma-ik)^2e^{(\gamma+ik)L}\right]\right)$$

$$\left(\frac{F}{A}\right)^2 = \left(\frac{4ik\gamma}{[(\gamma+ik)^2e^{(\gamma+ik)L}-(\gamma-ik)^2e^{(\gamma+ik)L}]}\right)\left(\frac{4ik\gamma}{[(\gamma+ik)^2e^{(\gamma+ik)L}-(\gamma-ik)^2e^{(\gamma+ik)L}]}\right)$$

After multiplying the numerator and denominator $e^{(\gamma-ik)L}$

$$\left(\frac{F}{A}\right)^2 = \left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma+ik)^2-(\gamma-ik)^2e^{2\gamma L}]}\right)\left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma+ik)^2-(\gamma-ik)^2e^{2\gamma L}]}\right)$$

$$= \left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma+ik)^2-(\gamma-ik)^2e^{2\gamma L}]}\right)\left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma+ik)^2-(\gamma-ik)^2e^{2\gamma L}]}\right)$$

$$= \left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\,(1+e^{2\gamma L})]}\right)\left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\,(1+e^{2\gamma L})]}\right) \qquad (15)$$

Putting the value of $\left(\frac{F}{A}\right)^2$ from above into equation (5), we get

$$\left(\frac{F}{A}\right)^2 = \left(\frac{(\gamma^2-k^2)(e^{2\gamma L}-1)}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\,(1+e^{2\gamma L})]}\right)\left(\frac{(\gamma^2-k^2)(e^{2\gamma L}-1)}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\,(1+e^{2\gamma L})]}\right) \qquad (16)$$

It may be mentioned here that in case one is interest in finding $C/A and D/A$, this can be achieved by substituting the value of $\left(\frac{F}{A}\right)^2$ from (15) into equations (11).

From (7), the reflection coefficient (or the probability of reflection) is given by

$$R = \frac{j_{ref}}{j_{inc}} = \left(\frac{\hbar k/m\,|B|^2}{\hbar k/m\,|A|^2}\right)^2 = (|B/A|^2)^2 = \left[\left(\frac{B}{A}\right)*\left(\frac{B}{A}\right)\right]^2$$

$$= \left(\frac{(\gamma^2-k^2)^2(e^{2\gamma L}-1)^2}{[(\gamma^2-k^2)(1-e^{2\gamma L})^2+4k^2\gamma^2(1+e^{2\gamma L})^2]}\right)\left(\frac{(\gamma^2-k^2)^2(e^{2\gamma L}-1)^2}{[(\gamma^2-k^2)(1-e^{2\gamma L})^2+4k^2\gamma^2(1+e^{2\gamma L})^2]}\right)$$

After dividing the numerator and denominator by $(1-e^{2\gamma L})^2$ one gets

$$R^2 = \left(\frac{(\gamma^2 - k^2)^2}{\left[(\gamma^2 - k^2)^2 + 4k^2\gamma^2 \left\{\frac{(1+e^{2\gamma L})}{(1-e^{2\gamma L})}\right\}^2\right]}\right)\left(\frac{(\gamma^2 - k^2)^2}{\left[(\gamma^2 - k^2)^2 + 4k^2\gamma^2 \left\{\frac{(1+e^{2\gamma L})}{(1-e^{2\gamma L})}\right\}^2\right]}\right)$$

Or

$$= \frac{(\gamma^2 - k^2)^2}{(\gamma^2 - k^2) + 4k^2\gamma^2\left(\frac{1+e^{4\gamma L}+2e^{2\gamma L}}{1+e^{4\gamma L}-2e^{2\gamma L}} - 1\right) + 4k^2\gamma^2}$$

$$= \left(\frac{(\gamma^2 - k^2)^2}{(\gamma^2 - k^2) + 4k^2\gamma^2\left\{\frac{4}{(e^{2\gamma L}+e^{-2\gamma L}-2)}\right\}}\right)\left(\frac{(\gamma^2 - k^2)^2}{(\gamma^2 - k^2) + 4k^2\gamma^2\left\{\frac{4}{(e^{2\gamma L}+e^{-2\gamma L}-2)}\right\}}\right)$$

$$R^2 = \frac{(\gamma^2 - k^2)^2}{(\gamma^2 - k^2) + \frac{4k^2\gamma^2\ 1}{\left(\frac{e^{\gamma L}-e^{-\gamma L}}{2}\right)^2}}$$

$$= \frac{(\gamma^2 - k^2)^2}{\left[(\gamma^2 - k^2) + \frac{4k^2\gamma^2\gamma^2}{\sin \hbar\gamma L}\right]} \quad (17)$$

After substituting the values of γ^2andk^2, one gets

$$R^2 = \left(\frac{V_O^2}{\left[V_O^2 + \frac{4E(V_O - E)}{\sin \hbar\alpha L}\right]}\right)\left(\frac{V_O^2}{\left[V_O^2 + \frac{4E(V_O - E)}{\sin \hbar\alpha L}\right]}\right)$$

$$= \left[1 + \frac{4E(V_O - E)}{V_O^2 \sin \hbar\alpha L}\right]^{-1} \times \left[1 + \frac{4E(V_O - E)}{V_O^2 \sin \hbar\alpha L}\right]^{-1} \quad (18)$$

The probability of finding the particle in a region$X > 0$, is given the name transmission coefficient T and using equation (15) we have

$$T^2 = \frac{j_{ref}}{j_{inc}} = \left(\frac{\hbar k/m\,|F|^2}{\hbar k/m\,|A|^2}\right)^2 = \left(\left|\frac{F}{A}\right|^2\right)^2 = \left[\left(\frac{F}{A}\right) * \left(\frac{F}{A}\right)\right]^2$$

$$= \left(\frac{16k^2\gamma^2 e^{2\gamma L}}{[(\gamma^2 - k^2)(1 - e^{2\gamma L})^2 + 4k^2\gamma^2(1 + e^{2\gamma L})^2]}\right)\left(\frac{16k^2\gamma^2 e^{2\gamma L}}{[(\gamma^2 - k^2)(1 - e^{2\gamma L})^2 + 4k^2\gamma^2(1 + e^{2\gamma L})^2]}\right)$$

$$= \left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 4k^2\gamma^2(e^{2\gamma L} + e^{2\gamma L} + 2)}\right)\left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 4k^2\gamma^2(e^{2\gamma L} + e^{2\gamma L} + 2)}\right)$$

Adding and subtracting $4k^2\gamma^2(e^{2\gamma L} + e^{-2\gamma L} - 2)$ from the denominator, one get

$$= \left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 16k^2\gamma^2}\right)\left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 16k^2\gamma^2}\right)$$

$$= \left(\frac{4k^2\gamma^2}{\left[(\gamma^2 - k^2)^2\left(\frac{e^{\gamma L}-e^{\gamma L}}{2}\right)^2 + 4k^2\gamma^2\right]}\right)\left(\frac{4k^2\gamma^2}{\left[(\gamma^2 - k^2)^2\left(\frac{e^{\gamma L}-e^{\gamma L}}{2}\right)^2 + 4k^2\gamma^2\right]}\right)$$

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$$= \left(\frac{4k^2\gamma^2}{(\gamma^2-k^2)^2 \sin\hbar^2\gamma L + 4k^2\gamma^2}\right)\left(\frac{4k^2\gamma^2}{(\gamma^2-k^2)^2 \sin\hbar^2\gamma L + 4k^2\gamma^2}\right) \quad (19)$$

Putting the value of γ^2 and k^2 one gets

$$T^2 = \left[1 + \frac{V_O^2 \sin\hbar^2\alpha L}{4E(V_O - E)}\right]^{-1} \times \left[1 + \frac{V_O^2 \sin\hbar^2\alpha L}{4E(V_O - E)}\right]^{-1} \quad (20)$$

One may, however check that$R + T = 1$. There are two interesting situations in which equations (17) to (20) become simpler considering the purely formal limit in which$\hbar \to 0$. The quantity $\hbar$ is a physical constant, but we can consider as a mathematical variable in order to examine the classical limit of our formulas. As$\hbar \to 0$, k and γ approach infinity and hence$T \to 0$, $R \to 1$, which is of course, the proper behavior of a classical particle with $E < V_O$. The other interesting limit occurs for high and wide barrier, that is, when $\gamma \gg 1$. In that case $\sin\hbar^2\gamma L \approx \frac{1}{2}e^{\gamma L}$, hence form (3.20) after neglecting 1 in comparison to the other which is very large, one gets

$$T^2 = \left(\frac{4E(V_O - E)}{V_O^2\left[\frac{1}{2}e^{-2\left\{\frac{2m(V_O-E)}{\hbar^2}\right\}^{\frac{1}{2}}L}\right]^2}\right)\left(\frac{4E(V_O - E)}{V_O^2\left[\frac{1}{2}e^{-2\left\{\frac{2m(V_O-E)}{\hbar^2}\right\}^{\frac{1}{2}}L}\right]^2}\right)$$

$$= \left(16\,\frac{E}{V_O}\left(1 - \frac{E}{V_O}\right)e^{-2\left\{\frac{2m(V_O-E)}{\hbar^2}\right\}^{\frac{1}{2}}L}\right)\left(16\,\frac{E}{V_O}\left(1 - \frac{E}{V_O}\right)e^{-2\left\{\frac{2m(V_O-E)}{\hbar^2}\right\}^{\frac{1}{2}}L}\right) \quad (21)$$

From equation (21) transmission coefficient would be given by

$$T^2 = \left(16\,\frac{E}{V(r_0)}\left[1 - \frac{E}{V(r_0)}\right]e^{-2\gamma L}\right)\left(16\,\frac{E}{V(r_0)}\left[1 - \frac{E}{V(r_0)}\right]e^{-2\gamma L}\right) \quad 22)$$

$\gamma L \gg 1$, the most important factor in the above equation is the exponential. The factor in front of the exponential which is of the order of 2 is not significant since its variation with V and E is negligible as compared to the variation in exponential itself(Chaddha, 1983). Hence we can write

$$InT^2 \simeq -4\gamma L \quad (23)$$

For a rectangular double thick potential barrier of thickness dx, we can write

$$InT^2 \simeq -4\gamma dx \quad (24)$$

Where

$$\gamma^4 = \left(\frac{2m}{\hbar^2}[V(x) - E]\right)\left(\frac{2m}{\hbar^2}[V(x) - E]\right)$$

$$= \left(\frac{2m}{\hbar^2}\left[\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right) - E\right]\right)\left(\frac{2m}{\hbar^2}\left[\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right) - E\right]\right) \quad (25)$$

makingγ a function of x

Equation (25) expression for the transmission coefficient or tunneling probability of a rectangular barrier. The actual barrier encountered by gamma particle has an exponential tail. We can approximate it as consisting of many rectangular barrier of decreasing height and obtain the total probability by summing the tunneling probability of each barrier the region between $r_0$andr_1. In this entire region, of course $E < V$. Hence taking the summation over all the rectangular potential barriers, we gets

$$In\, T^2 = \left(-2\int_{r_0}^{r_1}\gamma(x)dx\right)\left(-2\int_{r_0}^{r_1}\gamma(x)dx\right) \qquad (26)$$

From equation (3.25) that γ can be while is a function of x

$$\gamma = \left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right)^{\frac{1}{2}}\right)\left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right)^{\frac{1}{2}}\right) \qquad (27)$$

Substituting equation (27) in to equation (26)

$$In\, T^2 = \left(-2\int_{r_0}^{r_1}\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right)^{\frac{1}{2}}dx\right)\left(-2\int_{r_0}^{r_1}\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right)^{\frac{1}{2}}dx\right) \qquad (28)$$

Making use of equation (21), leads to

$$In\, T^2 = \left(-2\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\int_{r_0}^{r_1}\left(\frac{r_0}{x}-1\right)^{\frac{1}{2}}dx\right)\left(-2\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\int_{r_0}^{r_1}\left(\frac{r_0}{x}-1\right)^{\frac{1}{2}}dx\right) \qquad (29)$$

Putting $x = r_1\cos^2\theta, dx = r_1 2\cos\theta\ (-\sin\theta\ d\theta)$ and also changing the limits to $\theta(\,at\ x = r_0, \theta_0 = \cos^{-1}\left(\frac{r_0}{x}\right)^{\frac{1}{2}}\ and\ at\ x = r_0, \theta_0 = 0)$, one gets

$$In\, T^2 = \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}}r_1\int_0^{\theta_0}\left(\frac{1}{\cos^2\theta}-1\right)^{\frac{1}{2}}\sin\theta\cos\theta\,dx\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}}r_1\int_0^{\theta_0}\left(\frac{1}{\cos^2\theta}-1\right)^{\frac{1}{2}}\sin\theta\cos\theta\,dx\right) \qquad (30)$$

Since

$$\left(\left(\frac{1}{\cos^2\theta}-1\right)^{\frac{1}{2}}\right)^2 = \left(\frac{(1-\cos^2\theta)^{\frac{1}{2}}}{\cos\theta}\right)^2 = \left(\frac{\sin\theta}{\cos\theta}\right)^2$$

The double thick potential barrier is on the x coordinate

$$In\, T^2 = \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}}r_1\int_0^{\theta_0}\sin^2\theta\,d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}}r_1\int_0^{\theta_0}\sin^2\theta\,d\theta\right) \qquad (31)$$

Using trigonometric rule and integrating

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$$In\,T^2 = \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0} \frac{1-\cos 2\,\theta}{2}\,d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0} \frac{1-\cos 2\,\theta}{2}\,d\theta\right) \quad (32)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0} \left(\frac{1}{2} - \frac{\cos 2\,\theta}{2}\right) d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0} \left(\frac{1}{2} - \frac{\cos 2\,\theta}{2}\right) d\theta\right) \quad (33)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0} \frac{1}{2} d\theta - \frac{1}{2}\int_0^{\theta_0} \cos 2\,\theta\ d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0} \frac{1}{2} d\theta - \frac{1}{2}\int_0^{\theta_0} \cos 2\,\theta\ d\theta\right) \quad (34)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \left(\frac{1}{2}\int_0^{\theta_0} d\theta - \frac{1}{2}\int_0^{\theta_0} \cos 2\,\theta\ d\theta\right)\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \left(\frac{1}{2}\int_0^{\theta_0} d\theta - \frac{1}{2}\int_0^{\theta_0} \cos 2\,\theta\ d\theta\right)\right) \quad (35)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 (\theta_0 - (\cos^2\theta - \sin^2\theta))\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 (\theta_0 - (\cos^2\theta - \sin^2\theta))\right) \quad (36)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \left(\theta_0 - \left(\cos^2\theta - (1-\cos^2\theta)\right)\right)\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \left(\theta_0 - \left(\cos^2\theta - (1-\cos^2\theta)\right)\right)\right) \quad (37)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 (\theta_0 - (\cos^2\theta + \cos^2\theta - 1))\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 (\theta_0 - (\cos^2\theta + \cos^2\theta - 1))\right) \quad (38)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 (\theta_0 - (2\cos^2\theta - 1))\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 (\theta_0 - (2\cos^2\theta - 1))\right) \quad (39)$$

After putting the value of E

$$In\,T^2 = \left(-2\left[\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}} \frac{2Ze^2}{4\pi\epsilon_0 r_1}\right]^{1/2}\left[\cos^{-1}\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} - \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\left(1-\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right)\left(-2\left[\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}} \frac{2Ze^2}{4\pi\epsilon_0 r_1}\right]^{1/2}\left[\cos^{-1}\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} - \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\left(1-\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right) \quad (40)$$

Because of the fact that the potential barrier is relatively wide, $r_1 \gg r_0$,

$$\cos^{-1}\left(\frac{r_0}{x}\right)^{\frac{1}{2}} \approx \frac{\pi}{2} - \left(\frac{r_0}{x}\right)^{\frac{1}{2}}$$

As $\cos\left\{\frac{\pi}{2} - \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right\} = \sin\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} \simeq \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}$

If $\left(\frac{r_0}{r_1}\right) \ll 1$

Also

$$\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} \approx \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} \approx 1$$

Hence from equation (39)

$$In\,T^2 = \left(-2\left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right)^{\frac{1}{2}}\left[\pi/2 - 2\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right)\left(-2\left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right)^{\frac{1}{2}}\left[\pi/2 - 2\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right) \quad (41)$$

Replacing r_1 by $r_1 = \frac{2Ze^2}{4\pi\epsilon_0}$ and simplifying

$$InT^2 = \left(4\,\frac{e}{\hbar}\left(\frac{m}{\pi\epsilon_0}\right)^{\frac{1}{2}} Z^{\frac{1}{2}} r_0^{\frac{1}{2}} - \frac{e^2}{\hbar\epsilon_0}\left(\frac{m}{2}\right)^{\frac{1}{2}}\,Z\,E^{-\frac{1}{2}}\right)\left(4\,\frac{e}{\hbar}\left(\frac{m}{\pi\epsilon_0}\right)^{\frac{1}{2}} Z^{\frac{1}{2}} r_0^{\frac{1}{2}} - \frac{e^2}{\hbar\epsilon_0}\left(\frac{m}{2}\right)^{\frac{1}{2}}\,Z\,E^{-\frac{1}{2}}\right) \quad (42)$$

$$InT^2 = 4^2\left(\frac{e}{\hbar}\right)^2\left(\frac{m}{\pi\epsilon_0}\right) Z^{\frac{1}{4}} r_0^{\frac{1}{4}} - \frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right) Z^2 E^{-\frac{1}{4}} \quad (43)$$

Equation (43) gives the natural logarithm of the tunneling probability of the gamma particle.

Results

We assess the ability of gamma particle in tunneling through a barrier, its relationship with decay constant and half-life using equation (43)

$$In\,\underbrace{T^2}_{K_1} = \underbrace{\underbrace{4^2\frac{e^2}{\hbar^2}\left(\frac{m}{\pi\epsilon_0}\right)}_{I_1} Z^{\frac{1}{4}} r_0^{\frac{1}{4}}}_{J_1} - \underbrace{\underbrace{\frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right)}_{I_2} Z^2 E^{-\frac{1}{4}}}_{J_2} \quad 43$$

The constant I_1 and I_2 are to be calculated while:

Z = atomic number of the daughter nucleus (the gamma emitting nucleus)

$$r_o = 1.1\left(A_d^{\frac{1}{2}} + A_\gamma^{\frac{1}{2}}\right)\times 10^{-15}m \text{ (for each nucleus)} \quad 44$$

E = Potential energy of the emitted gamma particle

= or energy of decay for each nucleus

m = mass of gamma particle

1 atomic mass unit $= 1.66\times 10^{-27}kg$

$$\left.\begin{aligned} e &= 1.6\times 10^{-19}C \\ \hbar &= 1.05477\times 10^{-34}Js \\ \epsilon_o &= 8.85\times 10^{-12}Farad/m \end{aligned}\right\} \text{all are in S.I unit}$$

To keep equation (3.64) as simple as possible we calculate the constant I_1 and I_2

$$I_1 = 4^2\frac{e^2}{\hbar^2}\left(\frac{m}{\pi\epsilon_0}\right) \quad 45$$

$$I_1 = 8.792420946\times 10^{15} \quad 46$$

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$$I_2 = \frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right) \quad 47$$

$$I_2 = 2.496984634 \times 10^{-12} \quad 48$$

$$K_1 = T^2 \quad 49$$

Let T^2 be DT

$$K_1 = DT \quad 50$$

$$InDT = 8.792420946 \times 10^{15} Z^{\frac{1}{4}} {r_0}^{\frac{1}{4}} - 2.496984634 \times 10^{-12} Z^2 E^{-\frac{1}{4}} \quad 51$$

Equation (51) is used to get the result for tunneling for every γ emitting nucleus as show in Table 4.1

The decay probability per unit time or constant we write

$$\lambda = \Gamma T \quad 52$$

Where $\Gamma =$ number of time per second gamma particle within a nucleus strikes the potential barrier
$T =$ the probability of transmission through the barrier.

Assume only one gamma particle exists within a nucleus moving to and fro in the nuclear diameter

$$\Gamma = \frac{v}{2r_0} \quad 53$$

Where $v = \gamma$ particle velocity when it finally leaves the nucleus

$$\lambda = \frac{v}{2r_0} DT \quad 54$$

$$v = 10^7 ms^{-1}, r_0 = 10^{-14} m$$

$$\lambda = \frac{10^7}{2\times10^{-14}} DT \simeq 10^{-21} DT \quad 55$$

Equation (55) can be used to get the result for decay probability per unit time.

The half life $t_{\frac{1}{2}}$ is the time taken for half the original number of atom present to decay. Mathematically half-life $t_{\frac{1}{2}}$ can written as

$$t_{\frac{1}{2}} = \frac{In\,2}{\lambda} \quad 56$$

Substitute equation (56) into (55) gives

Table 1: ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ gamma particle emitting nuclei and their decay probability

S/N	Nucleus (name)	Mass No. (A)	Z	Mass Excess A(KeV)	r_0	E $\gamma(J)$	In DT(E12)	DT	Decay constant (E-20)	Half-life (E23) $t_{\frac{1}{2}}$
1	Kr	75	36	132.4	9.526279442	1.081469678E-14	6.728293536	29.53734267	2.953734267	2.046937849
2	Rb	76	37	2571.1	9.589577676	3.97566408E-13	6.785920598	29.54587108	2.954587108	2.047528866
3	Sr	80	38	589.0	9.838699101	2.146917582E-14	6.82001297	29.55088249	2.956088249	2.047846157
4	Y	80	39	385.9	9.838699101	4.524548695E-14	6.920046991	29.56544368	2.956544368	2.048885247

5	Y	81	39	124.2	9.90	1.357044173E-14	6.930800892	29.56699649	2.956699649	2.046937847
6	Y	87	39	484.5	10.26011696	5.34005694E-14	6.992986692	29.5792886	2.95792886	2.04961187
7	Zr	80	40	311.0	9.838699101	5.12696736E-15	6.963896005	29.57177313	2.957177313	2.049323878
8	Zr	85	40	416.5	10.1414989	2.552670702E-14	7.016960099	29.57935121	2.957935121	2.049849036
9	Zr	89	40	909.1	10.37737925	6.802844816E-14	7.056410711	29.58509935	2.958509935	2.050247385
10	Zr	90	40	2186.2	10.43551628	5.609222727E-14	7.067274417	29.58649601	2.958649601	2.050344173
11	Nb	84	41	540.0	10.08166653	6.584948703E-14	6.995280444	29.57625182	2.957265682	2.049634598
12	Nb	86	41	751.7	10.20098035	1.02411173E-13	7.070740754	29.58698636	2.988698636	2.050378155
13	Nb	88	41	1057.1	10.31891467	1.026354778E-13	7.09112895	29.58986568	2.958986568	2.050577692
14	Nb	89	41	1627.7	10.37737952	2.285505918E-13	7.101111936	29.5912725	2.95912725	2.050695184
15	Mo	106	42	465.7	11.32519316	4.381954916E-14	7.30182856	29.51915492	2.961914592	2.052606812
16	Mo	107	42	400.3	11.37848848	4.897856006E-14	7.310413426	29.62032094	2.962032094	2.052688241
17	Tc	88	43	741.0	10.31891467	1.10742495E-13	7.176027451	29.60176707	2.960176707	2.051402458
18	Tc	90	43	948.1	10.43551628	1.24525198E-13	7.19621402	29.6045747	2.960457617	2.051596817
19	Tc	91	43	653.0	10.94486181	8.558831137E-14	7.282460819	29.61648995	2.961468995	2.052422754
20	Ru	91	44	393.7	10.49333122	4.401181043E-14	7.247696298	29.61170478	2.961170478	2.052091141
21	Ru	97	44	215.7	10.83374358	1.858525668E-15	7.305774774	29.61968622	2.961968622	2.052644255
22	Ru	105	44	724.3	11.27164584	6.054628017E-14	7.378506638	29.62959239	2.962959239	2.053330753
23	Rh	92	45	893.0	10.55082935	1.183047718E-13	7.298493917	29.61868913	2.961868913	2.052575157
24	Rh	94	45	756.2	10.66489569	8.690209675E-14	7.318140675	29.62137741	2.962137741	2.052761454
25	Rh	96	45	832.6	10.77775487	7.459737509E-14	7.337425015	29.62400908	2.962400908	2.052943829
26	Rh	99	45	341.0	10.94486181	2.007528157E-14	7.365702458	29.62785554	2.962785554	2.053210389
27	Pd	115	46	749.0	11.79618582	6.410311377E-14	7.542677371	29.6207549	2.965207549	2.054888832
28	Pd	117	46	247.3	11.898831921	1.912999696E-14	7.56256872	29.65423203	2.965423203	2.055038279
29	Ag	95	47	1261.2	10.72147378	1.636634412E-13	7.407924692	29.63357145	2.963357145	2.053606501
30	Ag	99	47	342.6	10.94486181	1.238483053E-14	7.446213897	29.63872682	2.963472682	2.053963768
31	Cd	100	48	936.6	11.0	1.177600316E-14	7.494919158	29.64524646	2.964524646	2.054415558
32	Cd	105	48	961.8	11.27164584	1.350795682E-13	7.540768622	29.65134513	2.965134523	2.054838224
33	In	104	49	658.0	11.21784293	2.834251644E-14	7.570678971	29.65530387	2.965530387	2.055112558
34	In	106	49	632.6	11.32519316	8.797555554E-14	7.588726425	29.6576849	2.96576849	2.055277564
35	Sn	105	50	1281.7	11.27164584	1.752461531E-13	7.618119947	29.66155073	2.966155073	2.055545466
36	Sn	107	50	678.6	11.37848848	8.765512008E-14	7.63610896	29.66390929	2.966390929	2.055708914
37	Sb	108	51	1205.8	11.43153533	1.312663862E-13	7.682934965	29.67002275	2.967002275	2.056132577
38	Sb	112	51	1257.1	11.64130577	1.56548744E-13	7.717940754	29.6745687	2.96745687	2.056447611
39	Te	113	52	814.0	11.69316039	1.056956365E13	7.72651097	29.67567851	2.967567851	2.056524521
40	Te	115	52	770.4	11.79618582	4.364330965E-14	7.781166519	29.68272734	2.968272734	2.057013005
41	I	112	53	689.0	11.64130577	9.308650113E-14	7.79251889	29.68418527	2.968418527	2.057114039
42	I	114	53	708.8	11.74478608	5.380111373E-14	7.809778513	29.68639772	2.968639772	2.057267262
43	Xe	135	54	786.9	12.78084504	2.235037334E-14	8.013953247	29.71220529	2.971220529	2.059055827
44	Xe	140	54	805.6	13.01537552	3.471918209E-14	8.050467255	29.71675125	2.971675125	2.059370862
45	Cs	116	55	393.5	11.84736258	1.361850705E-14	7.8995902	29.697832	2.9697832	2.058059134
46	Cs	125	55	525.0	12.29837388	6.025788825E-14	7.973721585	29.70717245	2.970717245	2.05870705
47	Ba	126	56	233.6	12.34746938	3.52479006E-15	8.01770307	29.7126731	2.97126731	2.059088246
48	Ba	143	56	211.5	13.15405652	2.611548999E-14	8.145554522	29.72849344	2.972549344	2.060184595
49	La	126	57	256.0	12.34746938	1.385883365E-14	8.053259166	29.71709799	2.971709799	2.059394591
50	La	130	57	357.4	12.54192968	1.345828932E-14	7.533071049	29.65032392	2.965032392	2.054767448
51	Ce	127	58	120.4	12.39637044	8.3954090522E-15	7.30577475	29.61928622	2.961968622	2.052644255
52	Ce	133	58	477.2	12.68581885	5.913636414E-14	7.378506684	29.62959239	2.962959239	2.053330752
53	Pr	129	59	203.8	12.49359836	1.336215868E-14	7.2984939176	29.61868913	2.961868913	2.052575157
54	Pr	137	59	836.9	12.8751699	9.989575466E-14	7.318140675	29.62137741	2.962137741	2.052761455
55	Nd	133	60	402.8	12.68581885	4.24256549E-14	7.337425015	29.62400708	2.962400908	2.05293829
56	Nd	152	60	278.6	13.56171081	2.377661113E-14	7.365702458	29.62785564	2.962785564	2.053210389
57	Pm	136	61	373.7	12.82809417	3.375787571E-14	7.546278371	29.65207549	2.965207549	2.04888832
58	Sm	137	62	380.5	12.8751699	2.9111561154E-14	7.56256872	29.65423205	2.965423203	2.055638279
59	Eu	139	63	719.0	12.96880873	5.423370161E-14	7.407924692	29.63357145	2.963357145	2.053606501
60	Gd	159	64	363.0	13.87047223	4.792112304E-14	7.4462137979	29.63872782	2.963872728	2.053963758
61	Tb	144	65	284.0	13.20	1.683888342E-14	8.462141713	29.76662342	2.976662342	2.062827003

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62	Dy	145	66	578.2	13.24575404	8.542809364E-14	8.501853707	29.77130541	2.977130534	2.06315146
63	Ho	146	67	682.7	13.29135057	7.786581678E-14	8.541211008	29.77592392	2.977592392	2.063471528
64	Er	151	68	1140.2	13.5770263	1.691899229E-14	8.609065064	29.78383684	2.978383684	2.064019893
65	Tm	152	69	808.2	13.56171081	5.322432991E-14	8.647675105	29.78831163	2.978831163	2.064329996
66	Yb	157	70	231.1	1379296049	2.291113539E-14	8.714021135	29.79595447	2.979595447	2.064859645

$$t_{\frac{1}{2}} = \frac{In\ 2}{10^{-21}DT} \qquad 57$$

This equation gives the result for half-life of gamma emitting nucleus substitute equation (57) into (51)

$$t_{\frac{1}{2}} = \frac{In\ 2}{10^{-21}} e^{-\left[4^2\frac{e^2}{\hbar^2}\left(\frac{m}{\pi\epsilon_0}\right)Z^{\frac{1}{4}}r_0^{\frac{1}{4}} - \frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right)Z^2E^{-\frac{1}{4}}\right]} \qquad 58$$

$$t_{\frac{1}{2}} = 6.93 \times 10^{21} \times e^{-\left[8.792420946\times10^{15}Z^{\frac{1}{4}}r_0^{\frac{1}{4}} - 2.496984634\times10^{-12}Z^2E^{-\frac{1}{4}}\right]} \qquad 59$$

Table 2: ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ gamma particle emitting nuclei and their calculated and experimental half lives

S/N	Nucleus (name)	Mass No. (A)	Z	E $\gamma(J)$	In DT (E12)	DT	Log Decay constant	Log Half-life $t_{\frac{1}{2}}$	Log Half-life $t_{\frac{1}{2}}$ (from chart)
1	Kr	75	36	1.081469678E-14	6.728293536	29.53734267	-19.52962858	23.31110466	2.411619406
2	Rb	76	37	3.975664408E-13	6.785920598	29.54587108	-19.529032	23.31123003	1.568201724
3	Sr	80	38	2.146917582E-14	6.82001297	29.55088249	-19.52942955	23.31130369	3.804275767
4	Y	80	39	4.524548695E-14	6.920046991	29.56544368	-19.5292156	23.31151764	0.6812412374
5	Y	81	39	1.357044173E-14	6.930800892	29.56699649	-19.5219279	23.31110466	1.908485019
6	Y	87	39	5.34005694E-14	6.992986692	29.5792886	-19.52901228	23.31167163	3.683407299
7	Zr	80	40	5.12696736E-15	6.963896005	29.57177313	-19.52912263	23.3116106	0.6989700043
8	Zr	85	40	2.552670702E-14	7.016960099	29.57935121	-19.52901136	23.31172188	1.037426498
9	Zr	89	40	6.802844816E-14	7.056410711	29.58509935	-19.52892697	23.31180627	3.672910245
10	Zr	90	40	5.609222727E-14	7.067274417	29.58649601	-19.58728819	23.31182677	2.907945522
11	Nb	84	41	6.584948703E-14	6.995280444	29.57625182	-19.52905679	23.31167644	1.09181246
12	Nb	86	41	1.02411173E-13	7.070740754	29.58698636	-19.52889927	23.31183397	1.942504106
13	Nb	88	41	1.026354778E-13	7.09112895	29.58986568	-19.52885701	23.31187623	2.66464976
14	Nb	89	41	2.285505918E-13	7.101111936	29.5912725	-19.52883836	23.31189688	3.857332496
15	Mo	106	42	4.381954916E-14	7.30182856	29.51915492	-19.52842747	23.31230577	0.9395192526
16	Mo	107	42	4.897856006E-14	7.310413426	29.62032094	-19.52841020	23.31232299	0.5440680444
17	Tc	88	43	1.10742495E-13	7.176027451	29.60176707	-19.52868286	23.31205087	0806179974
18	Tc	90	43	1.24525198E-13	7.19621402	29.6045747	-19.52864115	23.31209202	1.691965103
19	Tc	91	43	8.558831137E-14	7.282460819	29.61648995	-19.528466	23.31226682	2.29666519
20	Ru	91	44	4.401181043E-14	7.247696298	29.61170478	-19.52853659	23.31219665	2.346352974
21	Ru	97	44	1.858525668E-15	7.305774774	29.61968622	-19.52841955	23.31231369	3.619260335
22	Ru	105	44	6.054628017E-14	7.378506638	29.62959239	-19.52827432	23.31245891	4.203685471
23	Rh	92	45	1.183047718E-13	7.298493917	29.61868913	-19.52843417	23.31229907	0.6989700043
24	Rh	94	45	8.690209675E-14	7.318140675	29.62137741	-19.52839475	23.31233848	1.411619706
25	Rh	96	45	7.459737509E-14	7.337425015	29.62400908	-19.52835617	23.31237707	1957128198
26	Rh	99	45	2.007528157E-14	7.365702458	29.62785554	-19.52829978	23.3124345	4.228400359
27	Pd	115	46	6.410311377E-14	7.542677371	29.6207549	-19.5279449	23.31278833	1.698970004
28	Pd	117	46	1.912999696E-14	7.56256872	29.65423203	-19.5279132	23.31281992	0.6434526765
29	Ag	95	47	1.636634412E-13	7.407924692	29.63357145	-19.528216	23.31251723	0.27875601
30	Ag	99	47	1.238483053E-14	7.446213897	29.63872682	-19.52841047	23.31259278	1.041392685
31	Cd	100	48	1.177600316E-14	7.494919158	29.64524646	-19.52804493	23.3126883	1691081492
32	Cd	105	48	1.350795682E-13	7.540768622	29.65134513	-19.5279556	23.31277764	3.522444234

33	In	104	49	2.834251644E-14	7.570678971	29.65530387	-19.52789762	23.31283561	2.035829825
34	In	106	49	8.797555554E-14	7.588726425	29.6576849	-19.52786275	23.31287048	2.502427212
35	Sn	105	50	1.752461531E-13	7.618119947	29.66155073	-19.52780615	23.31292709	1.53147819
36	Sn	107	50	8.765512008E-14	7.63610896	29.66390929	-19.52777162	23.31296162	2.243534107
37	Sb	108	51	1.312663862E-13	7.682934965	29.67002275	-19.5276212	23.31305111	0.8692317197
38	B	112	51	1.56548744E-13	7.717940754	29.6745687	-19.52761558	23.31311765	1.1710963119
39	Te	113	52	1.056956365E13	7.72651097	29.67567851	-19.52759934	23.31313389	2.008600172
40	Te	115	52	4.364330965E-14	7.781166519	29.68272734	-19.5274962	23.31232704	2.604226053
41	I	112	53	9.308650113E-14	7.79251889	29.68418527	-19.52747487	23.31325837	0.531478917
42	I	114	53	5.380111373E-14	7.809778513	29.68639772	-19.5274425	23.31329074	0.7923916895
43	Xe	135	54	2.235037334E-14	8.013953247	29.71220529	-19.52706511	23.31366812	2.962842681
44	Xe	140	54	3.471918209E-14	8.050467255	29.71675125	-19.52699867	23.31373456	1.133538908
45	Cs	116	55	1.361850705E-14	7.8995902	29.697832	-19.52727559	23.31345785	0.84509804
46	Cs	125	55	6.025788825E-14	7.973721585	29.70717245	-19.52713868	23.31354455	3.431363764
47	Ba	126	56	3.52479006E-15	8.01770307	29.7126731	-19.52705828	23.31367496	3.773786445
48	Ba	143	56	2.611548999E-14	8.145554522	29.72849344	-19.5268271	23.31390614	1.155336037
49	La	126	57	1.385883365E-14	8.053259166	29.71709799	-19.5269936	23.31373963	1.698970004
50	La	130	57	1.345828932E-14	7.533071049	29.65032392	-19.52797056	23.31276268	2.717670503
51	Ce	127	58	8.395409052E-15	7.305774775	29.61928622	-19.52841955	23.31231369	1.531478917
52	Ce	133	58	5.913636414E-14	7.378506684	29.62959239	-19.52827432	23.31245891	2.51054501
53	Pr	129	59	1.336215868E-14	7.2984939176	29.61868913	-19.52843417	23.31229907	1.477121255
54	Pr	137	59	9.989575466E-14	7.318140675	29.62137741	-19.52839475	23.31233843	1.88536122
55	Nd	133	60	4.24256549E-14	7.337425015	29.62400708	-19.52835617	23.31237589	1.84509804
56	Nd	152	60	2.377661113E-14	7.365702458	29.62785564	-19.52829978	23.31243345	2.835056102
57	Pm	136	61	3.375787571E-14	7.546278371	29.65207549	-19.5279449	23.31278833	1.672097858
58	Sm	137	62	2.911156154E-14	7.56256872	29.65423205	-19.52791332	23.3129467	1.653212514
59	Eu	139	63	5.423370161E-14	7.407924692	29.63357145	-19.528216	23.31251723	1.255272505
60	Gd	159	64	4.792112304E-14	7.4462137979	29.63872782	-19.52814044	23.31251275	3.045322979
61	Tb	144	65	1.683888342E-14	8.462141713	29.76662342	-19.52627043	23.31446281	0.6232492904
62	dy	145	66	8.542809364E-14	8.501853707	29.77130541	-19.52620212	23.31453111	1.146128036
63	Ho	146	67	7.786581678E-14	8.541211008	29.77592392	-19.526123475	23.31489848	0.5785139399
64	Er	151	68	1.691899229E-14	8.609065064	29.78383684	-19.526019366	23.31471388	0.7781512504
65	Tm	152	69	5.322432991E-14	8.647675105	29.78831163	-19.52595411	23.31477912	0.6989700043
66	Yb	157	70	2.291113539E-14	8.714021135	29.79595447	-19.5258427	23.31489054	1.5910064607

Discussion

The results of tunneling probabilities of gamma particle for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ nuclei are shown in Tables 1 and 2.Table 1 and 2 have atomic number Z = 36 to 70 for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ gamma nuclei. The tables that indicate the medium gamma particle has an appropriate result obtained which shows that gamma decay is possible.The calculated tunnel probability in equation (4.8) indicate input data in Table 2. The isotopes of gamma particle emitter with Z = 36 to 70 that is ${}^{75}_{36}Kr - {}^{157}_{70}Yb$ for medium gamma particle and Z = 71 to 101 that is ${}^{158}_{71}Lu - {}^{256}_{101}Md$ for heavy gamma particle are shown.The half-life varies from one nucleus to another which indicates that from Table 2 observes that the values of calculated half-lives are so small but also match with the experimental half-lives.In general, the gamma particle half-life $t_{\frac{1}{2}}$ presented in the Table 2 are in agreement with the experimental result (see chart of Nuclides Edwards et al., 2002).

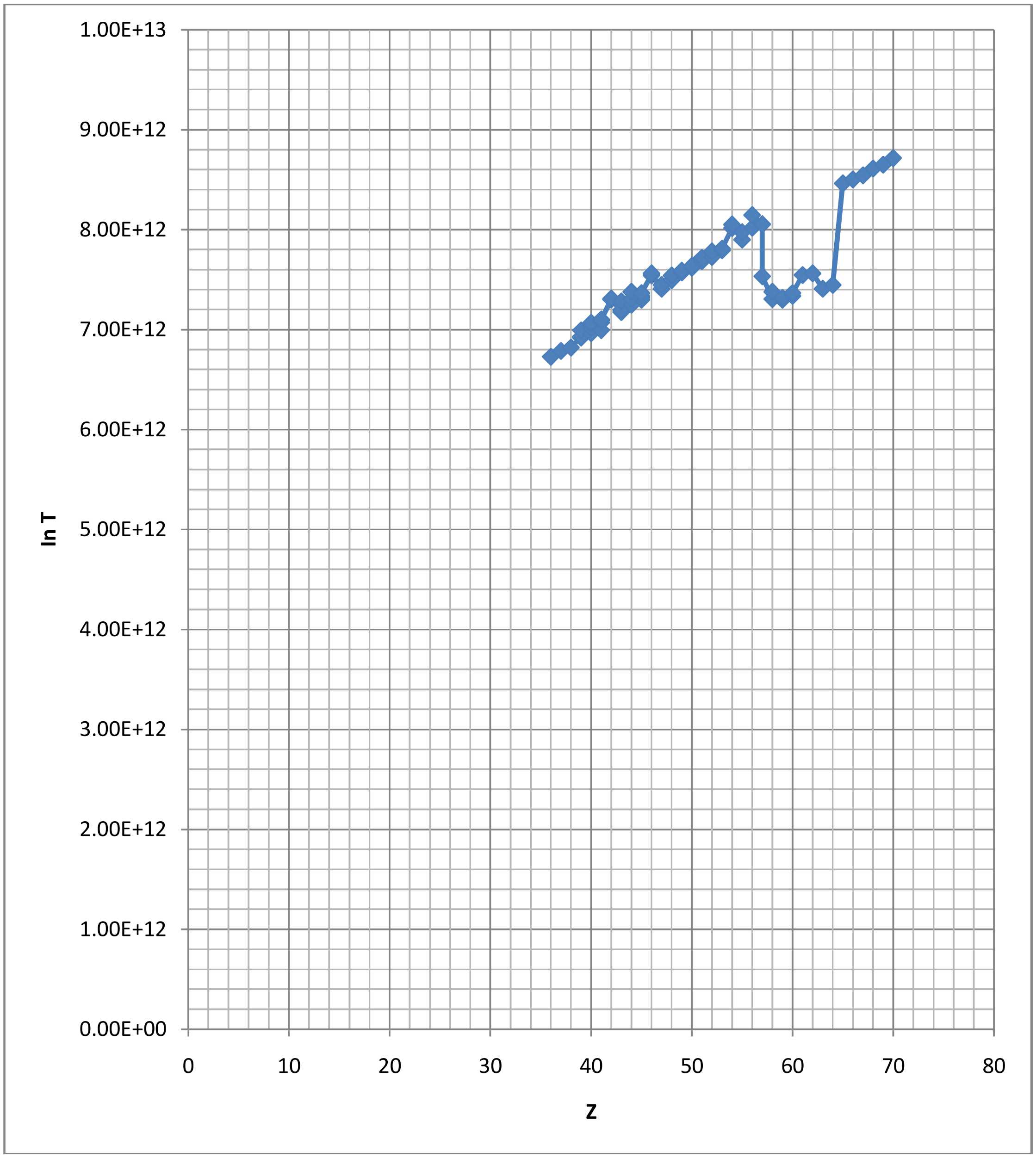

Figure 1: Natural logarithm of Tunneling probability versus Atomic number for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ Gamma Particle emitting nuclei.

Figure 1 represents the natural logarithm of tunneling probability versus atomic number Z for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass gamma particle emitters respectively.Figure 1, the anomaly lies with high atomic number Z values for the medium gamma particle nuclei. From atomic number Z= 42, 44, 46, 48, 52, 54 and 56 are slightly high than the orders also from atomic number Z= 57 to 65 makes a shape of " w" and from the atomic number Z= 65 diminishes with increasing value of natural logarithm of tunneling probability. The reason that anomaly lies at low atomic number Z is as result of different energy of gamma particle emitters with atomic number Z. The shape "w" is as a result from one nucleus to another, that the nuclei have either very small tunneling probability or the nuclei are stable and are depicted by points lying at the bottom for each isotope which even-even is with even-odd (even neutron and odd proton or even proton and odd neutron) the figure shows that the probability of gamma emission is higher than even-even nuclei. The atomic number Z= 42, 44, 46, 52, 54 and 56 it shows that even-even nuclei have the slightly high probability of gamma emission.

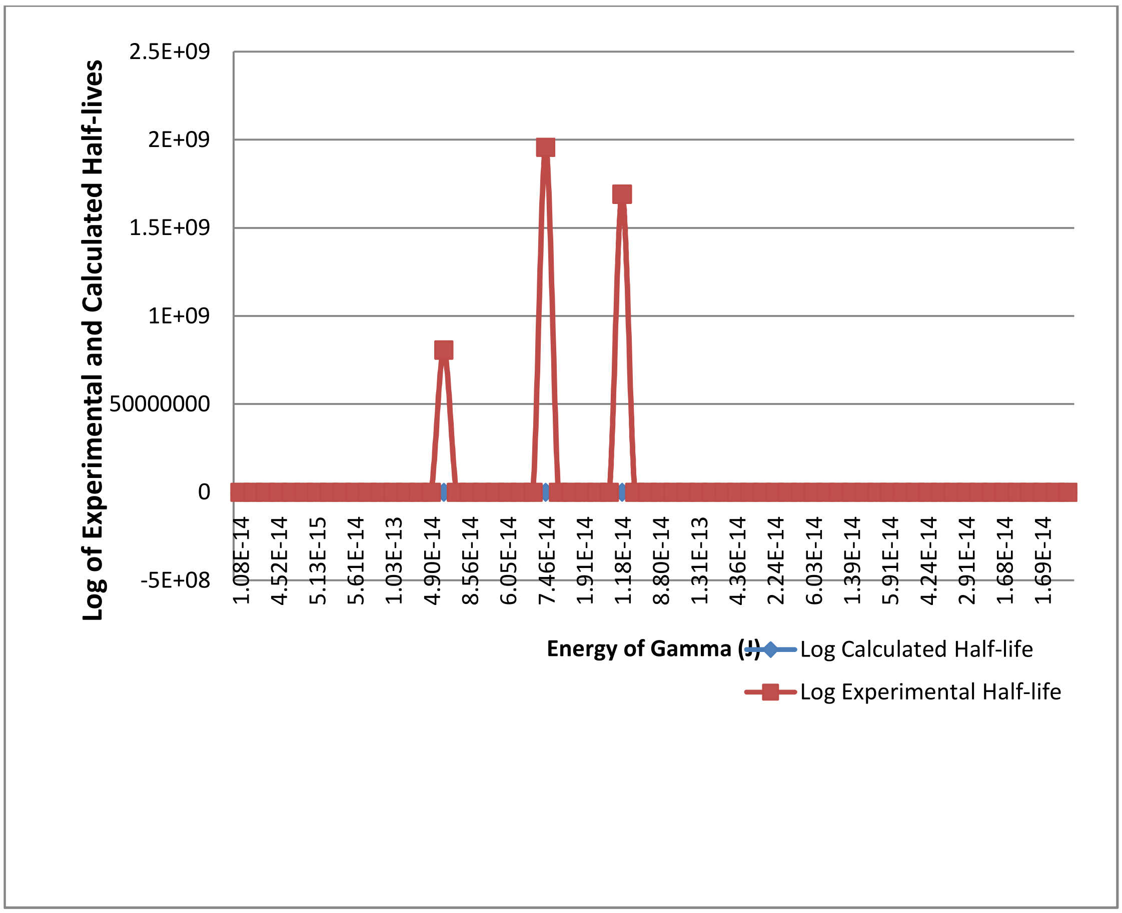

Figure 2: Logarithmic plot of Experimental and Calculated Half-lives versus Energy of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 2 shows the logarithm of experimental and calculated half-lives versus energy of gamma particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.Figure 2, shows the anomaly lies with high energy of gamma particle values of experimental and calculated half-lives except for the three anomalies nuclei of the experimental half-lives that are high energy of gamma particle emitter prove that they have high half-lives experimentally as in the anomalies of the energy of gamma particle are 4.90 E-14 J, 7.46 E-14 J and 1.18 E-14 J.

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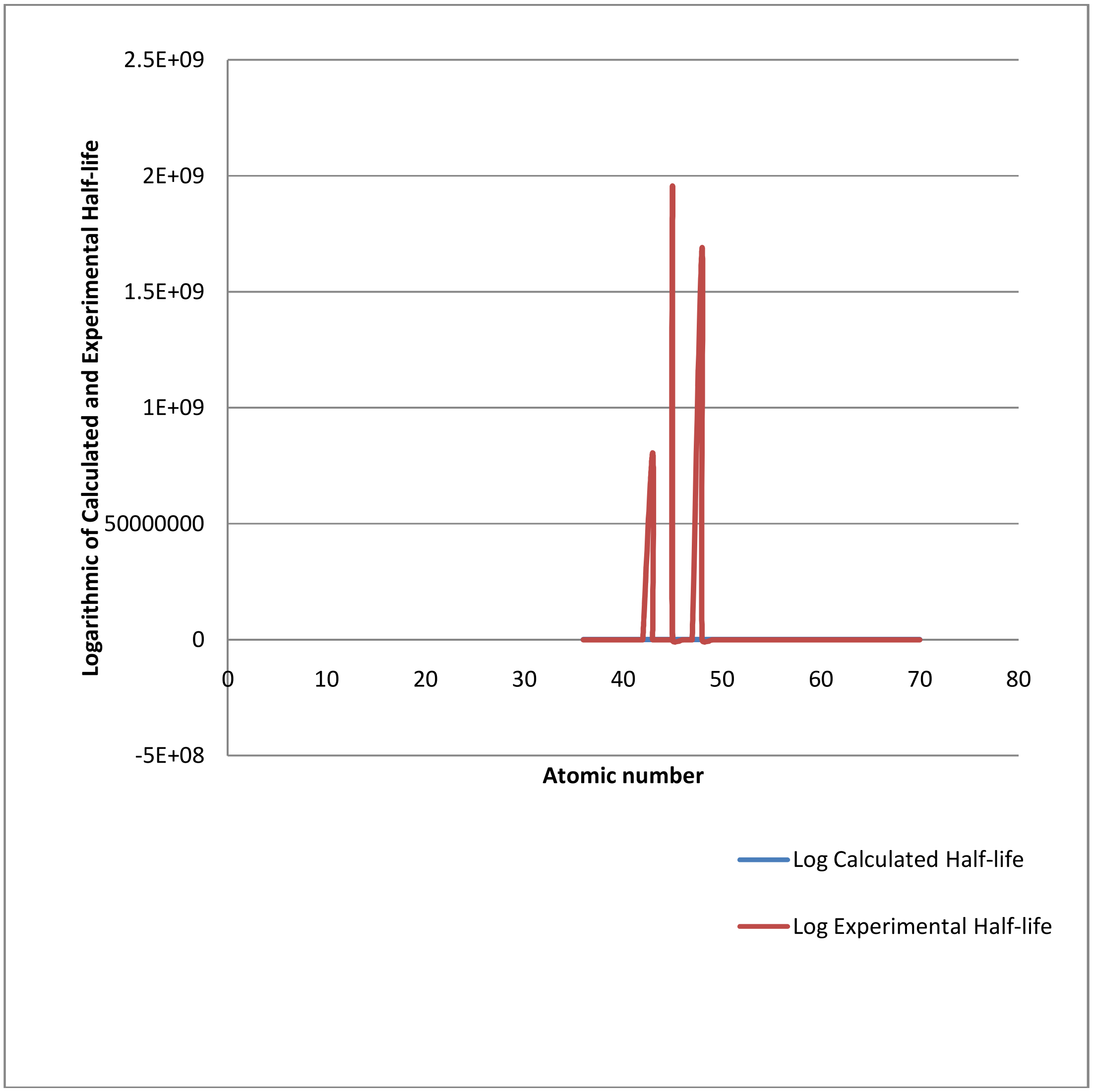

Figure 3: Logarithmic plot of Calculated and Experimental Half-lives versus Atomic Number for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 3 shows the logarithmic calculated and experimental half-lives versus Mass number A for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei respectively.
Figure 3 shows the anomaly lays with high mass number A values for the medium mass number A nuclei sustain a straight line of the value of calculated and experimental half-lives except for the three anomaly of the experimental half-lives that are high that is for the isotopes of the nuclei with mass number A = 94 to 99. These reveal that those low mass number A have a low rate of calculated and experimental half-lives while the three mass number A indicates that they have high experimental half-lives.

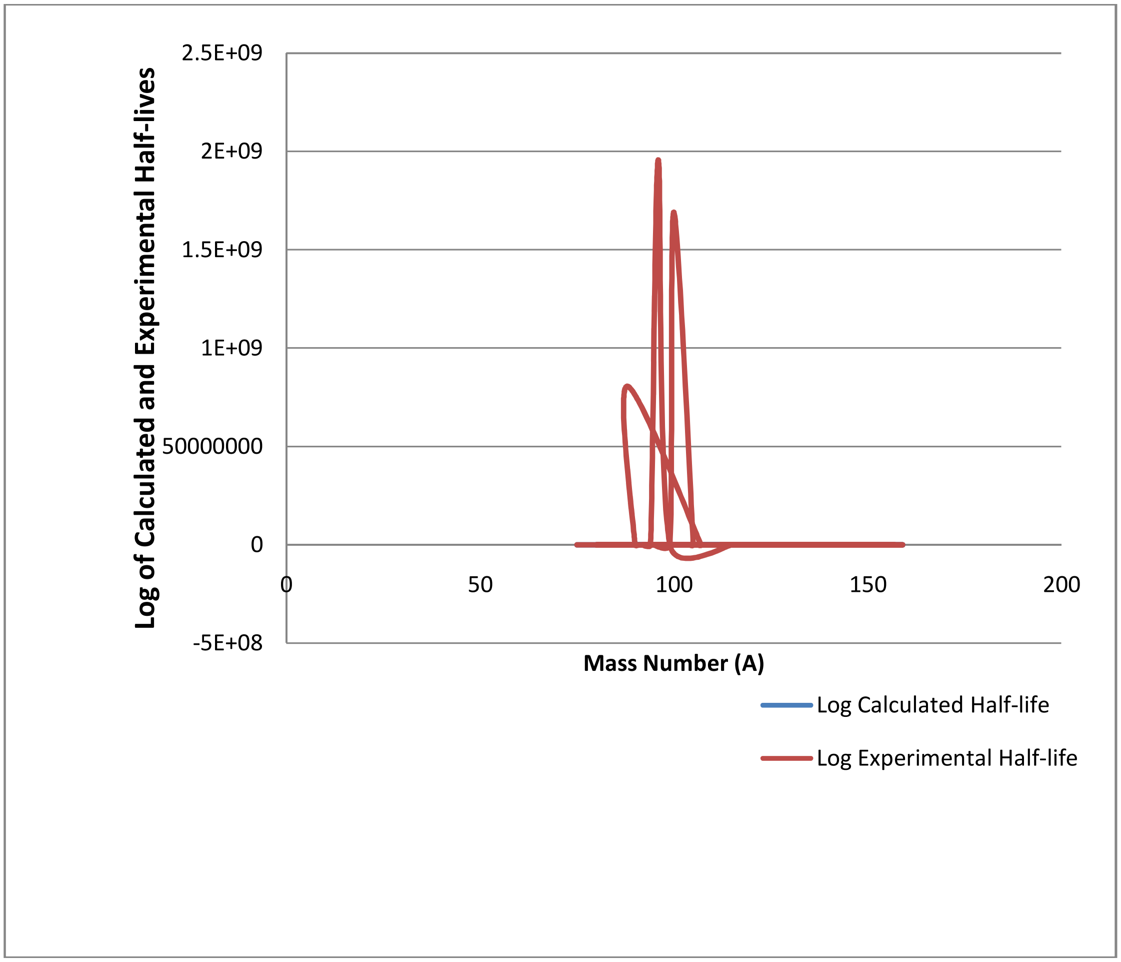

Figure 4: Logarithmic plot of Calculated and Experimental Half-lives versus Mass Number (A) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ Mass Nuclei.

Figure 4 shows the logarithm of experimental and calculated half-lives versus mass number of gamma particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei. Figure 4, indicates the anomaly lies with low mass number of gamma particle values of experimental and calculated half-lives except for the three anomalies nuclei of the experimental half-lives that are high mass number of gamma particle either prove that they have high half-lives experimental as in the anomalies of the mass number of gamma particle are 90, 99 and 101.

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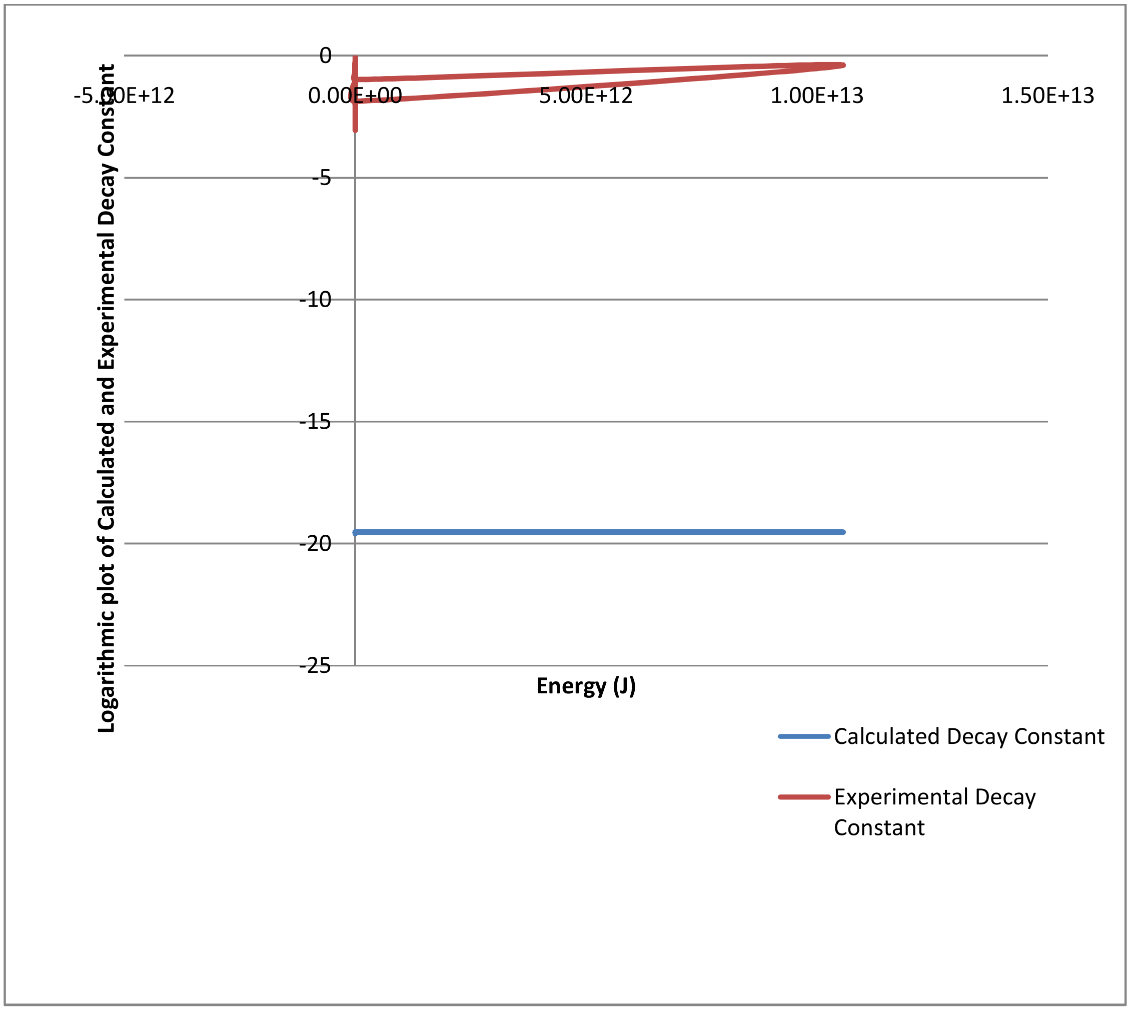

Figure 5: Logarithmic plot of Calculated and Experimental Decay constant versus Energy (J) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ Mass Nuclei.

Figure 5 represents the logarithm calculated decay constant versus Energy (J) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass gamma particle emitters respectively.Figure 5, the anomaly lays with low Energy (J) values for the medium gamma particles emitting nuclei. for the Energy (J) value of 0.00 is having a vertical line on the logarithm calculated decay constant from 0.00 to around -3.5 which also the figure it has a shape if cone on the position of neutral equilibrium. The cone neutral equilibrium position lies on the low Energy (J) than the order vertical line. The figure also shows a horizontal line on the Energy (J) from 0.00E+00 to 1.00E+13

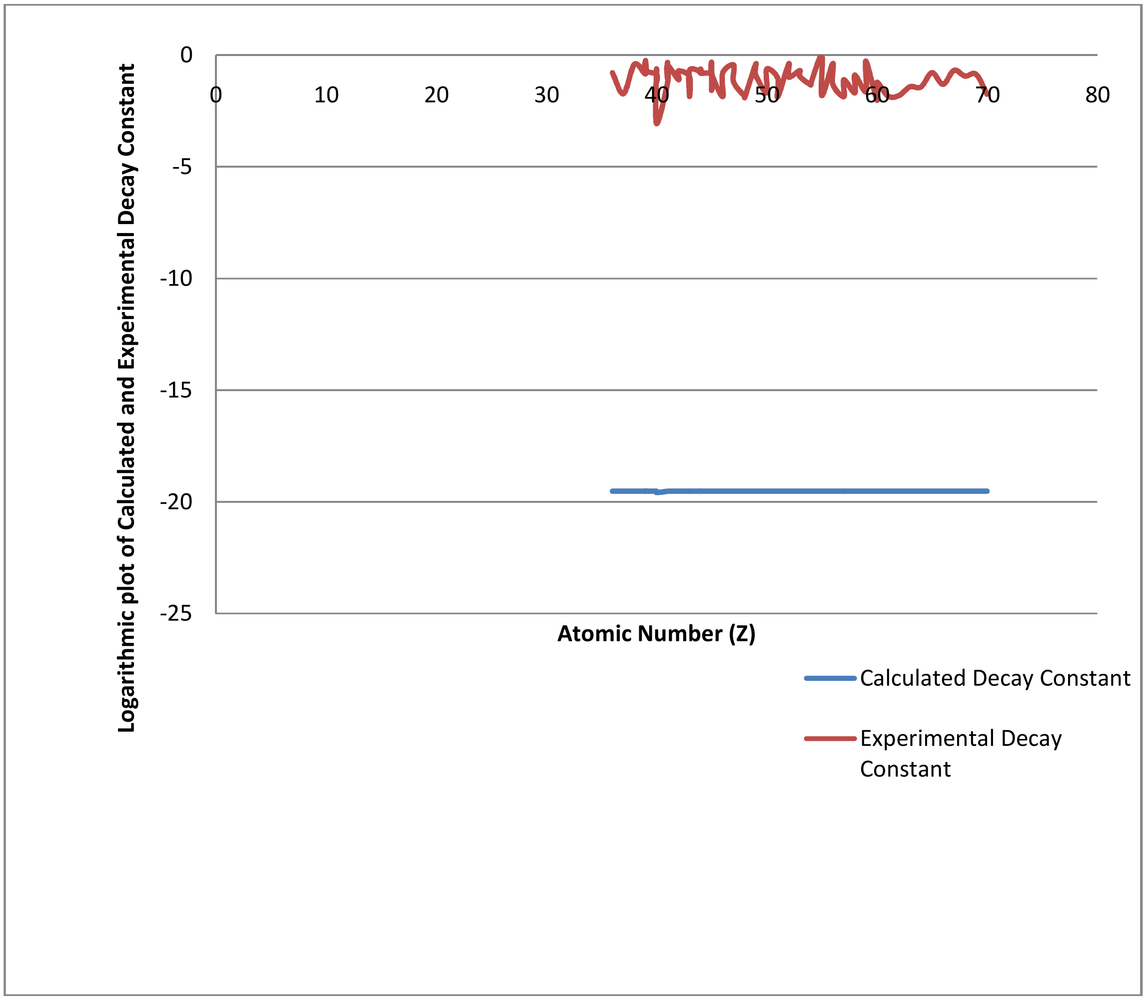

Figure 6: Logarithmic plot of Calculated and Experimental Decay constant versus Atomic Number (Z) of Gamma Particle for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ Mass Nuclei.

Figure 6 shows the logarithm of calculated and experimental decay constant versus atomic number (z) of gamma particles for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass nuclei.

Figure 6, shows the anomaly lies with low atomic number (Z) Of gamma particle values of calculated and experimental decay constant for the medium gamma particle nuclei. The figure shows a zigzag and horizontal line. For the zigzag value on atomic number (Z) = 40 has the lowest value on the zigzag while from the atomic number (Z) = 60 to 70 it diminishes.

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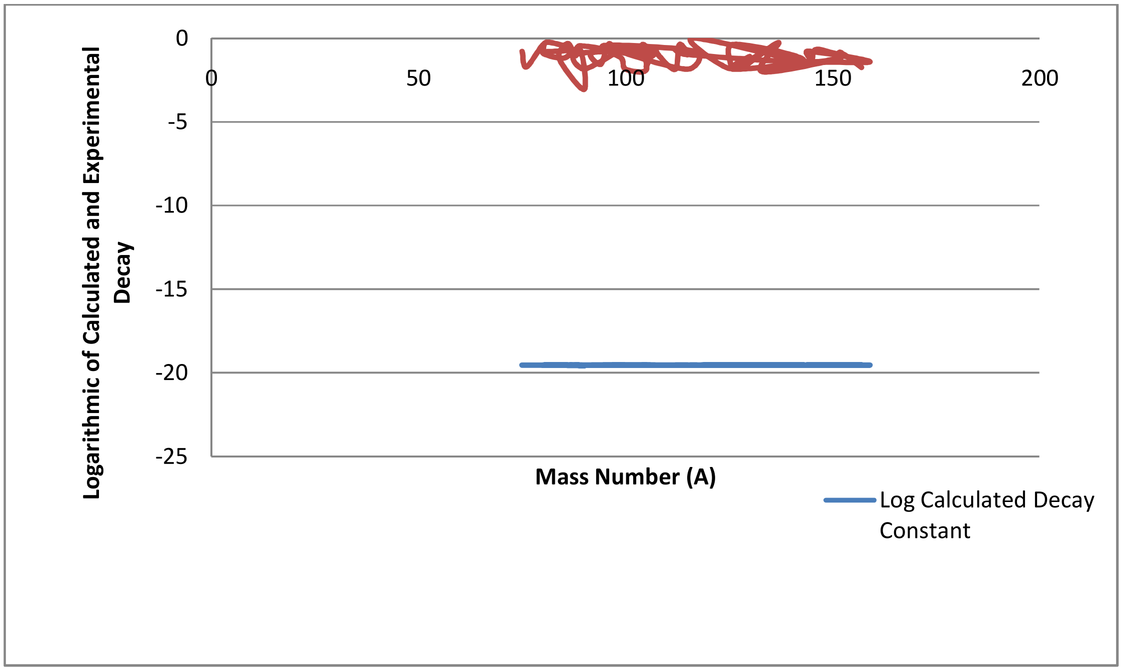

Figure 7: Logarithmic plot of Calculated and Experimental Decay constant versus Mass Number (A) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ Mass Nuclei.

Figure 7 shows the logarithm of calculated and experimental decay constant versus mass number (A) of gamma particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.
Figure 7, shows the anomaly lies with low mass number (A) values for the medium gamma particle nuclei. It shows the shapes of cones and a horizontal line. The shapes of cones are like in the position of neutral and also unstable equilibrium. The shapes lie in between mass number (A) = 71-160.

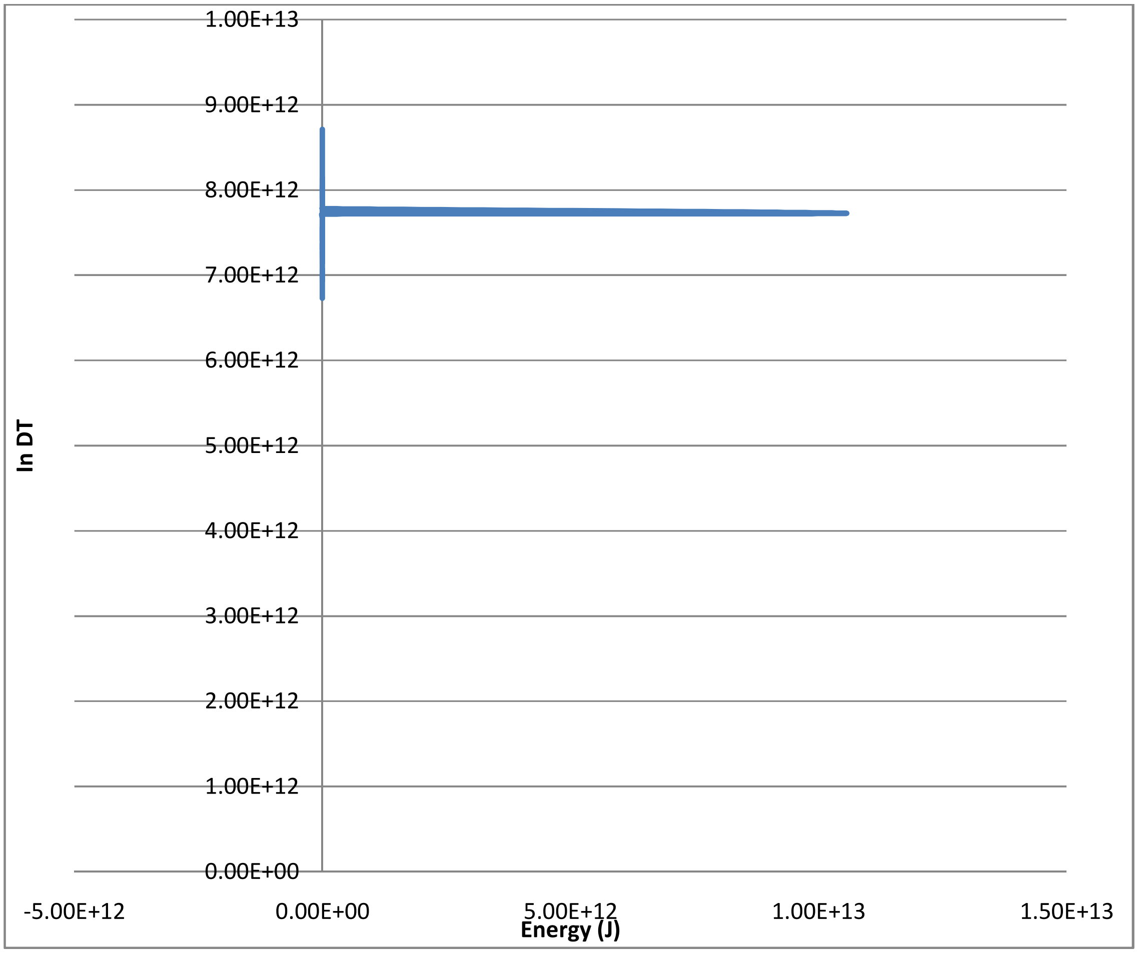

Figure 8: Natural logarithm of Tunneling probability versus Energy (J) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ Gamma Particle emitting nuclei.

Figure 8 represent the natural logarithm of tunneling probability versus Energy (J) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle emitters respectively.

Figure 8, the anomaly lie with high Energy (J) values for the medium Gamma particle emitting nuclei. From natural logarithms of tunneling probability axis that lies 0.00 E+00 on Energy (J) axis while from two different points that meet at a point that make a narrow space between the two point from the at a distance less than 5.00 E+12 Energy (J) it continuous up to a distance above 1.00 E+13. These means that the nuclides that Energy (J) to tunnel through the Double thick barrier, the Energy of the nuclides that have 0.00 E+00 lies in between a distance close to 7.00 E+12 to distance close to 9.00 E+12 and the two points are close 8.00 E+12 of the natural logarithm of the tunneling probability.

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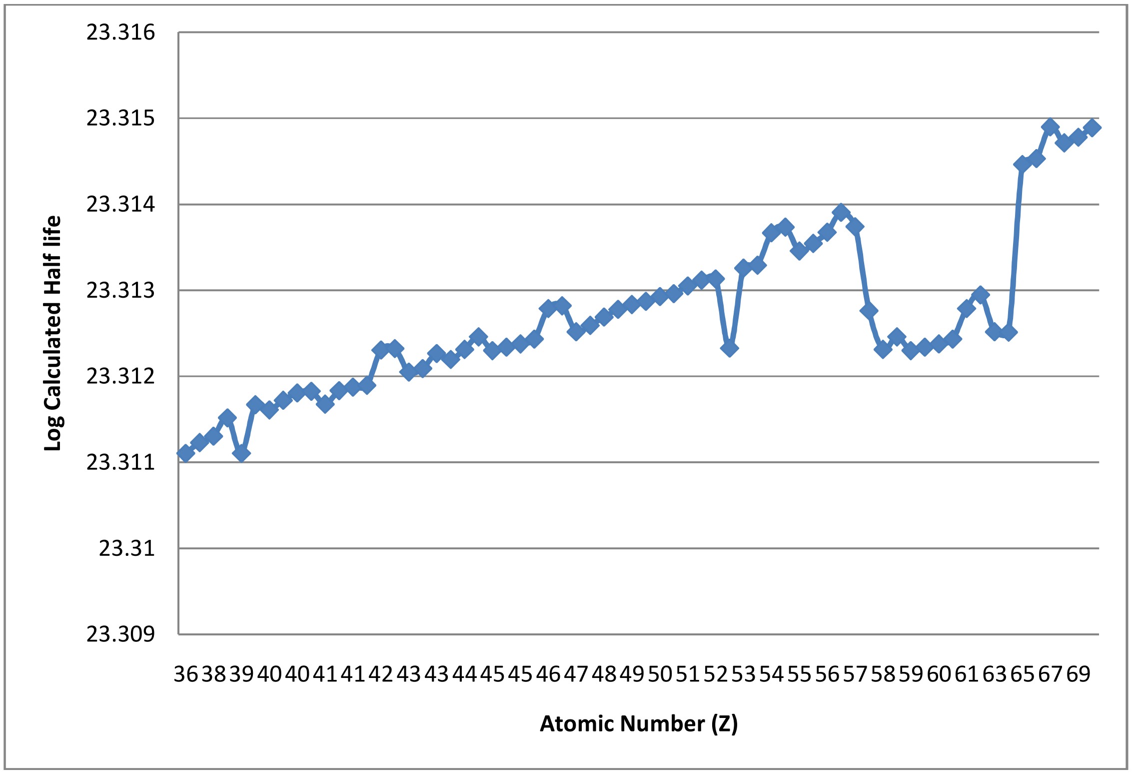

Figure 9: Logarithmic plot of Calculated Half-lives versus Atomic Number (Z) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 9 represents the logarithms of calculated half-life verses Atomic number Z for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle respectively.Figure 9, the anomaly lies with high atomic number Z value for the medium gamma particle emitting nuclei. From atomic number Z = 42, 44, 46, 48, 52,54 and 56 are slightly high than the orders also from atomic number Z = 57 to 65 makes a shape of w and from the atomic Z = 65 diminishes with increasing value of natural logarithm of tunneling probability. The reason that anomaly lies at low atomic number Z is as a result of different logarithms of calculated half-life. The shape w is as a result of a different time of tunneling to the other, that the nuclei have either very small time tunneling probability or the nuclei are stable are depicted by points lying at the bottom for each isotopes which even-even is with even-old (even number and odd proton or even proton and odd neutron) the figure shows that the time taken for the probability of gamma emission is high than even-even nuclei. The atomic number Z = 42, 44, 46, 48, 52,54 and 56 it shows that even-even nuclei have the slit high logarithms of calculated half-life of gamma emission.

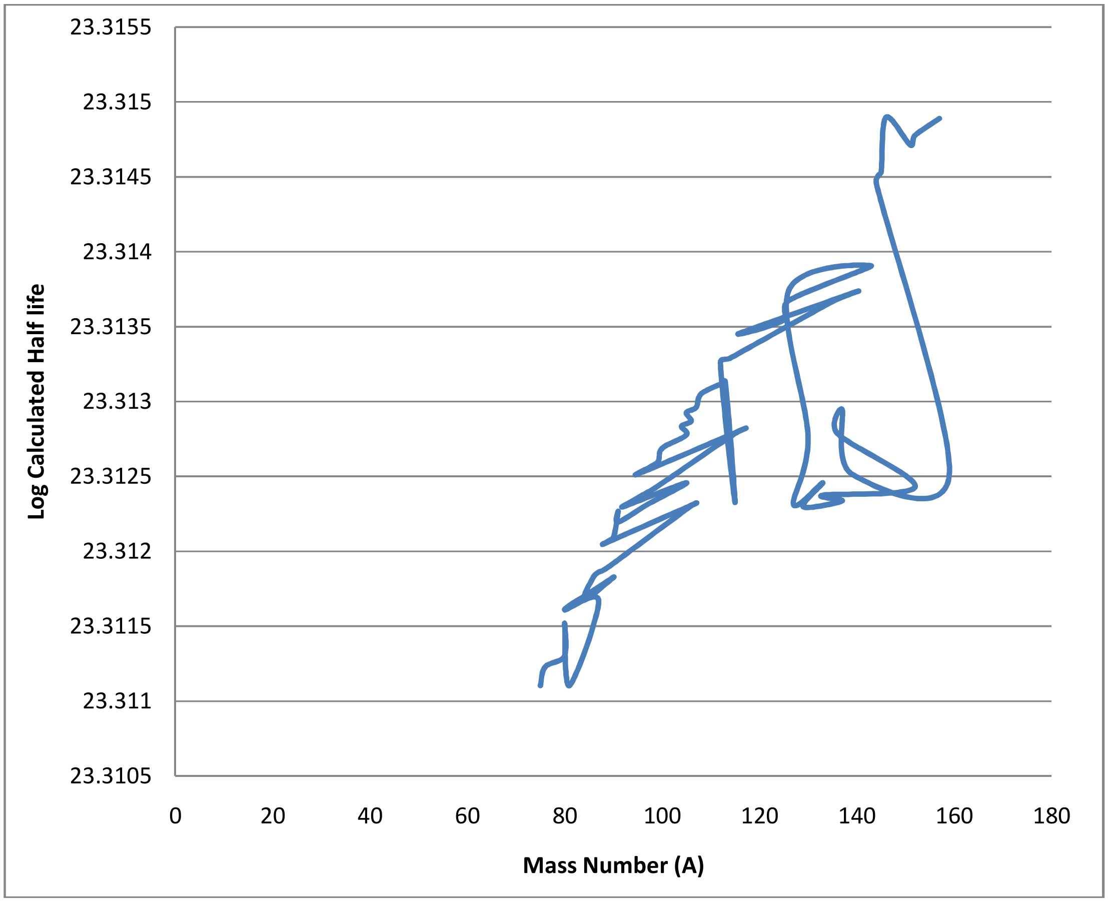

Figure 10: Logarithmic plot of Calculated Half-life versus Mass Number (J) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 10 represents the logarithms of calculated half-life versus mass number (A) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle emitters respectively.

Figure 10, the anomaly lies with high mass number (A) values for the medium gamma particle emitting nuclei. The figure shows the shape of v, zigzag shape in the ascending order and also a shape of w. The reason for the shape of v is as a result of low in logarithms of calculated half-life taken for tunneling probability, zigzag shape is as a result of different value of logarithm of calculated half-life which is not at a close distance and also shape w is as a result from one nucleus to another of the logarithm half-life for the tunneling probability.

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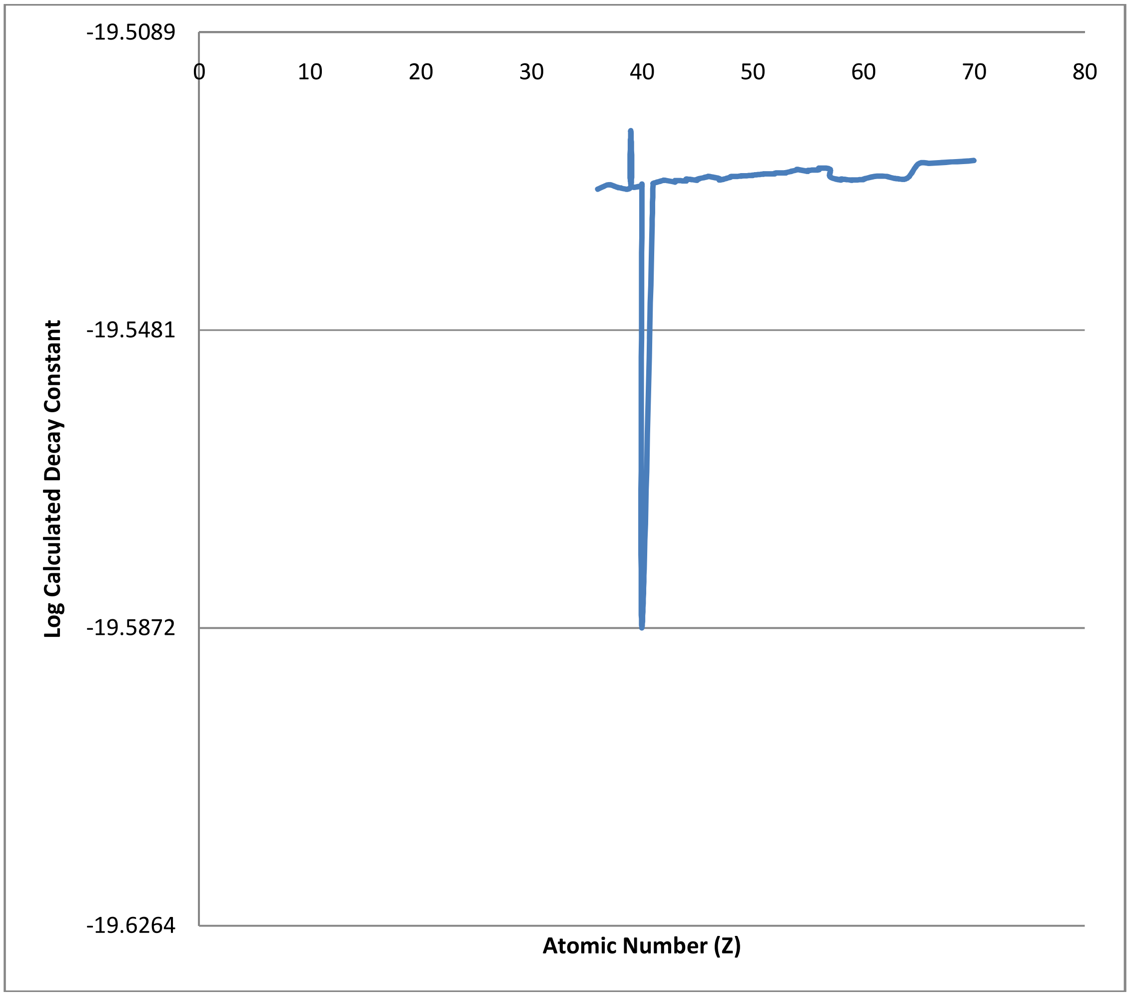

Figure 11: Logarithmic plot of Calculated Decay Constant versus Atomic Number (Z) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 11 represents the logarithms calculated Decay constant versus atomic number Z for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle emitters respectively.

Figure 11, the anomaly lies with low atomic number Z values for the medium gamma particle emitting nuclei. For the atomic number Z = 39 slightly high than the orders, also for atomic number Z = 40 is lower than the orders from atomic number Z = 57 to 65 makes a shape of w and also from atomic number Z = 65 diminishes with increasing value of logarithms of calculated decay constant. This shows that atomic number Z = 39 having a high logarithms calculated decay constant than order after the tunneling probability.

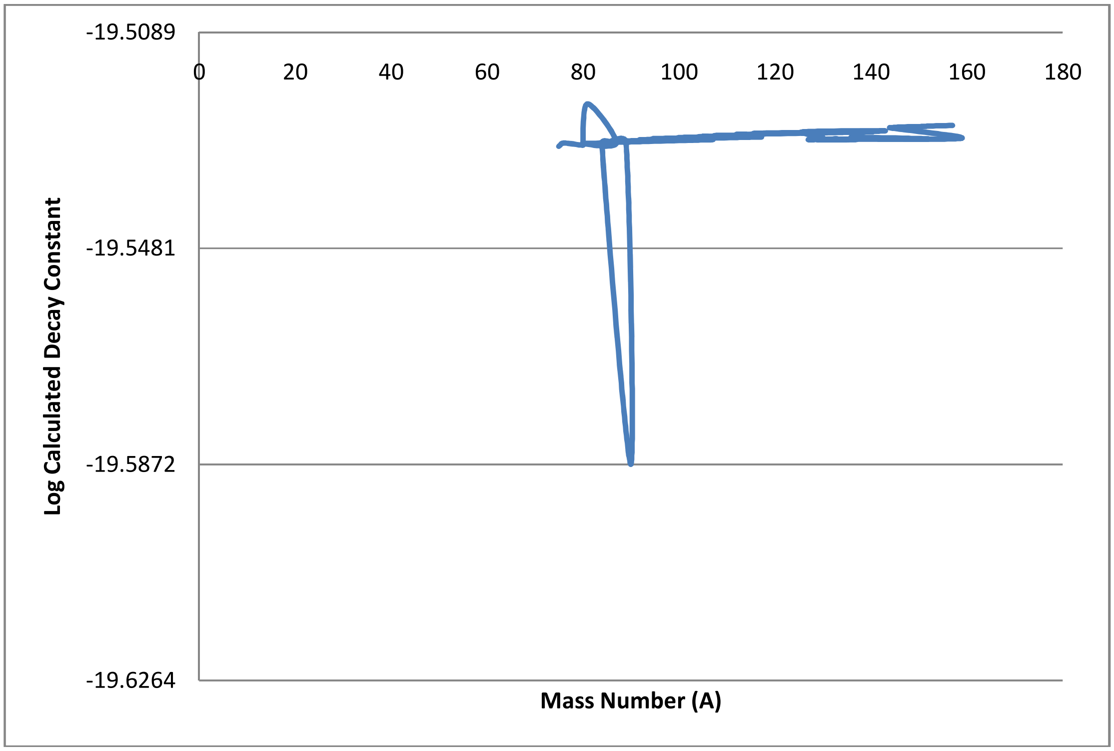

Figure 12:Logarithmic plot of Calculated Decay Constant versus Mass Number (A) of Gamma Particle for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass nuclei.

Figure 12 represents the logarithm decay constant versus mass number (A) for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass Gamma particle emitters respectively.
Figure12, the anomaly lies with low mass number (A) values for the medium gamma particle nuclei. The figure shows a shape of a cone, shape of an upside down cone, closed distance zigzag and also a shape of letter S. The reason for the shape of a cone is one of the mass number (A) have lower logarithm decay constant value than the orders, for the upside down cone is as the result of the middle valve of mass number (A) is having a high value of logarithm decay constant than orders, closed distance zigzag shape is as a result of fluctuation of values of logarithm decay constant at a closed distance and also for the shape of letter "S" is as a result of fluctuation of values of logarithm decay constant at a distance after the tunneling probability.

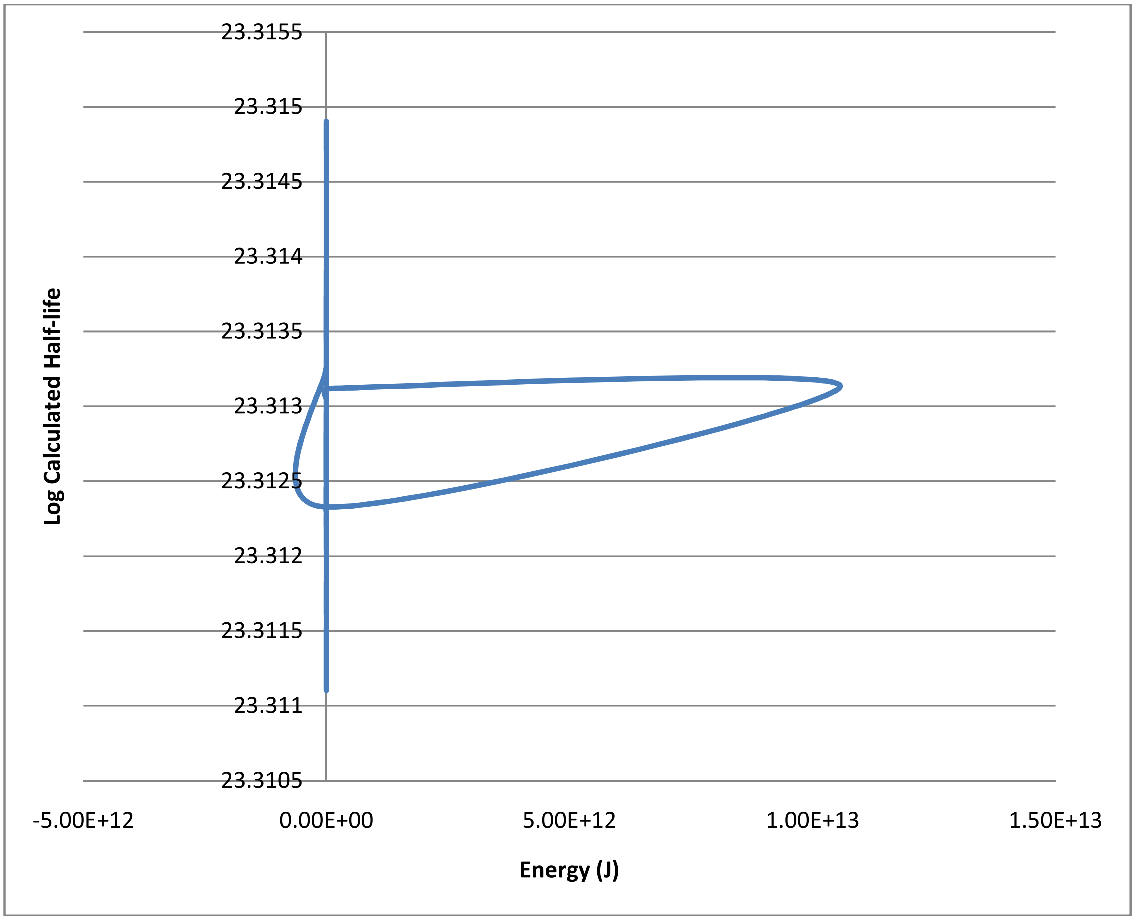

Figure 13: Logarithmic plot of Calculated Half-life versus Energy (J) of Gamma Particle for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass nuclei.

Figure 13represents the logarithm calculated Half-life versus Energy (J) for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass Gamma particle emitters respectively.Figure 13, the anomaly lies with high Energy (J) values for the medium gamma particle emitting nuclei. For the Energy (J) value 0.00 E+00 is having a vertical line on the logarithm calculated half-life from above 23.311 seconds to close to 23.315 seconds and also a shape of cone on the position of neutral equilibrium. The cone neutral equilibrium position lies on the higher Energy (J) than the order vertical line. The cone lies in between close to 23.3125 seconds to above 23.313 seconds of logarithm calculated half-life that is taken to tunneling probability.

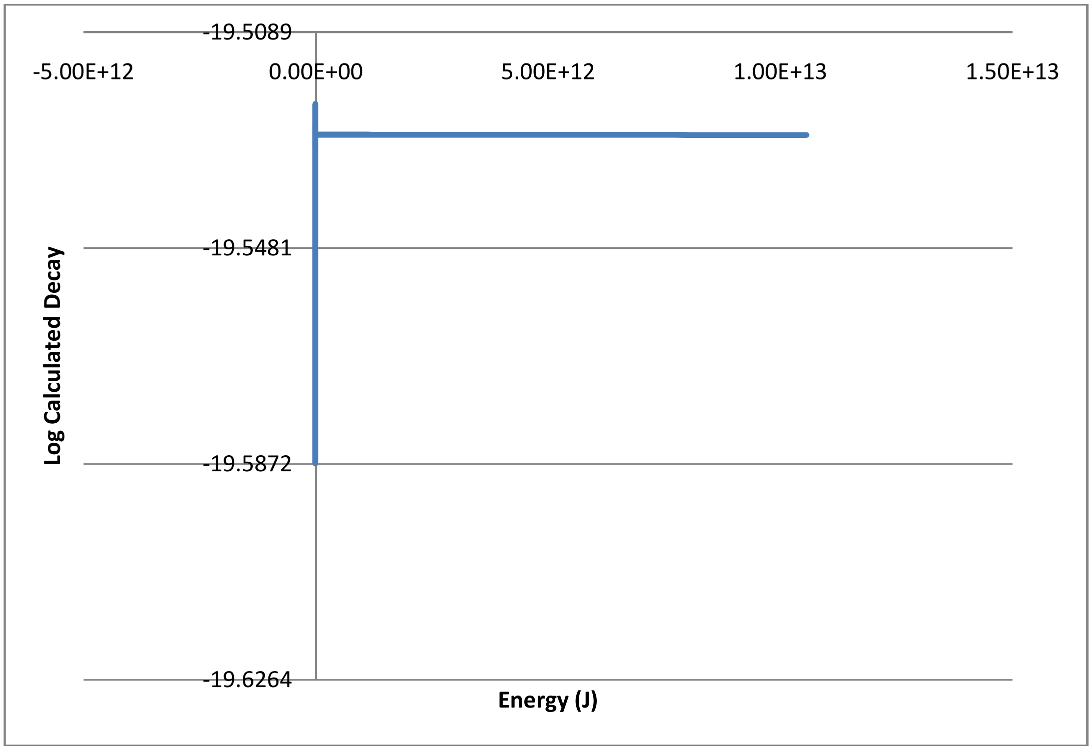

Figure 14: Logarithmic plot of Calculated Decay Constant versus Energy (J) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 14 represents the logarithm calculated decay versus Energy (J) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle emitters respectively.Figure 14, the anomaly lies with low Energy (J) values for the medium gamma particle emitting nuclei. The figure shows the vertical and horizontal lines. The values of Energy (J) at 0.00 E+00 is having vertical line on the logarithm of calculated decay which lies below -19.52 to above -19.58, the horizontal lines is as a result of the Energy (J) that have 1.00 E+13 that is after the tunneling probability.

CONCLUSION

It has been calculated analytically the quantum mechanical emission probability of barrier penetration $^{75}_{36}Kr$ to $^{157}_{70}Yb$ of the gamma particle decay of atomic nuclei. The SchrÖdinger's time-independent equation has been applied to a potential barrier whose height is greater than the gamma particle's energy. However, on application of barrier emission theory, the probability of the gamma particle crossing the barrier is in non-zero and this probability has been calculated.

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