International Journal of Science and Advanced Innovative Research
ISSN: 2536-7315 (Print) 2536-7323 (Online)
Volume 6, Number 1, March 2021
http://www.casirmediapublishing.com

AN ANALYSIS OF STEPS APPROACHING ZERO EMISSION THROUGH DOUBLE THICK BARRIER OF HEAVY GAMMA PARTICLE

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ABSTRACT

The goal of this work is to obtain tunneling probability of a gamma particle. The application of Schrödinger's equation in barrier penetration has been applied to gamma particle decay for light, medium and heavy nuclei. Gamma particle tunneling probability has been calculated analytically. Decay probability computed for each gamma particle emitting nucleus shows interesting variations. Log plot of calculated Decay constant plotted against atomic number (Z), mass number (A) and Energy for gamma particle emitting nucleus shows the variations interesting. Half-life which is a function of decay probability plotted against gamma particle energy or against atomic number of gamma particle emitting nucleus shows the variations of decay probabilities. Log plot of Calculated Half-life plotted against atomic number (Z), mass number (A) and Energy for gamma particle emitting nucleus shows interesting variations of decay probabilities. Calculated half-lives compared with experimental half-lives for each gamma particle emitting nucleus shows results which are in good agreement.

Key word: Emission, Gamma, Schrödinger's equation, Decay constant and Half-life.

INTRODUCTION

Gamma rays are produced during gamma decay, which normally occurs after other forms of decay occur, such as alpha or beta decay. An excited nucleus can decay by the emission of α or β particle. The daughter nucleus that results is usually left in an excited state. It can then decay to a lower energy state by emitting a gamma ray photon, in a process called gamma decay. The emission of a gamma ray from an excited nucleus typically requires only 10 −12 seconds (Rutherford, 1903). Gamma decay may also follow nuclear reactions such as neutron capture, nuclear fission, or nuclear fusion. Gamma decay is also a mode of relaxation of many excited states of atomic nuclei following other types of radioactive decay, such as beta decay, so long as these states possess the necessary component of nuclear spin. When high-energy gamma rays, electrons, or protons bombard materials, the excited atoms emit characteristic "secondary" gamma rays, which are products of the creation of excited nuclear states in the bombarded atoms (Villard, 1900a). Such transitions, a form of nuclear gamma fluorescence, form a topic in nuclear physics called gamma spectroscopy. Formations of fluorescent gamma rays are a rapid subtype of radioactive gamma decay. In certain cases, the excited nuclear state that follows the emission of a beta particle or other type of excitation may be more stable than average, and is termed a meta-stable excited state, if its decay takes (at least) 100 to 1000 times longer than the average 10−12 seconds. Such relatively long-lived excited nuclei are termed nuclear isomers, and their decays are termed isomeric transitions. Such nuclei have half-life that are more easily measurable, and rare nuclear isomers are able to stay in their excited state for minutes, hours, days, or occasionally far longer, before emitting a gamma ray. The process of isomeric transition is therefore similar to any gamma emission, but differs in that it involves the intermediate meta-stable excited state(s) of the nuclei. Meta-stable states are often characterized by high nuclear spin, requiring a change in spin of several units or more with gamma decay, instead of a single unit transition that occurs in only 10 −12 seconds. The rate of gamma decay is also slowed when the energy of excitation of the nucleus is small (Michael, 2007). An emitted gamma ray from any type of excited state may transfer its energy directly to any electrons, but most probably to one of the K shell electrons of the atom, causing it to be ejected from that atom in a process generally termed the photoelectric effect (external gamma rays and ultraviolet rays may also cause this effect) (Villard, 1900b). The photoelectric effect should not be confused with the internal conversion process, in which a gamma ray photon is not produced as an intermediate particle. Gamma rays are produced in many processes of particle physics. Typically, gamma rays are the products of neutral systems which decay through electromagnetic interactions (rather than a weak or strong interaction).

MATERIALS AND METHOD

Materials

The materials used are the*Schrödinger*'s equation.

Method

We now consider the beam of a particle incident upon a square potential barrier of height V_o presumed positive for now and width a. As mentioned above, this geometry is particularly important as it includes the simplest example of scattering phenomenon in which a beam of particles is 'deflected' by a local potential. Moreover, this one-dimensional geometry also provides a plat form to explore a phenomenon peculiar to quantum mechanics quantum tunneling (Dyson, 1951).

The potential energy variation in the case of a rectangular potential barrier shown in figure 1 is given by

$$\mathrm{V(x)} \quad = \left.\begin{matrix} 0, & x < 0 \\ V_o, & 0 < x < L \end{matrix}\right\}$$

$$\mathrm{V(x)} \quad = \left.\begin{matrix} 0, & x < 0 \\ V_o, & 0 < x < L \end{matrix}\right\} \qquad (1)$$

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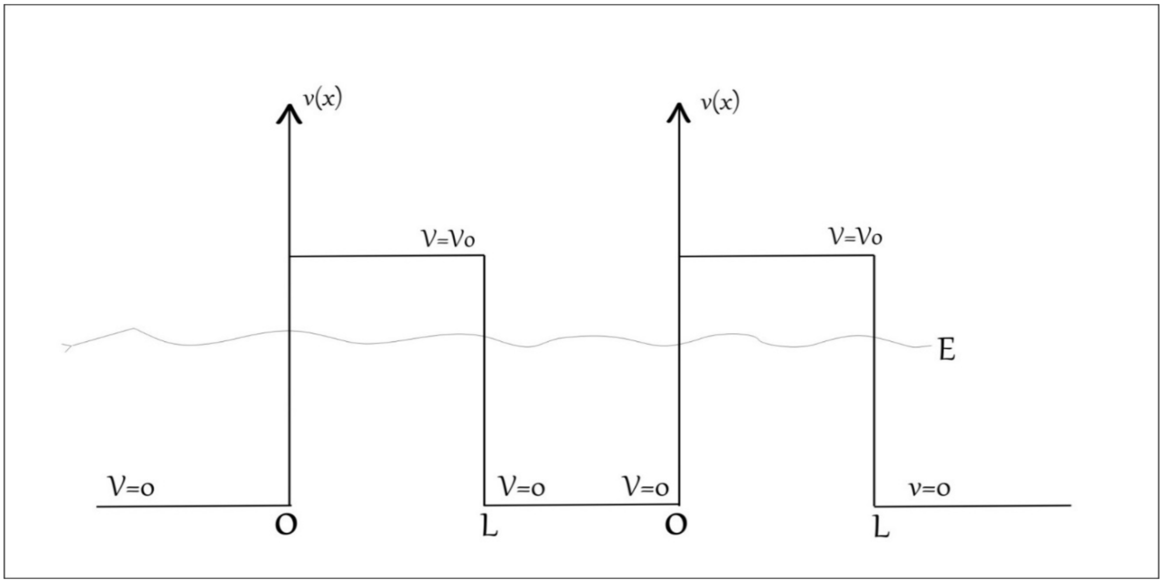

Fig. 1: a rectangular double thick potential barrier of width L and height V_0.

Let us consider two cases

(i) $0 < E < V_0$ Classically a particle of energy E if incident from the left would be reflected at the double thick barriers as it cannot enter $(0 < x < L)$ in which its K.E is negative. To describe the behavior of particle quantum mechanically, we will have to solve the $Schrödinger$ equation,

$$\left(\frac{d^2\varphi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\varphi(x)\right)\left(\frac{d^2\varphi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\varphi(x)\right) = 0$$

Or

$$\left(\frac{d^2\varphi(x)}{dx^2} + k^2\varphi(x)\right)\left(\frac{d^2\varphi(x)}{dx^2} + k^2\varphi(x)\right) = 0, k^2 = \frac{2m\,E}{\hbar^2}, x < 0 \; and \; x > L \quad (2)$$

And

$$\left(\frac{d^2\varphi(x)}{dx^2} + \gamma^2\varphi(x)\right)\left(\frac{d^2\varphi(x)}{dx^2} + \gamma^2\varphi(x)\right) = 0, \gamma^2 = \frac{2m(V_0 - E)}{\hbar^2}, 0 < x < L \quad (3)$$

The general solutions of these equations are given by

$$\varphi^2(x) = (A\,e^{ikx} + Be^{-ikx})(A\,e^{ikx} + Be^{-ikx}), x < 0 \quad (4)$$

$$\varphi^2(x) = (Ce^{\alpha x} + De^{-\alpha x})(Ce^{\alpha x} + De^{-\alpha x}), 0 < x < L \quad (5)$$

$$\varphi^2(x) = (F\,e^{ikx} + Ge^{-ikx})(F\,e^{ikx} + Ge^{-ikx}), x < L \quad (6)$$

Notice that we allow for waves traveling in both the directions for $x < 0$ representing the incident and reflected waves. We must also allow for $e^{\gamma x}$ and $e^{-\gamma x}$term in the region $0 < x < L$ because x is finite and there is no danger of φ becoming infinite. We have only a wave traveling from left to right of $x > L$ as there cannot be any wave travelling from right to left (reflected wave) since there is no discontinuity in the potential. Hence we must set G=0. The solution, therefore would be

$$\varphi^2(x) = \left(F\,e^{ikx}\right)\left(F\,e^{ikx}\right), x > L \quad (7)$$

The continuity conditions (that is, φ *and* $d\varphi/dx$ *be* continuous) at x = 0 and at x = L yield

At x = 0, $A + B = C + D$ *and* $ik(A - B) = \alpha(C + D)$ (8)

At $x > L$,

$$(Ce^{\gamma L} + De^{-\gamma L})(Ce^{\gamma L} + De^{-\gamma L}) = \left(F\,e^{ikL}\right)^2 \text{ and } \gamma(Ce^{\gamma L} + De^{-\gamma L})\gamma(Ce^{\gamma L} + De^{-\gamma L}) = \left(ikF\,e^{ikL}\right)^2 \quad (9)$$

There are number of ways of solving these equations. If solution leads to

$$\left.\begin{aligned} C^2 &= \left(\frac{[(\gamma+ik)A+(\gamma-ik)B]}{2\gamma}\right)\left(\frac{[(\gamma+ik)A+(\gamma-ik)B]}{2\gamma}\right) \\ D^2 &= \left(\frac{[(\gamma-ik)A+(\gamma+ik)B]}{2\gamma}\right)\left(\frac{[(\gamma-ik)A+(\gamma+ik)B]}{2\gamma}\right) \end{aligned}\right\} x = 0 \quad (10)$$

Similarly

$$\left.\begin{aligned} C^2 &= \left(\frac{[(\gamma+ik)Ae^{-(\gamma-ik)L}F]}{2\gamma}\right)\left(\frac{[(\gamma+ik)Ae^{-(\gamma-ik)L}F]}{2\gamma}\right) \\ D^2 &= \left(\frac{[(\gamma-ik)Ae^{(\gamma+ik)L}F]}{2\gamma}\right)\left(\frac{[(\gamma-ik)Ae^{(\gamma+ik)L}F]}{2\gamma}\right) \end{aligned}\right\} x = L \quad (11)$$

Equating the values of C^2 and D^2 to each other yield

$$\left((\gamma + ik)A + (\gamma - ik)B\right)^2 = \left((\gamma + ik)Ae^{-(\gamma-ik)L}F\right)\left((\gamma + ik)Ae^{-(\gamma-ik)L}F\right) \quad (12)$$

And

$$\left((\gamma - ik)A + (\gamma + ik)B\right)^2 = \left((\gamma - ik)Ae^{(\gamma+ik)L}F\right)\left((\gamma - ik)Ae^{(\gamma+ik)L}F\right) \quad (13)$$

And so

$$(B/A)^2 = \left(\frac{(\gamma-ik)}{(\gamma+ik)}\left[e^{(\gamma+ik)L}\,F/A - 1\right]\right)\left(\frac{(\gamma-ik)}{(\gamma+ik)}\left[e^{(\gamma+ik)L}\,F/A - 1\right]\right) \quad (14)$$

Putting the above value of $(B/A)^2$ in to (3.14) yields

$$\left(\frac{(\gamma + ik)+(\gamma - ik)^2}{(\gamma + ik)}\left[e^{(\gamma+ik)L}\frac{F}{A} - 1\right]\right)^2 = \left((\gamma + ik)e^{(\gamma+ik)}\frac{F}{A}\right)\left((\gamma + ik)e^{(\gamma+ik)}\frac{F}{A}\right)$$

Or

$$\left(\frac{(\gamma + ik)^2 + (\gamma - ik)^2}{(\gamma + ik)}\left[e^{(\gamma+ik)L}\frac{F}{A} - 1\right]\right)^2 = \left((\gamma + ik)^2e^{(\gamma+ik)}\frac{F}{A}\right)\left((\gamma + ik)^2e^{(\gamma+ik)}\frac{F}{A}\right)$$

Or

$$\begin{aligned}((\gamma + ik)^2 - (\gamma - ik)^2)^2 \\ = \left(\frac{F}{A}\left[(\gamma + ik)^2e^{(\gamma+ik)L} - (\gamma - ik)^2e^{(\gamma+ik)L}\right]\right)\left(\frac{F}{A}\left[(\gamma + ik)^2e^{(\gamma+ik)L}\right.\right. \\ \left.\left. - (\gamma - ik)^2e^{(\gamma+ik)L}\right]\right)\end{aligned}$$

$$\left(\frac{F}{A}\right)^2 = \left(\frac{4ik\gamma}{[(\gamma + ik)^2e^{(\gamma+ik)L} - (\gamma - ik)^2e^{(\gamma+ik)L}]}\right)\left(\frac{4ik\gamma}{[(\gamma + ik)^2e^{(\gamma+ik)L} - (\gamma - ik)^2e^{(\gamma+ik)L}]}\right)$$

After multiplying the numerator and denominator $e^{(\gamma-ik)L}$

$$\left(\frac{F}{A}\right)^2 = \left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma+ik)^2-(\gamma-ik)^2e^{2\gamma L}]}\right)\left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma+ik)^2-(\gamma-ik)^2e^{2\gamma L}]}\right)$$

$$= \left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma+ik)^2-(\gamma-ik)^2e^{2\gamma L}]}\right)\left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma+ik)^2-(\gamma-ik)^2e^{2\gamma L}]}\right)$$

$$= \left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\ (1+e^{2\gamma L})]}\right)\left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\ (1+e^{2\gamma L})]}\right) \quad (15)$$

Putting the value of $\left(\frac{F}{A}\right)^2$ from above into equation (5), we get

$$\left(\frac{F}{A}\right)^2 = \left(\frac{(\gamma^2-k^2)(e^{2\gamma L}-1)}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\ (1+e^{2\gamma L})]}\right)\left(\frac{(\gamma^2-k^2)(e^{2\gamma L}-1)}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\ (1+e^{2\gamma L})]}\right) \quad (16)$$

It may be mentioned here that in case one is interest in finding $C/A$and D/A, this can be achieved by substituting the value of $\left(\frac{F}{A}\right)^2$ from (15) into equations (11).

From (7), the reflection coefficient (or the probability of reflection) is given by

$$R = \frac{j_{ref}}{j_{inc}} = \left(\frac{\hbar k/m\,|B|^2}{\hbar k/m\,|A|^2}\right)^2 = (|B/A|^2)^2 = \left[\left(\frac{B}{A}\right)*\left(\frac{B}{A}\right)\right]^2$$

$$= \left(\frac{(\gamma^2-k^2)^2(e^{2\gamma L}-1)^2}{[(\gamma^2-k^2)(1-e^{2\gamma L})^2+4k^2\gamma^2(1+e^{2\gamma L})^2]}\right)\left(\frac{(\gamma^2-k^2)^2(e^{2\gamma L}-1)^2}{[(\gamma^2-k^2)(1-e^{2\gamma L})^2+4k^2\gamma^2(1+e^{2\gamma L})^2]}\right)$$

After dividing the numerator and denominator by $(1-e^{2\gamma L})^2$ one gets

$$R^2 = \left(\frac{(\gamma^2-k^2)^2}{\left[(\gamma^2-k^2)^2+4k^2\gamma^2\left\{\frac{(1+e^{2\gamma L})}{(1-e^{2\gamma L})}\right\}^2\right]}\right)\left(\frac{(\gamma^2-k^2)^2}{\left[(\gamma^2-k^2)^2+4k^2\gamma^2\left\{\frac{(1+e^{2\gamma L})}{(1-e^{2\gamma L})}\right\}^2\right]}\right)$$

Or

$$= \frac{(\gamma^2-k^2)^2}{(\gamma^2-k^2)+4k^2\gamma^2\left(\frac{1+e^{4\gamma L}+2e^{2\gamma L}}{1+e^{4\gamma L}-2e^{2\gamma L}}-1\right)+4k^2\gamma^2}$$

$$= \left(\frac{(\gamma^2-k^2)^2}{(\gamma^2-k^2)+4k^2\gamma^2\left\{\frac{4}{(e^{2\gamma L}+e^{-2\gamma L}-2)}\right\}}\right)\left(\frac{(\gamma^2-k^2)^2}{(\gamma^2-k^2)+4k^2\gamma^2\left\{\frac{4}{(e^{2\gamma L}+e^{-2\gamma L}-2)}\right\}}\right)$$

$$R^2 = \frac{(\gamma^2-k^2)^2}{(\gamma^2-k^2)+\frac{4k^2\gamma^2\ 1}{\left(\frac{e^{\gamma L}-e^{-\gamma L}}{2}\right)^2}}$$

$$= \frac{(\gamma^2-k^2)^2}{\left[(\gamma^2-k^2)+\frac{4k^2\gamma^2\gamma^2}{\sin\hbar\gamma L}\right]} \quad (17)$$

After substituting the values of $\gamma^2$and k^2, one gets

$$R^2 = \left(\frac{V_O^2}{\left[V_O^2 + \frac{4E(V_O - E)}{\sin\hbar\,\alpha L}\right]}\right)\left(\frac{V_O^2}{\left[V_O^2 + \frac{4E(V_O - E)}{\sin\hbar\,\alpha L}\right]}\right)$$

$$= \left[1 + \frac{4E(V_O-E)}{V_O^2 \sin\hbar\alpha L}\right]^{-1} \times \left[1 + \frac{4E(V_O-E)}{V_O^2 \sin\hbar\alpha L}\right]^{-1} \qquad (18)$$

The probability of finding the particle in a region $X > 0$, is given the name transmission coefficient T and using equation (15) we have

$$T^2 = \frac{j_{ref}}{j_{inc}} = \left(\frac{\hbar k/m\,|F|^2}{\hbar k/m\,|A|^2}\right)^2 = \left(\left|\frac{F}{A}\right|^2\right)^2 = \left[\left(\frac{F}{A}\right) * \left(\frac{F}{A}\right)\right]^2$$

$$= \left(\frac{16k^2\gamma^2 e^{2\gamma L}}{[(\gamma^2 - k^2)(1 - e^{2\gamma L})^2 + 4k^2\gamma^2(1 + e^{2\gamma L})^2]}\right)\left(\frac{16k^2\gamma^2 e^{2\gamma L}}{[(\gamma^2 - k^2)(1 - e^{2\gamma L})^2 + 4k^2\gamma^2(1 + e^{2\gamma L})^2]}\right)$$

$$= \left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 4k^2\gamma^2(e^{2\gamma L} + e^{2\gamma L} + 2)}\right)\left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 4k^2\gamma^2(e^{2\gamma L} + e^{2\gamma L} + 2)}\right)$$

Adding and subtracting $4k^2\gamma^2(e^{2\gamma L} + e^{-2\gamma L} - 2)$ from the denominator, one get

$$= \left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 16k^2\gamma^2}\right)\left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 16k^2\gamma^2}\right)$$

$$= \left(\frac{4k^2\gamma^2}{\left[(\gamma^2 - k^2)^2\left(\frac{e^{\gamma L} - e^{\gamma L}}{2}\right)^2 + 4k^2\gamma^2\right]}\right)\left(\frac{4k^2\gamma^2}{\left[(\gamma^2 - k^2)^2\left(\frac{e^{\gamma L} - e^{\gamma L}}{2}\right)^2 + 4k^2\gamma^2\right]}\right)$$

$$= \left(\frac{4k^2\gamma^2}{(\gamma^2-k^2)^2 \sin\hbar^2\gamma L + 4k^2\gamma^2}\right)\left(\frac{4k^2\gamma^2}{(\gamma^2-k^2)^2 \sin\hbar^2\gamma L + 4k^2\gamma^2}\right) \qquad (19)$$

Putting the value of γ^2 and k^2 one gets

$$T^2 = \left[1 + \frac{V_O^2 \sin\hbar^2\alpha L}{4E(V_O-E)}\right]^{-1} \times \left[1 + \frac{V_O^2 \sin\hbar^2\alpha L}{4E(V_O-E)}\right]^{-1} \qquad (20)$$

One may, however check that $R + T = 1$. There are two interesting situations in which equations (17) to (20) become simpler considering the purely formal limit in which $\hbar \to 0$. The quantity $\hbar$ is a physical constant, but we can consider as a mathematical variable in order to examine the classical limit of our formulas. As $\hbar \to 0$, k and γ approach infinity and hence $T \to 0$, $R \to 1$, which is of course, the proper behavior of a classical particle with $E < V_O$. The other interesting limit occurs for high and wide barrier, that is, when $\gamma \gg 1$. In that case $\sin\hbar^2 \gamma L \approx \frac{1}{2}e^{\gamma L}$, hence form (3.20) after neglecting 1 in comparison to the other which is very large, one gets

$$T^2 = \left(\frac{4E(V_O - E)}{V_O^2 \left[\frac{1}{2} e^{-2\left\{ \frac{2m(V_O - E)}{\hbar^2} \right\}^{\frac{1}{2}} L} \right]^2} \right) \left(\frac{4E(V_O - E)}{V_O^2 \left[\frac{1}{2} e^{-2\left\{ \frac{2m(V_O - E)}{\hbar^2} \right\}^{\frac{1}{2}} L} \right]^2} \right)$$

$$= \left(16 \frac{E}{V_O} (1 - \frac{E}{V_O}) e^{-2\left\{ \frac{2m(V_O - E)}{\hbar^2} \right\}^{\frac{1}{2}} L} \right) \left(16 \frac{E}{V_O} (1 - \frac{E}{V_O}) e^{-2\left\{ \frac{2m(V_O - E)}{\hbar^2} \right\}^{\frac{1}{2}} L} \right) \quad (21)$$

From equation (21) transmission coefficient would be given by

$$T^2 = \left(16 \frac{E}{V(r_0)} \left[1 - \frac{E}{V(r_0)} \right] e^{-2\gamma L} \right) \left(16 \frac{E}{V(r_0)} \left[1 - \frac{E}{V(r_0)} \right] e^{-2\gamma L} \right) \quad (22)$$

$\gamma L \gg 1$, the most important factor in the above equation is the exponential. The factor in front of the exponential which is of the order of 2 is not significant since its variation with V and E is negligible as compared to the variation in exponential itself(Chaddha, 1983). Hence we can write

$$InT^2 \simeq -4\gamma L \quad (23)$$

For a rectangular double thick potential barrier of thickness dx, we can write

$$InT^2 \simeq -4\gamma dx \quad (24)$$

Where

$$\gamma^4 = \left(\frac{2m}{\hbar^2} [V(x) - E] \right) \left(\frac{2m}{\hbar^2} [V(x) - E] \right)$$

$$= \left(\frac{2m}{\hbar^2} \left[\left(\frac{2Ze^2}{4\pi\epsilon_0 x} \right) - E \right] \right) \left(\frac{2m}{\hbar^2} \left[\left(\frac{2Ze^2}{4\pi\epsilon_0 x} \right) - E \right] \right) \quad (25)$$

makingγ a function of x

Equation (25) expression for the transmission coefficient or tunneling probability of a rectangular barrier. The actual barrier encountered by gamma particle has an exponential tail. We can approximate it as consisting of many rectangular barrier of decreasing height and obtain the total probability by summing the tunneling probability of each barrier the region between $r_0$andr_1. In this entire region, of course $E < V$. Hence taking the summation over all the rectangular potential barriers, we gets

$$In\, T^2 = \left(-2 \int_{r_0}^{r_1} \gamma(x) dx \right) \left(-2 \int_{r_0}^{r_1} \gamma(x) dx \right) \quad (26)$$

From equation (3.25) that γ can be while is a function of x

$$\gamma = \left(\left(\frac{2m}{\hbar} \right)^{\frac{1}{2}} \left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x} \right) - E \right)^{\frac{1}{2}} \right) \left(\left(\frac{2m}{\hbar} \right)^{\frac{1}{2}} \left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x} \right) - E \right)^{\frac{1}{2}} \right) \quad (27)$$

Substituting equation (27) in to equation (26)

$$In\, T^2 = \left(-2 \int_{r_0}^{r_1} \left(\frac{2m}{\hbar} \right)^{\frac{1}{2}} \left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x} \right) - E \right)^{\frac{1}{2}} dx \right) \left(-2 \int_{r_0}^{r_1} \left(\frac{2m}{\hbar} \right)^{\frac{1}{2}} \left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x} \right) - E \right)^{\frac{1}{2}} dx \right)$$

(28)

Making use of equation (21), leads to

$$In\,T^2 = \left(-2\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\int_{r_0}^{r_1}\left(\frac{r_0}{x}-1\right)^{\frac{1}{2}} dx\right)\left(-2\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\int_{r_0}^{r_1}\left(\frac{r_0}{x}-1\right)^{\frac{1}{2}} dx\right) \quad (29)$$

Putting $x = r_1\cos^2\theta, dx = r_1 2\cos\theta\ (-\sin\theta\ d\theta)$ and also changing the limits to $\theta(\,at\ x = r_0, \theta_0 = \cos^{-1}\left(\frac{r_0}{x}\right)^{\frac{1}{2}}\ and\ at\ x = r_0, \theta_0 = 0)$, one gets

$$In\,T^2 = \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\left(\frac{1}{\cos^2\theta}-1\right)^{\frac{1}{2}}\sin\theta\cos\theta\, dx\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\left(\frac{1}{\cos^2\theta}-1\right)^{\frac{1}{2}}\sin\theta\cos\theta\, dx\right) \quad (30)$$

Since

$$\left(\left(\frac{1}{\cos^2\theta}-1\right)^{\frac{1}{2}}\right)^2 = \left(\frac{(1-\cos^2\theta)^{\frac{1}{2}}}{\cos\theta}\right)^2 = \left(\frac{\sin\theta}{\cos\theta}\right)^2$$

The double thick potential barrier is on the x coordinate

$$In\,T^2 = \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\sin^2\theta\, d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\sin^2\theta\, d\theta\right) \quad (31)$$

Using trigonometric rule and integrating

$$In\,T^2 = \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\frac{1-\cos 2\,\theta}{2}\, d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\frac{1-\cos 2\,\theta}{2}\, d\theta\right) \quad (32)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\left(\frac{1}{2}-\frac{\cos 2\,\theta}{2}\right) d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\left(\frac{1}{2}-\frac{\cos 2\,\theta}{2}\right) d\theta\right) \quad (33)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\frac{1}{2}d\theta - \frac{1}{2}\int_0^{\theta_0}\cos 2\,\theta\ d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\int_0^{\theta_0}\frac{1}{2}d\theta - \frac{1}{2}\int_0^{\theta_0}\cos 2\,\theta\ d\theta\right) \quad (34)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\frac{1}{2}\int_0^{\theta_0}d\theta - \frac{1}{2}\int_0^{\theta_0}\cos 2\,\theta\ d\theta\right)\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\frac{1}{2}\int_0^{\theta_0}d\theta - \frac{1}{2}\int_0^{\theta_0}\cos 2\,\theta\ d\theta\right)\right) \quad (35)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1(\theta_0 - (\cos^2\theta - \sin^2\theta))\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1(\theta_0 - (\cos^2\theta - \sin^2\theta))\right) \quad (36)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\theta_0 - \left(\cos^2\theta - (1-\cos^2\theta)\right)\right)\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\theta_0 - \left(\cos^2\theta - (1-\cos^2\theta)\right)\right)\right) \quad (37)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\theta_0 - (\cos^2\theta + \cos^2\theta - 1)\right)\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\theta_0 - (\cos^2\theta + \cos^2\theta - 1)\right)\right) \quad (38)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\theta_0 - (2\cos^2\theta - 1)\right)\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\theta_0 - (2\cos^2\theta - 1)\right)\right) \quad (39)$$

After putting the value of E

$$In\,T^2 = \left(-2\left[\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right]^{1/2}\left[\cos^{-1}\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} - \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\left(1 - \frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right)\left(-2\left[\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right]^{1/2}\left[\cos^{-1}\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} - \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\left(1 - \frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right) \quad (40)$$

Because of the fact that the potential barrier is relatively wide, $r_1 \gg r_0$,

$$\cos^{-1}\left(\frac{r_0}{x}\right)^{\frac{1}{2}} \approx \frac{\pi}{2} - \left(\frac{r_0}{x}\right)^{\frac{1}{2}}$$

As $\cos\left\{\frac{\pi}{2} - \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right\} = \sin\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} \simeq \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}$

If $\left(\frac{r_0}{r_1}\right) \ll 1$

Also

$$\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} \approx \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} \approx 1$$

Hence from equation (39)

$$In\,T^2 = \left(-2\left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right)^{\frac{1}{2}}\left[\pi/2 - 2\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right)\left(-2\left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right)^{\frac{1}{2}}\left[\pi/2 - 2\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right) (41)$$

Replacing r_1 by $r_1 = \frac{2Ze^2}{4\pi\epsilon_0}$ and simplifying

$$InT^2 = \left(4\,\frac{e}{\hbar}\left(\frac{m}{\pi\epsilon_0}\right)^{\frac{1}{2}} Z^{\frac{1}{2}} r_0^{\frac{1}{2}} - \frac{e^2}{\hbar\epsilon_0}\left(\frac{m}{2}\right)^{\frac{1}{2}} Z\,E^{-\frac{1}{2}}\right)\left(4\,\frac{e}{\hbar}\left(\frac{m}{\pi\epsilon_0}\right)^{\frac{1}{2}} Z^{\frac{1}{2}} r_0^{\frac{1}{2}} - \frac{e^2}{\hbar\epsilon_0}\left(\frac{m}{2}\right)^{\frac{1}{2}} Z\,E^{-\frac{1}{2}}\right) \quad (42)$$

$$InT^2 = 4^2\left(\frac{e}{\hbar}\right)^2\left(\frac{m}{\pi\epsilon_0}\right) Z^{\frac{1}{4}} r_0^{\frac{1}{4}} - \frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right) Z^2 E^{-\frac{1}{4}} \quad (43)$$

Equation (43) gives the natural logarithm of the tunneling probability of the gamma particle.

RESULTS

We assess the ability of gamma particle in tunneling through a barrier, its relationship with decay constant and half-life using equation (43)

$$\underbrace{In\,T^2}_{K_1} = \underbrace{\underbrace{4^2\frac{e^2}{\hbar^2}\left(\frac{m}{\pi\epsilon_0}\right)}_{I_1} Z^{\frac{1}{4}} r_0^{\frac{1}{4}}}_{J_1} - \underbrace{\underbrace{\frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right)}_{I_2} Z^2 E^{-\frac{1}{4}}}_{J_2} \quad 43$$

The constant I_1 and I_2 are to be calculated while:
Z = atomic number of the daughter nucleus (the gamma emitting nucleus)
$r_o = 1.1\left(A_d^{\frac{1}{2}} + A_\gamma^{\frac{1}{2}}\right) \times 10^{-15} m$ (for each nucleus) 44
E = Potential energy of the emitted gamma particle
= or energy of decay for each nucleus
m = mass of gamma particle
1 atomic mass unit $= 1.66 \times 10^{-27} kg$

$$\left.\begin{aligned} e &= 1.6 \times 10^{-19} C \\ \hbar &= 1.05477 \times 10^{-34} Js \\ \epsilon_o &= 8.85 \times 10^{-12} Farad/m \end{aligned}\right\} \text{all are in S.I unit}$$

To keep equation (3.64) as simple as possible we calculate the constant I_1 and I_2
$I_1 = 4^2 \frac{e^2}{\hbar^2}\left(\frac{m}{\pi\epsilon_0}\right)$ 45
$I_1 = 8.792420946 \times 10^{15}$ 4.6
$I_2 = \frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right)$ 47
$I_2 = 2.496984634 \times 10^{-12}$ 48
$K_1 = T^2$ 49
Let T^2 be DT
$K_1 = DT$ 50
$InDT = 8.792420946 \times 10^{15} Z^{\frac{1}{4}} r_0^{\frac{1}{4}} - 2.496984634 \times 10^{-12} Z^2 E^{-\frac{1}{4}}$ 51
Equation (51) is used to get the result for tunneling for every γ emitting nucleus as show in Table 4.1
The decay probability per unit time or constant we write
$\lambda = \Gamma T$ 52
Where Γ = number of time per second gamma particle within a nucleus strikes the potential barrier
T = the probability of transmission through the barrier.
Assume only one gamma particle exists within a nucleus moving to and fro in the nuclear diameter
$\Gamma = \frac{v}{2r_0}$ 53
Where $v = \gamma$ particle velocity when it finally leaves the nucleus
$\lambda = \frac{v}{2r_0} DT$ 54

$$v = 10^7 ms^{-1}, r_0 = 10^{-14} m$$

$\lambda = \frac{10^7}{2\times10^{-14}} DT \simeq 10^{-21} DT$ 55
Equation (55) can be used to get the result for decay probability per unit time.
The half life $t_{\frac{1}{2}}$ is the time taken for half the original number of atom present to decay.
Mathematically half-life $t_{\frac{1}{2}}$ can written as

$$t_{\frac{1}{2}} = \frac{In\ 2}{\lambda} \qquad 56$$

Substitute equation (56) into (55) gives

Table 1: Heavy gamma particle emitting nuclei and their decay probability

S/N	Nucleus (name)	Mass No. (A)	Z	Mass Excess A(KeV)	r_0	E $\gamma(J)$	In DT (E12)	DT	Decay constant (E-20)	Half-life $t_{\frac{1}{2}}$ (E23)
1	Lu	158	71	358.2	13.8267856	3.893290839E-14	6.347922152	29.47914566	2.9479134866	2.042905002
2	Lu	160	71	243.4	13.9140217	2.710883991E-14	6.383794036	29.48478371	2.948478371	2.043295511
3	Lu	163	71	163.1	14.04385987	1.360248528E-14	8.786070449	29.80418868	2.980418868	2.065430276
4	Hf	165	72	180.0	14.12975584	2.707679637E-14	8.830295977	29.80920965	2.980920965	2.065778229
5	Hf	167	72	315.2	14.21513278	3.117837026E-14	7.417436994	29.63485469	2.963485469	2.05369543
6	Ta	163	73	396.0	14.04385987	4.16566098E-14	8.84701125	29.81110081	2.981110081	2.065909286
7	Ta	166	73	158.7	14.1725086	5.863968918E-15	8.867280019	29.81338922	2.981338922	2.066067873
8	W	170	74	316.2	14.34224529	1.157986948E-14	8.924228643	29.81979101	2.981979101	2.066511517
9	W	175	74	270.3	14.55163221	1.57135931E-14	8.956623723	29.82341485	2.982341485	2.066762649
10	Re	168	75	363.2	14.25762954	3.80356891E-14	8.940989353	29.8216674	2.98216674	2.066641551
11	Re	174	75	243.4	14.50999655	3.276452579E-14	8.980290438	29.82605334	2.982605334	2.066945496
12	Os	171	76	189.9	14.38456651	1.421131265E-14	8.990514697	29.82719121	2.982714121	2.070123509
13	Os	178	76	969.0	14.67583047	1.167025945E-13	9.035715368	29.83220621	2.983220621	2.06737189
14	Os	183	76	1101.9	14.88052418	1.179683146E-13	9.067058708	29.83566904	2.983566904	2.067611864
15	Ir	177	77	184.1	14.63454817	9.468867845E-15	9.05889862	29.83476866	2.983476866	2.067549468
16	Ir	182	77	273.0	14.83981132	4.051906342E-14	9.090509969	29.83825212	2.983825212	2.067790872
17	Pt	176	78	227.0	14.59314908	1.04588925E-14	9.081746552	29.83728764	2.983728764	2.067724033
18	Pt	186	78	687.2	15.00199987	6.814060057E-14	9.144698938	29.84419548	2.984419548	2.068202747
19	Au	189	79	166.7	15.12249979	9.645707346E-15	9.192235492	29.84938028	2.984938028	2.068562053
20	Au	195	79	261.8	15.36066405	3.992625832E-14	9.225215819	29.85328684	2.98532868654	2.068832778
21	Hg	180	80	301.0	14.75804865	4.727704777E-14	9.161693857	29.8460522	2.98460522	2.068331417
22	Hg	184	80	236.4	14.92112596	1.3057745E-14	9.190335546	29.84917356	2.9849177356	2.068547728
23	Ti	184	81	366.5	14.92112596	3.797160201E-14	9.218921694	29.85227919	2.985227919	2.068762948
24	Ti	187	81	299.3	15.04227376	4.377148384E-14	9.237577585	29.8543008	2.9854308	2.068903045

25	Ti	192	81	422.9	15.24204711	4.490902772E-14	9.058940589	29.8347733	2.98347733	2.06754979
26	Pb	192	82	1195.4	15.24204711	1.248096117E-13	9.090509989	29.83825213	2.983825213	2.067790872
27	Pb	202	82	960.7	15.63393741	9.496104857E-14	9.081746551	29.83728764	2.983728764	2.067721034
28	Pb	204	82	899.2	15.71114254	7.36360687E-14	9.144698939	29.84419548	2.984419548	2.068202747
29	Bi	196	83	1049.0	15.40	7.485372346E-14	1.19223549	29.84938028	2.984938023	2.068562053
30	Bi	198	83	1063.5	15.47837201	8.980203767E-14	9.228215886	29.85328685	2.985328685	2.068832779
31	Po	205	84	82.4	16.74960317	8.347343735E-15	9.165121001	29.4864262	2.98464262	2.068357336
32	Po	216	84	805.0	16.1666323	3.135460996E-14	9.91033572	29.84917357	2.984917357	2.068547728
33	At	210	85	1181.4	15.94051442	1.425296926E-14	9.218921695	29.85227919	2.985227919	2.068762948
34	Rn	211	86	674.1	15.97842295	2.068410814E-14	9.237565519	29.85429949	2.985429949	2.068902955
35	Rn	219	86	271.23	16.27851345	2.039571703E-15	9.563938406	29.88902073	2.988902073	2.071309136
36	Rn	222	86	510.0	16.38963087	4.309856937E-14	9.563898592	29.8890168	2.98890165	2.071308848
37	Fr	208	87	635.8	15.86442561	2.21901581E-14	9.5630029434	29.88546892	2.988546892	2.071062996
38	Fr	229	87	310.1	16.602055	2.369620227E-14	9.618272975	29.89468584	2.98946858	2.071701729
39	Ra	222	88	324.2	16.38963087	4.479687731E-14	9.619043633	29.89476516	2.989476596	2.071707281
40	Ra	223	88	269.4	16.42650298	8.203147776E-15	9.785231677	29.91189539	2.991185939	2.072891856
41	Ac	223	89	191.3	16.42650498	1.623005605E-14	9.673788857	29.90044116	2.990044116	2.072100573
42	Ac	228	89	911.0	16.60963576	1.16125807E-13	9.69353558	29.90248034	2.99048034	2.072241888
43	Th	225	90	322.0	16.50	2.094045731E-14	9.705982822	29.9037636	2.99037636	2.072330817
44	Th	226	90	111.1	16.53662602	1.379474655E-14	9.711377264		2.990431923	2.072369322
45	Pa	228	91	911.20	16.60963576	1.281902058E-13	9.745516857	29.90782849	2.9908782849	2.072612514
46	Pa	230	91	314.8	16.68232598	3.69146499E-14	9.7485209	29.90813669	2.990813669	2.072633873
47	U	237	92	59.5	16.93428475	4.710401262E-15	9.823067038	29.9157452	2.99157452	2.073161788
48	Np	232	93	326.8	16.75470083	4.079143406E-14	9.823434585	29.91579193	2.991579193	2.072633873
49	Np	233	93	312.0	16.79077127	4.58783699E-14	9.828717443	29.91632957	2.991632957	2.073201639
50	Np	234	93	1558.7	16.82676439	2.40470719E-13	9.833930645	29.91685983	2.991685983	2.073238386
51	Pu	237	94	280.4	16.39428475	4.08234776E-14	9.876023816	29.9211311	2.99211311	2.073534385
52	Pu	239	94	51.8	17.00558732	3.58887152E-15	9.886402893	29.92218148	2.992218148	2.073676477

53	Am	237	95	280.2	16.93428475	3.701029563E-14	9.902186619	29.9237746	2.99237746	2.07371758
54	Am	240	95	987.7	17.07112672	1.512775807E-13	9.917756	29.9253478	2.99253478	2.073826603
55	Cm	241	96	471.8	17.01659217	3.111428317E-14	9.948934739	29.9284866	2.99284866	2.074044121
56	Cm	243	96	277.5	17.14730299	3.492746514E-14	9.959217933	29.92951966	2.992951966	2.074115713
57	Bk	244	97	892.0	17.18254929	1.358486133E-13	9.990179819	29.93262371	2.993262371	2.074330823
58	Bk	246	97	798.8	17.25282586	1.211246059E-13	10.00037914	29.93364412	2.993364412	2.074401538
59	Cf	245	98	616.3	17.21772343	8.47391574E-14	10.02095064	29.93569908	2.993569908	2.074543946
60	Cf	249	98	388.19	17.35770722	4.873663129E-14	10.04125696	29.93772342	2.993772342	2.074684233
61	Es	252	99	52.3	17.46195865	1.506046662E-15	10.08185646	29.94175853	2.994175853	2.074963866
62	Es	254	99	648.8	17.5311152	9.699581379E-14	10.09182374	29.94274668	2.994274668	2.075032345
63	Fm	253	100	271.8	17.4965109	1.509251017E-14	10.1122463	29.94476831	2.994476831	2.075172444
64	Md	256	101	634.1	17.60	8.57004637E-14	10.15235938	29.94872725	2.994872725	2.075446798

$$t_{\frac{1}{2}} = \frac{In\ 2}{10^{-21} DT} \quad 57$$

This equation gives the result for half-life of gamma emitting nucleus substitute equation (57) into (51)

$$t_{\frac{1}{2}} = \frac{In\ 2}{10^{-21}} e^{-\left[4^2 \frac{e^2}{\hbar^2}\left(\frac{m}{\pi\epsilon_0}\right) Z^{\frac{1}{4}} r_0^{\frac{1}{4}} - \frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right) Z^2 E^{-\frac{1}{4}}\right]} \quad 58$$

$$t_{\frac{1}{2}} = 6.93 \times 10^{21} \times e^{-\left[8.792420946 \times 10^{15} Z^{\frac{1}{4}} r_0^{\frac{1}{4}} - 2.496984634 \times 10^{-12} Z^2 E^{-\frac{1}{4}}\right]} \quad 59$$

Table 2: Heavy gamma particle emitting nuclei and their calculated and experimental half lives

S/N	Nucleus (name)	Mass No. (A)	Z	E $\gamma(J)$	In DT (E12)	DT	Log Decay constant	Log Half-life $t_{\frac{1}{2}}$	Log Half-life $t_{\frac{1}{2}}$ (from chart)
1	Lu	158	71	3.893290839E-14	6.347922152	29.47914566	-19.53048506	23.31024877	1.01703333
2	Lu	160	71	2.710883991E-14	6.383794036	29.48478371	-19.53040205	23.31033118	1.602059991
3	Lu	163	71	1.360248528E-14	8.786070449	29.80418868	-19.5257227	23.31501054	2.380211240
4	Hf	165	72	2.707679637E-14	8.830295977	29.80920965	-19.52564954	23.3150837	1.878521796
5	Hf	167	72	3.117837026E-14	7.417436994	29.63485469	-19.5281972	23.3153604	2.079181246
6	Ta	163	73	4.16566098E-14	8.84701125	29.81110081	-19.52562199	23.31511125	1.025305865
7	Ta	166	73	5.863968918E-15	8.867280019	29.81338922	-19.52558865	23.31514458	1.531478917
8	W	170	74	1.157986948E-14	8.924228643	29.81979101	-19.5252954	23.31523783	2.158362492
9	W	175	74	1.57135931E-14	8.956623723	29.82341485	-19.52544263	23.3152906	3.322219295
10	Re	168	75	3.80356891E-14	8.940989353	29.8216674	-19.52546808	23.31526516	0.6020599913
11	Re	174	75	3.276452579E-14	8.980290438	29.82605334	-19.52540421	23.31532902	2.158362492
12	Os	171	76	1.421131265E-14	8.990514697	29.82719121	-19.52538764	23.31599626	0.903089987
13	Os	178	76	1.167025945E-13	9.035715368	29.83220621	-19.52531463	23.31541861	2.477121255
14	Os	183	76	1.179683146E-13	9.067058708	29.83566904	-19.52526424	23.31546902	4.551937695
15	Ir	177	77	9.468867845E-15	9.05889862	29.83476866	-19.52527733	23.31545591	1.477121255
16	Ir	182	77	4.051906342E-14	9.090509969	29.83825212	-19.52522662	23.31550661	2.954242509
17	Pt	176	78	1.04588925E-14	9.081746552	29.83728764	-19.52524066	23.31549258	0.8129133566
18	Pt	186	78	6.814060057E-14	9.144698938	29.84419548	-19.52514012	23.31559311	3.874365836
19	Au	189	79	9.645707346E-15	9.192235492	29.84938028	-19.52506468	23.31566855	2.4409099082

20	Au	195	79	3.992625832E-14	9.225215819	29.85328684	-19.52800811	23.31572539	1.484299839
21	Hg	180	80	4.727704777E-14	9.161693857	29.8460522	-19.52511311	23.31562013	0.414973348
22	Hg	184	80	1.3057745E-14	9.190335546	29.84917356	-19.52506769	23.31566555	1.489958479
23	Ti	184	81	3.797160201E-14	9.218921694	29.85227919	-19.52502251	23.31571073	1.041392685
24	Ti	187	81	4.377148384E-14	9.237577585	29.8543008	-19.52503674	23.31574014	1.193124598
25	Ti	192	81	4.490902772E-14	9.058940589	29.8347733	-19.52527726	23.31545598	2.811575006
26	Pb	192	82	1.248096117E-13	9.090509989	29.83825213	-19.52522662	23.31550661	2.322219295
27	Pb	202	82	9.496104857E-14	9.081746551	29.83728764	-19.52524066	23.31549258	4.104077206
28	Pb	204	82	7.36360687E-14	9.144698939	29.84419548	-19.52514012	23.31559311	3.605520523
29	Bi	196	83	7.485372346E-14	1.19223549	29.84938028	-19.52506468	23.31566855	2.380211245
30	Bi	198	83	8.980203767E-14	9.228215886	29.85328685	-19.52500785	23.31572539	3.886941706
31	Po	205	84	8.347343735E-15	9.165121001	29.4864262	-19.52510766	23.31562557	3.786751422
32	Po	216	84	3.135460996E-14	9.91033572	29.84917357	-19.52506764	23.31566555	2.161368002
33	At	210	85	1.425296926E-14	9.218921695	29.85227919	-19.52502251	23.31571073	4.46478752
34	Rn	211	86	2.068410814E-14	9.237565519	29.85429949	-19.52499311	23.31574012	1.164352856
35	Rn	219	86	2.039571703E-15	9.563938406	29.88902073	-19.52448831	23.31624492	0.591064607
36	Rn	222	86	4.309856937E-14	9.563898592	29.8890168	-19.52448838	23.31624486	5.578974837
37	Fr	208	87	2.21901581E-14	9.5630029434	29.88546892	-19.52453993	23.31619331	8.9590413923
38	Fr	229	87	2.369620227E-14	9.618272975	29.89468584	-19.52440601	23.31632723	1.698970004
39	Ra	222	88	4.479687731E-14	9.619043633	29.89476516	-19.52440484	23.31632839	1.579783594
40	Ra	223	88	8.203147776E-15	9.785231677	29.91189539	-19.52415659	23.31657665	5.994749911
41	Ac	223	89	1.623005605E-14	9.673788857	29.90044116	-19.5243224	23.31641083	2.100370545

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42	Ac	228	89	1.16125807E-13	9.69353558	29.90248034	-19.52429279	23.31644045	4.345177617
43	Th	225	90	2.094045731E-14	9.705982822	29.9037636	-19.52427445	23.3164591	2.718667735
44	Th	226	90	1.379474655E-14	9.711377264		-19.5226608	23.31646715	3.263872677
45	Pa	228	91	1.281902058E-13	9.745516857	29.90782849	-19.52421512	23.31651812	3.8576332496
46	Pa	230	91	3.69146499E-14	9.7485209	29.90813669	-19.52421064	23.31652259	6.177062991
47	U	237	92	4.710401262E-15	9.823067038	29.9157452	-19.52410017	23.3166332	5.765817575
48	Np	232	93	4.079143406E-14	9.823434585	29.91579193	-19.5240995	23.31652259	2.945468585
49	Np	233	93	4.58783699E-14	9.828717443	29.91632957	-19.52409169	23.31664154	3.336859821
50	Np	234	93	2.40470719E-13	9.833930645	29.91685983	-19.52408399	23.31664924	5.579966419
51	Pu	237	94	4.08234776E-14	9.876023816	29.9211311	-19.52402199	23.31671124	6.591652177
52	Pu	239	94	3.58887152E-15	9.886402893	29.92218148	-19.52400675	23.316741	11.87963212
53	Am	237	95	3.701029563E-14	9.902186619	29.9237746	-19.52398363	23.31674961	3.642662331
54	Am	240	95	1.512775807E-13	9.917756	29.9253478	-19.52396079	23.31677245	5.262849602
55	Cm	241	96	3.111428317E-14	9.948934739	29.9284866	-19.52391522	23.31681799	6.452387586
56	Cm	243	96	3.492746514E-14	9.959217933	29.92951966	-19.52390025	23.31683298	8.961508115
57	Bk	244	97	1.358486133E-13	9.990179819	29.93262371	-19.52385521	23.31687802	4.199755177
58	Bk	246	97	1.211246059E-13	10.00037914	29.93364412	-19.52384041	23.31689283	5.191786248
59	Cf	245	98	8.47391574E-14	10.02095064	29.93569908	-19.5238106	23.31692264	3.440909082
60	Cf	249	98	4.873663129E-14	10.04125696	29.93772342	-19.52378123	23.31695201	10.04292224
61	Es	252	99	1.506046662E-15	10.08185646	29.94175853	-19.5237227	23.31701054	10.15351195
62	Es	254	99	9.699581379E-14	10.09182374	29.94274668	-19.52370836	23.31702487	5.151357591

63	Fm	253	100	1.509251017E-14	10.1122463	29.94476831	-19.52367094	23.31705419	5.628478845
64	Md	256	101	8.57004637E-14	10.15235938	29.94872725	-19.52362163	23.31711161	3.666892211

Discussion

The results of tunneling probabilities of gamma particle for heavy nuclei are shown in Table 1. For the heavy nuclei Table 1 indicates good result. The calculated tunnel probability in equation (51) indicates input data in Table 2. The isotopes of gamma particle emitter with Z = 71 to 101 that is ${}^{158}_{71}Lu - {}^{256}_{101}Md$ for heavy gamma particle are shown. The half-life varies from one nucleus to another which indicates that from Table 2 observes that the values of calculated half-lives are so small but also match with the experimental half-lives. In general, the gamma particle half-life $t_{\frac{1}{2}}$ presented in the Table 2 are in agreement with the experimental result (see chart of Nuclides Edwards et al., 2002).

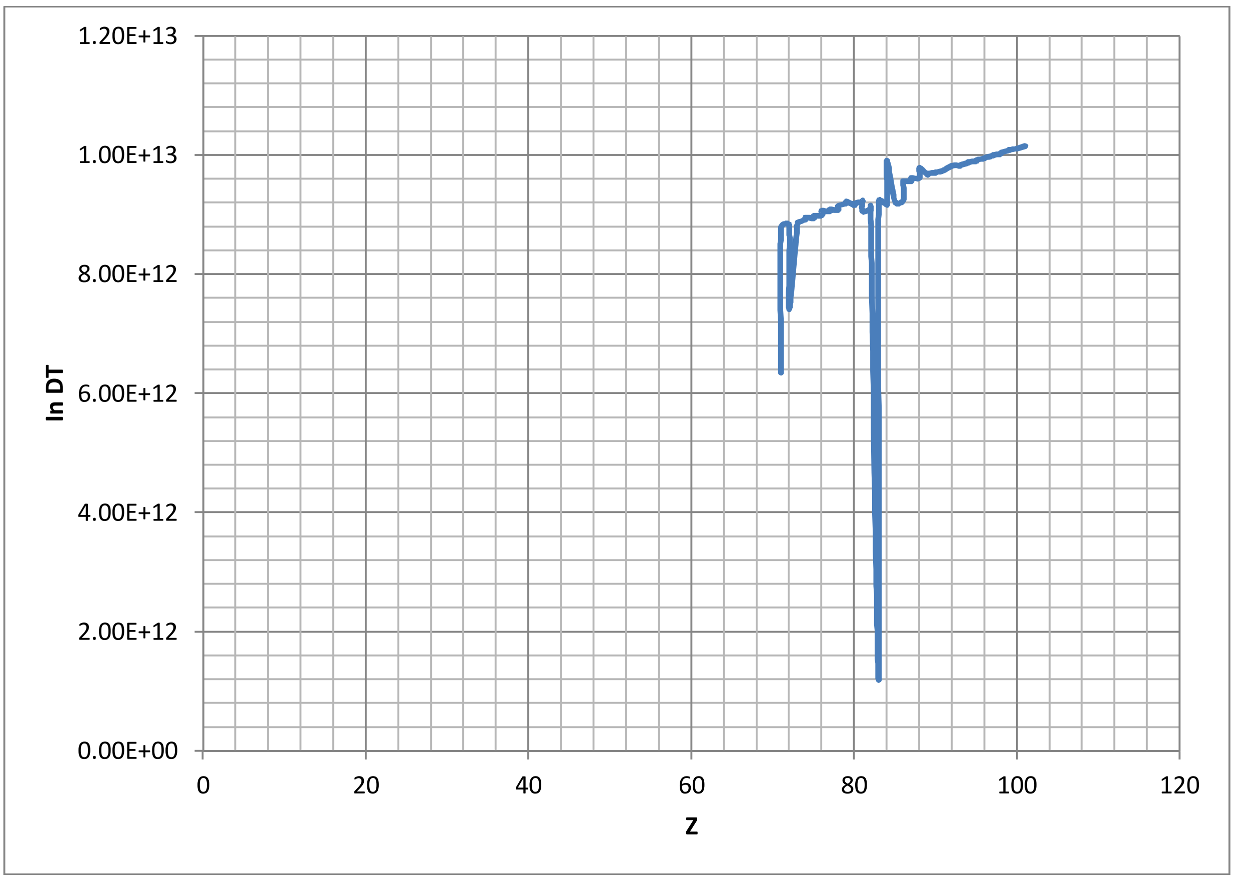

Figure 1: Natural logarithm of Tunneling probability versus Atomic number for Heavy Gamma Particle emitting nuclei.

Figure 1 represents the natural logarithm of tunneling probability versus atomic number Z for heavy mass gamma particle emitters respectively. Figure 1, for the heavy gamma particle emitter that the shapes of 'v, w and upside down v' at a high atomic number Z for heavy gamma particle emitting nuclei. The reason for the shape of ' v' is as result of low in natural logarithm of tunneling, for the 'w' shape is as a result of fluctuation in the natural logarithm of tunneling and for the upside down 'v' is as a result of slight value of natural logarithm. Thereafter, from Z≫90 the logarithms tunneling probability to be increasing uniformly as atomic number Z increases.

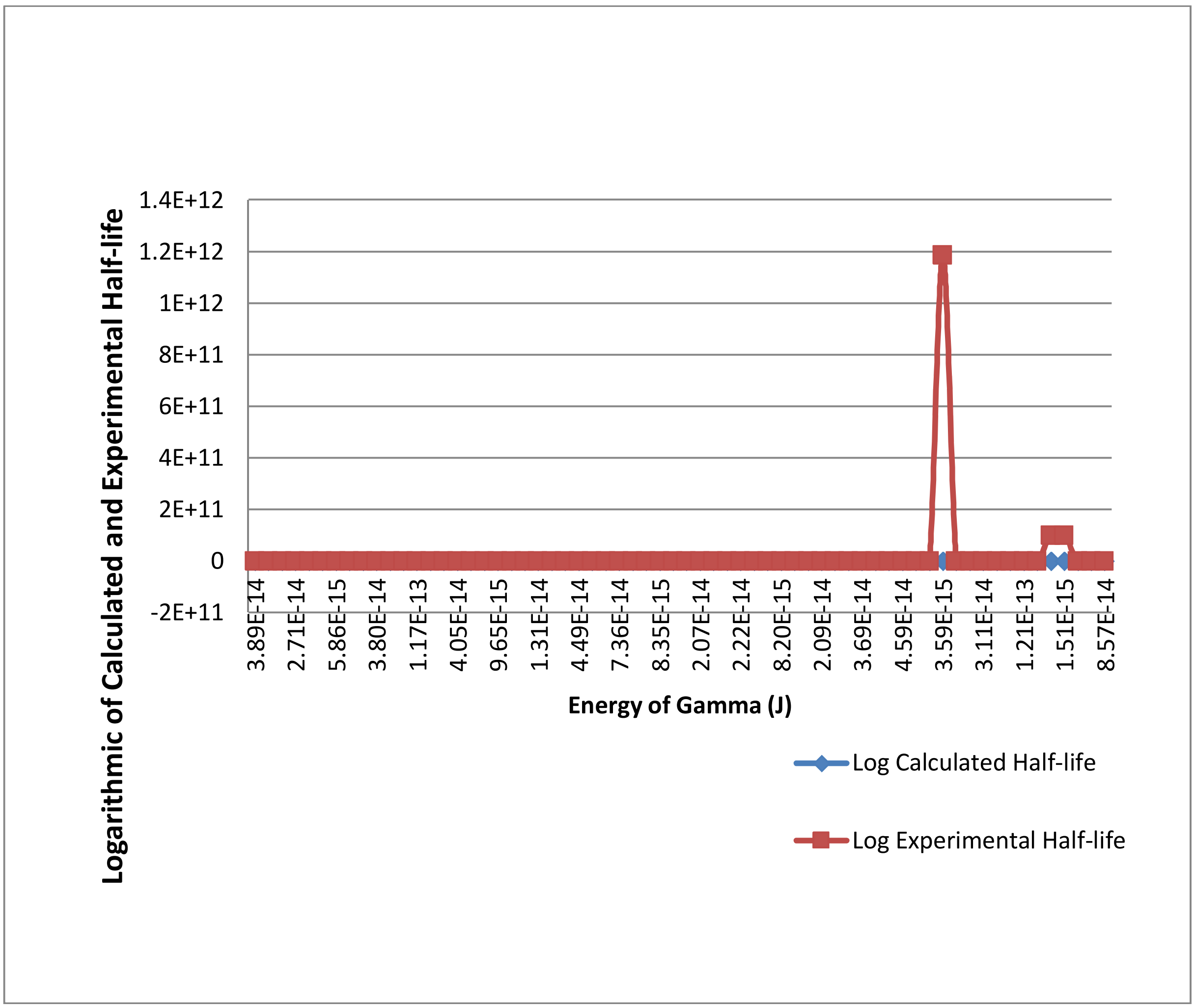

Figure 2: Logarithmic plot of Experimental and Calculated Half-lives versus Energy of Gamma Particle for Heavy mass nuclei.

Figure 2 shows the logarithm of experimental and calculated half-lives versus energy of gamma particle for heavy mass nuclei. Figure 2 reveal anomaly that lies with high energy of gamma particle values for heavy mass nuclei which sustain a straight line of values of experimental and calculated half-lives except for the values of three anomalies of both experimental and calculated half-lives that are high. Also the figure shows that those low energy of gamma particle emitters have low rate of experimental and calculated half-lives while the three anomaly nuclei have high energy of gamma particle emitters proves that there have high experimental and calculated half-lives.

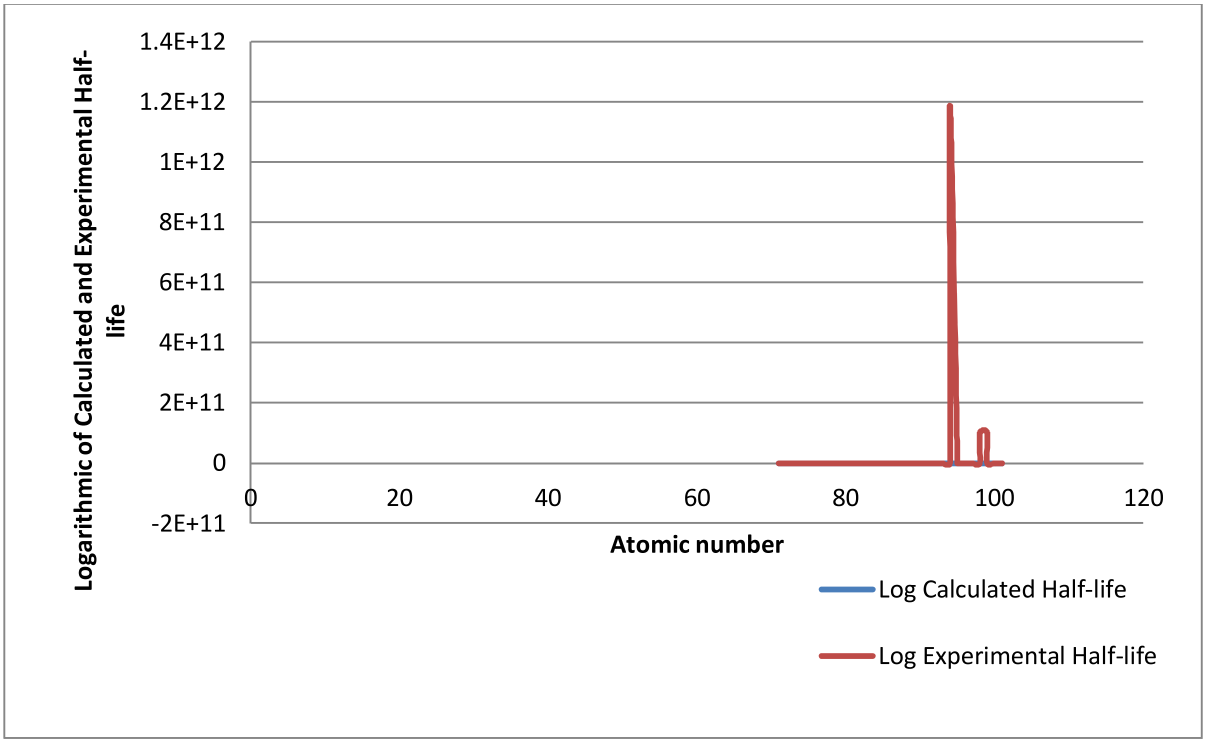

Figure 3: Logarithmic plot of Calculated and Experimental Half-lives versus Atomic Number for Heavy mass nuclei.

Figure 3 shows the logarithmic calculated and experimental half-lives versus Mass number A for heavy mass nuclei respectively. It can also be observed that figure 3 shows the logarithmic plot of Calculated and experimental half-lives versus Atomic Number for Heavy mass nuclei. This indicates that the anomaly lies with low atomic number Z values of calculated and experimental half-lives except for the three anomaly of atomic number for the both calculated and experimental half-lives that are high.

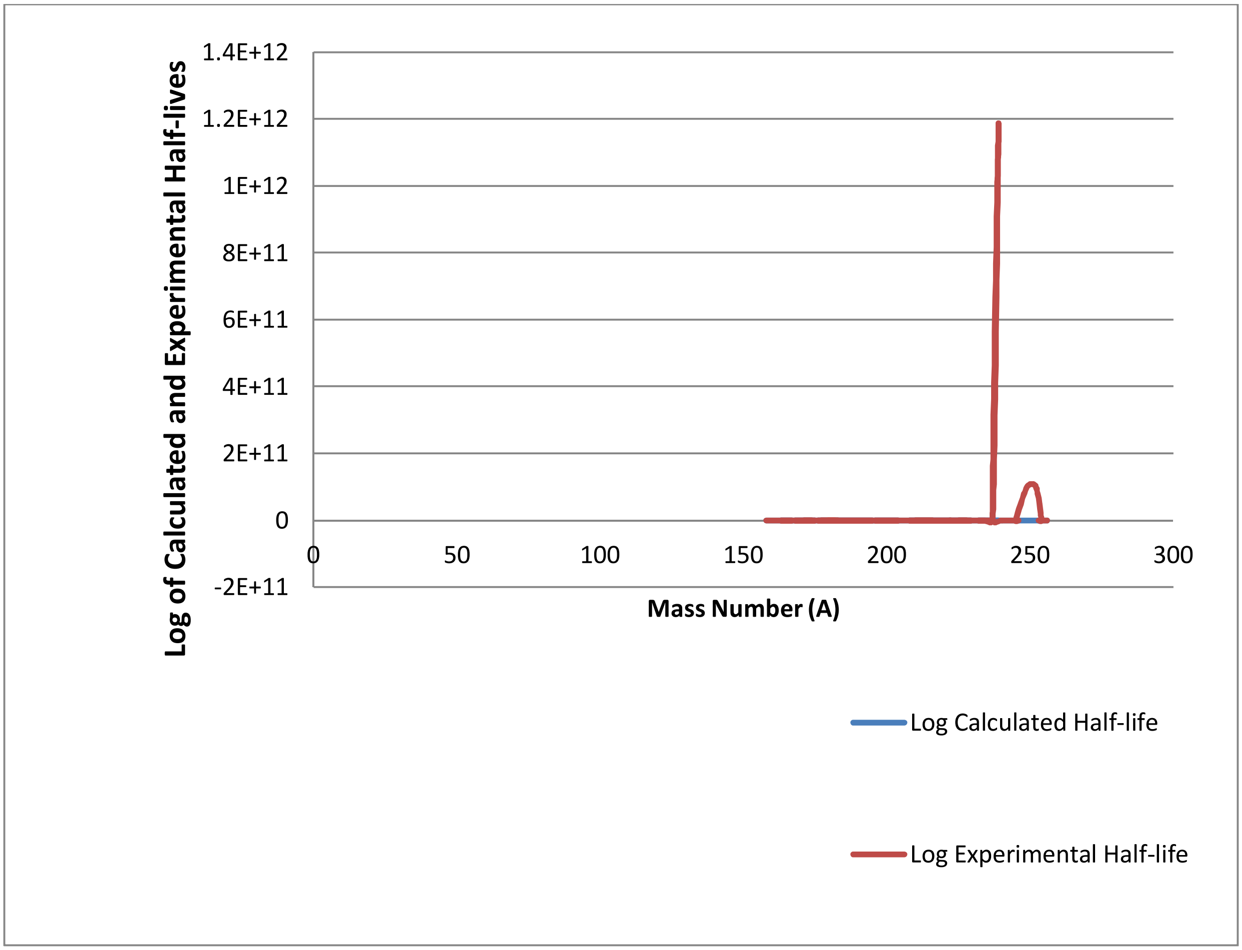

Figure 4: Logarithmic plot of Calculated and Experimental Half-lives versus Mass Number (A) of Gamma Particle for Heavy Mass Nuclei.

Figure 4 shows the logarithm of experimental and calculated half-lives versus mass number of gamma particle for heavy mass nuclei. Figure 4, shows anomaly that lies with high mass number of gamma particle values for heavy mass nuclei which sustain a straight line of values of experimental and calculated except for the values of three anomalies of both experimental and calculated except for the values of three anomalies of both experimental and calculated half-lives that are high. Also the figure indicates that those low mass number of gamma particle emitters have low rate of experimental and calculated half-lives while the three anomaly nuclei have high mass number of gamma particle emitters proves that there have high experimental and calculated half-lives.

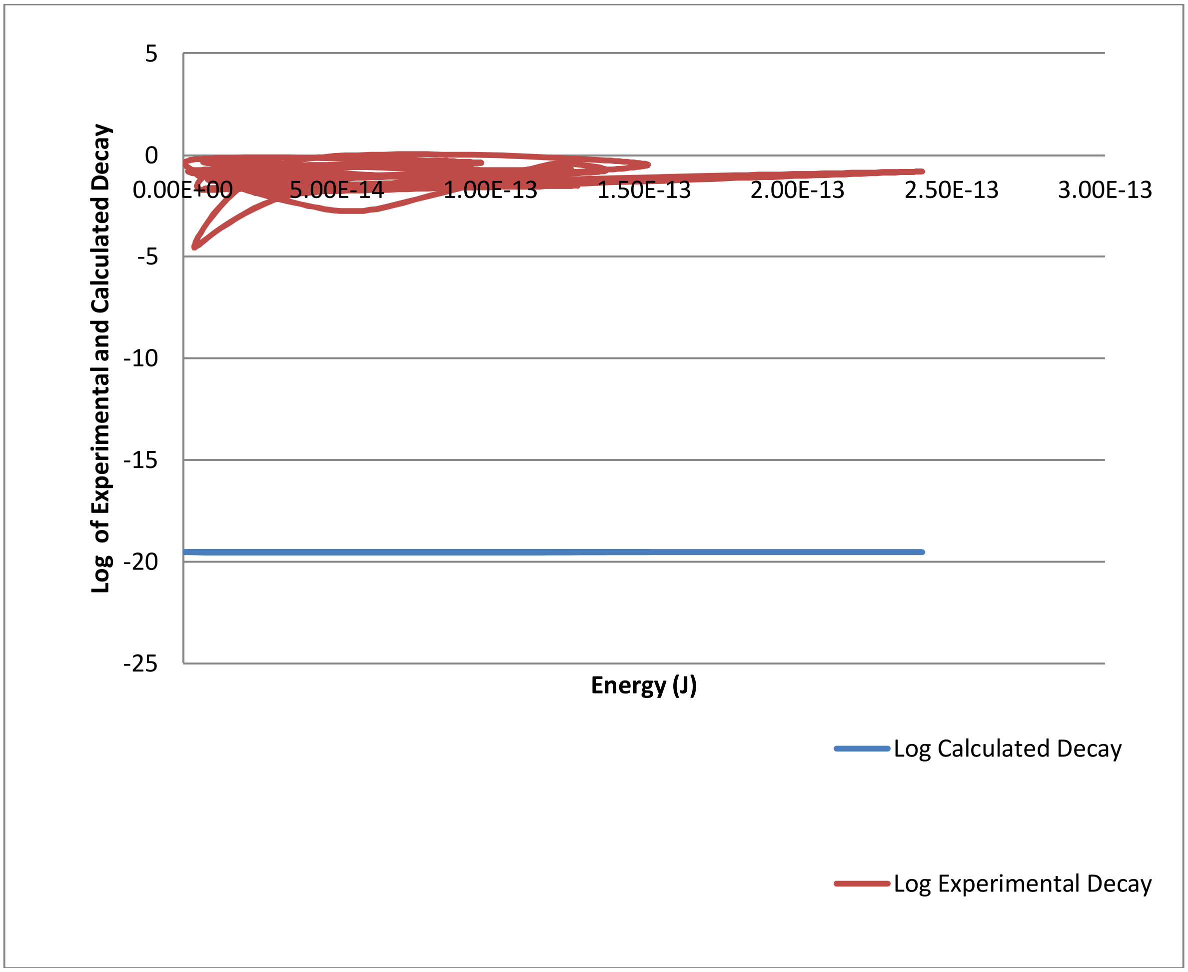

Figure 5: Logarithmic plot of Calculated and Experimental Decay constant versus Energy (J) of Gamma Particle for Heavy Mass Nuclei.

Figure 5 represents the logarithm calculated decay constant versus Energy (J) for heavy mass gamma particle emitters respectively. Figure 5, for the heavy gamma particle emitters the anomaly lies with low Energy (J) values. The figure shows a shape of closed distance zigzag cone and a horizontal line. The reason for the shape of zigzag is as a result of different value of Energy(J) that fluctuate, the cone shape lies on the position of unstable equilibrium and a horizontal line on the Energy (J) from 0.00 E+00 to 2.50E -13.

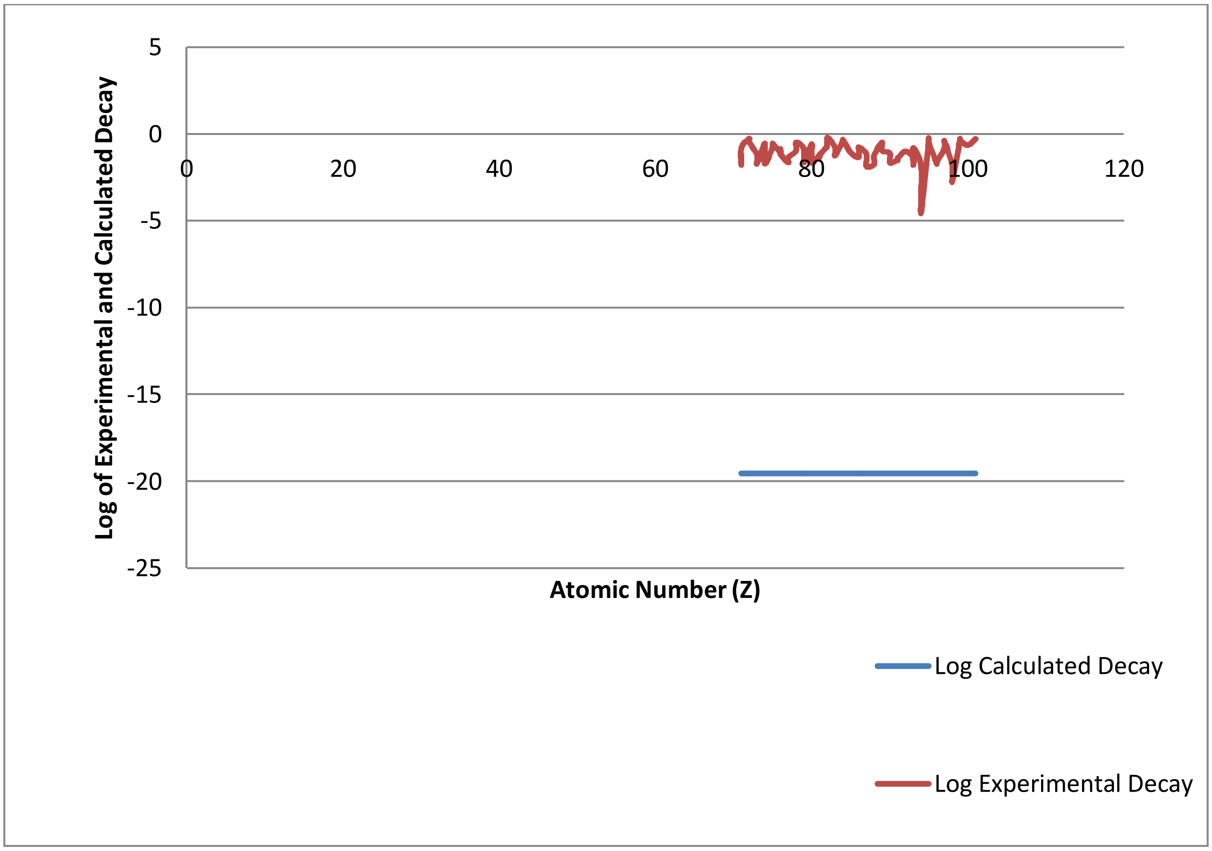

Figure 6: Logarithmic plot of Calculated and Experimental Decay constant versus Atomic Number (Z) of Gamma Particle for Heavy Mass Nuclei.

Figure 6 shows the logarithm of calculated and experimental decay constant versus atomic number (z) of gamma particles for heavy mass nuclei. Figure 6, reveal anomaly that lies with low atomic number (Z) of gamma particle values for heavy mass nuclei. The figure indicates a zigzag and horizontal line. The zigzag value on the atomic number (Z) = 95 and 99 have low value on the zigzag.

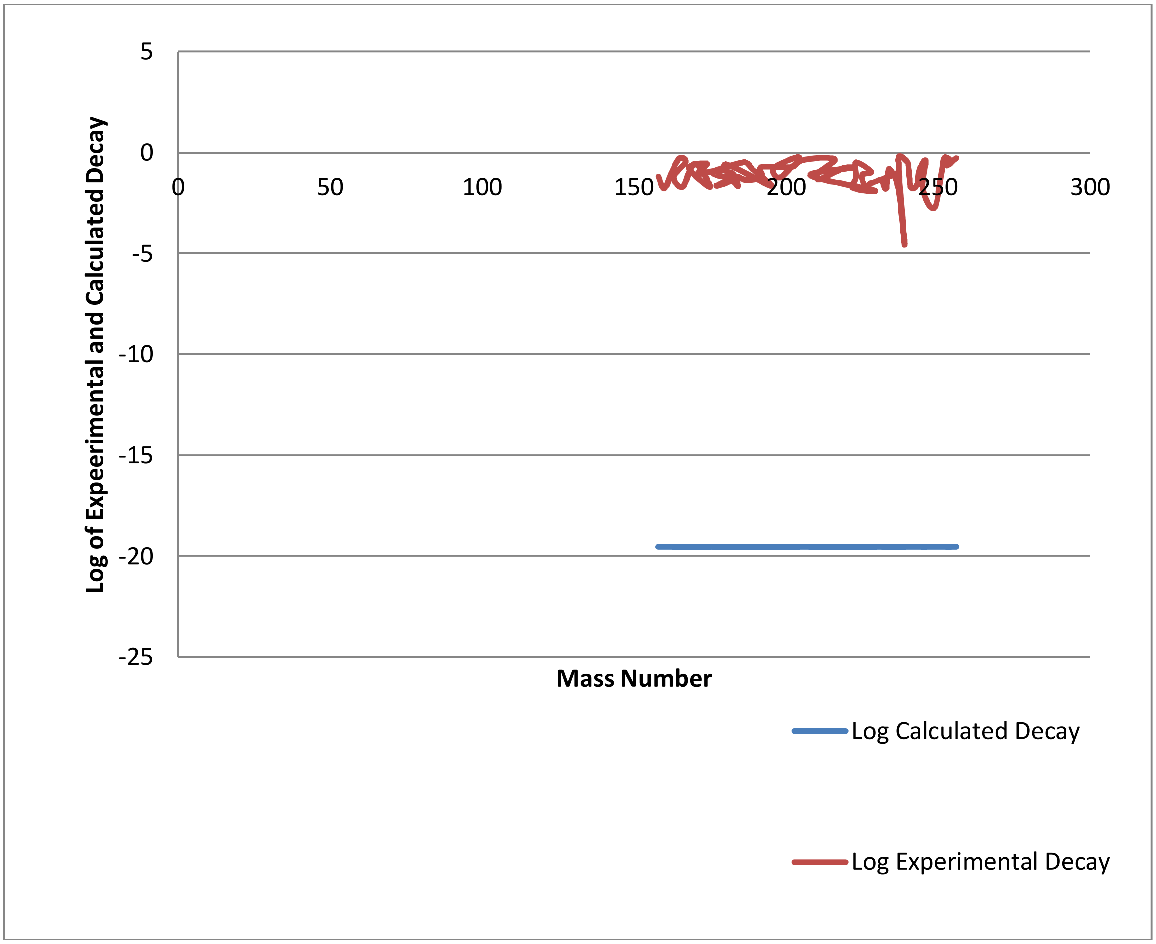

Figure 7b: Logarithmic plot of Calculated and Experimental Decay constant versus Mass Number (A) of Gamma Particle for Heavy Mass Nuclei.

Figure 7 shows the logarithm of calculated and experimental decay constant versus mass number (A) of gamma particle for heavy mass nuclei. Figure 7, reveal anomaly that lies with low mass number (A) of gamma particle values for heavy mass nuclei. The figure indicates the shape of cones, zigzag and also a horizontal line. The cones are like in the position of unstable while on the zigzag the mass number (A) = 240 have the lowest values of decay.

International Journal of Science and Advanced Innovative Research
ISSN: 2536-7315 (Print) 2536-7323 (Online)
Volume 6, Number 1, March 2021
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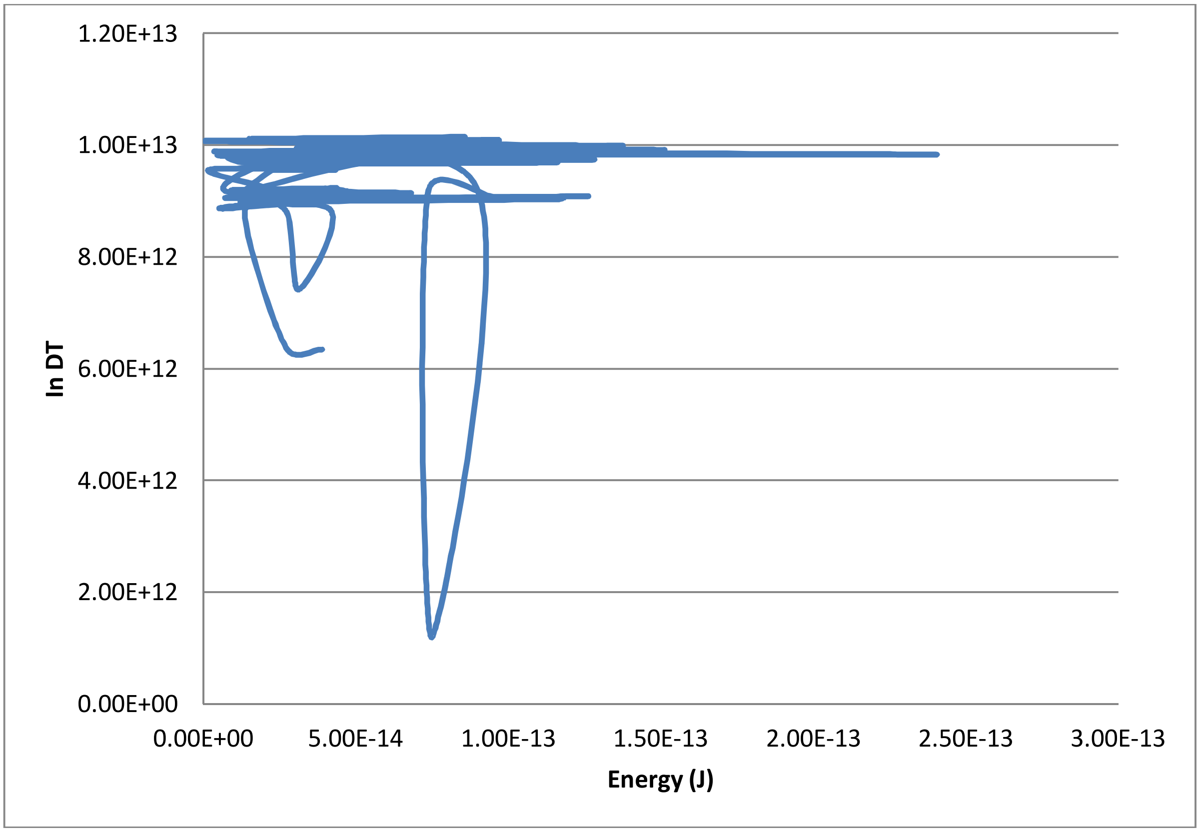

Figure 8: Natural logarithm of Tunneling probability versus Energy (J) for Heavy Gamma Particle emitting nuclei.

Figure 8 represent the natural logarithm of tunneling probability versus Energy (J) for heavy mass Gamma particle emitters respectively. Figure 8, for the heavy Gamma particle emitters that it has a closed distance zigzag, a shape of figure 8, triangle and also a shape of a cone for heavy Gamma particle emitting nuclei. The reason for the shape zigzag and figure 8 is as a result of different value of Energy (J) while for triangle and also triangle and also a shape of a cone is as a result of some nuclides that have less than the Energy (J) than orders for tunneling that occurred.

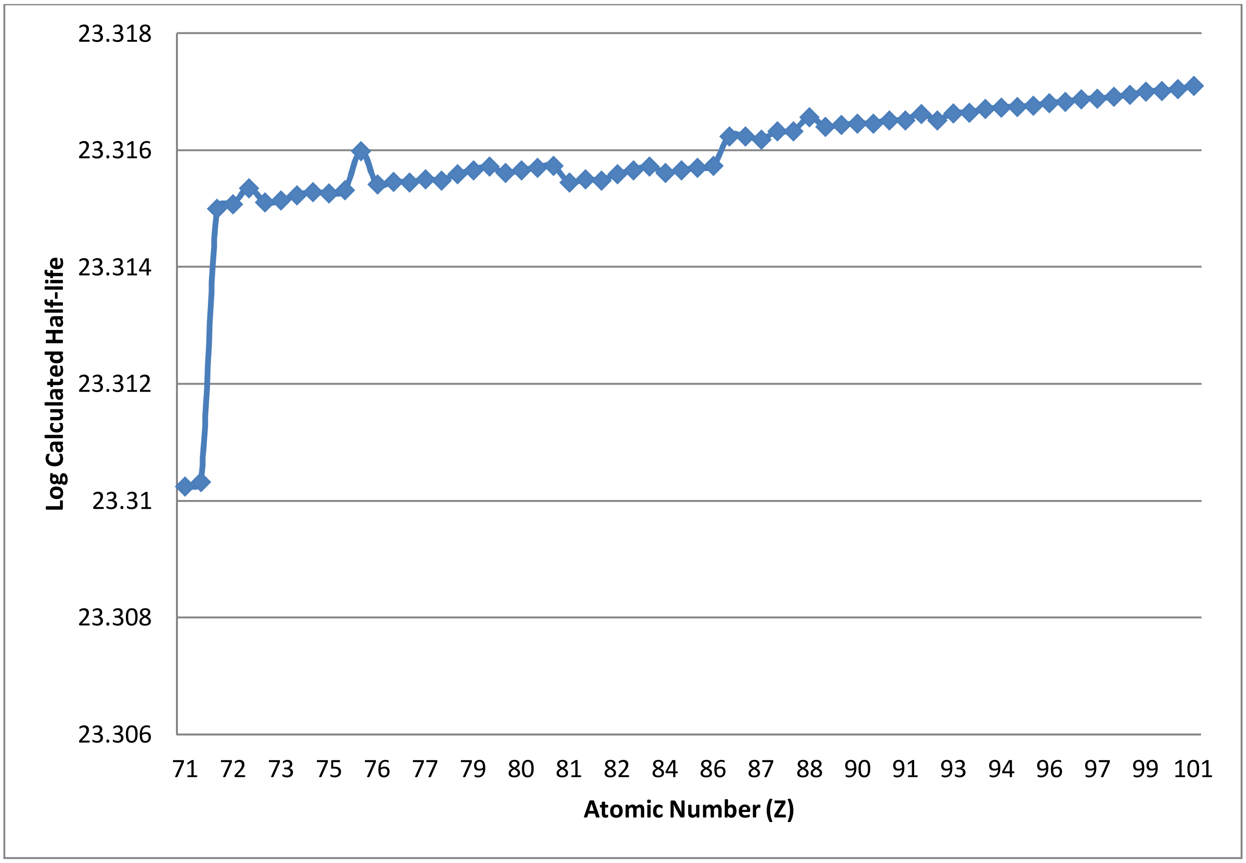

Figure 9: Logarithmic plot of Calculated Half-lives versus Atomic Number (Z) of Gamma Particle for Heavy mass nuclei.

Figure 9 represents the logarithms of calculated half-life verses Atomic number Z for heavy mass Gamma particle respectively. Figure 9, for the heavy gamma particle emitter that the shape of v, w and upside down v at a high atomic number Z for heavy gamma particle emitting nuclei. The reason for the shape of v is as a result of low in logarithms of calculated half-life of tunneling for the w shape is as result of fluctuation in the logarithms of calculated half-life of tunneling and for the upside down v is as a result as the result of slit value of logarithms of calculated half-life taken for tunneling. Thereafter, from Z≫90 the logarithm of calculated half-life taken for tunneling probability to be increasing uniformly as atomic number Z increases.

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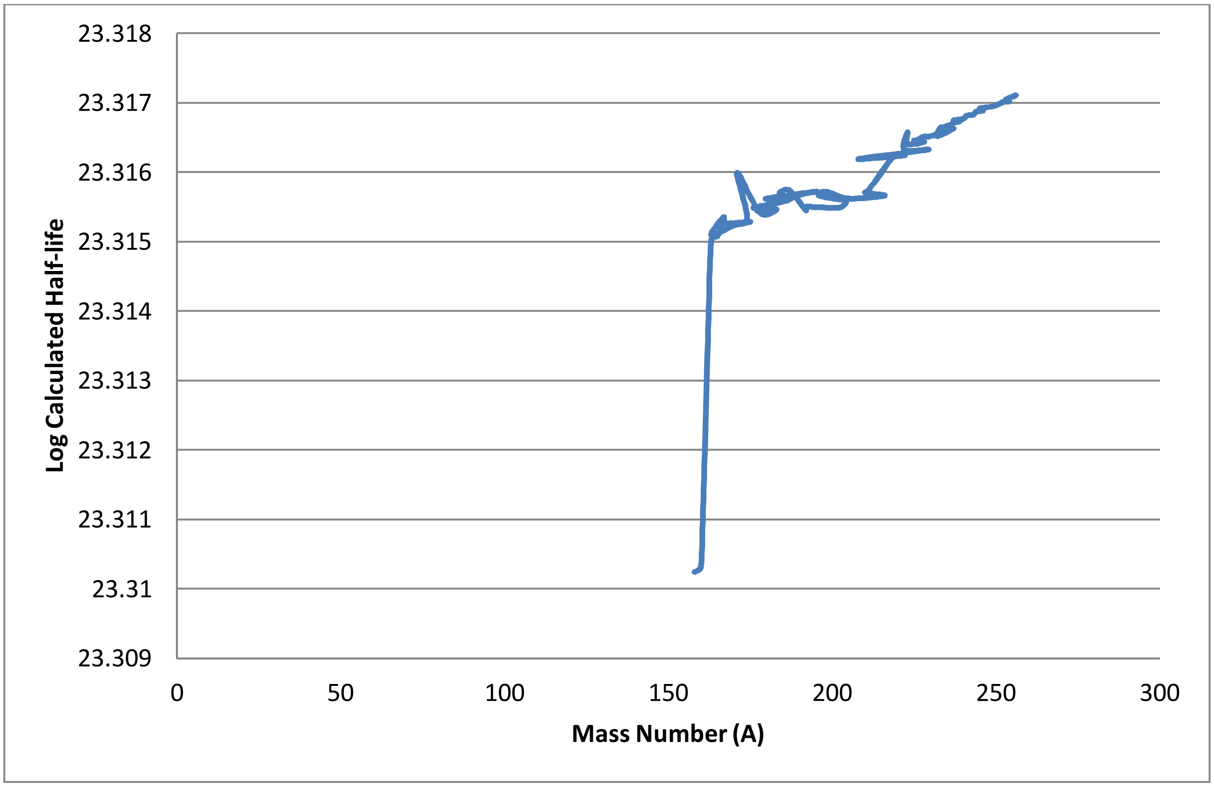

Figure 10: Logarithmic plot of Calculated Half-life versus Mass Number (A) for Heavy mass nuclei.

Figure 10 represents the logarithms of calculated half-life versus mass number (A) for heavy mass Gamma particle emitters respectively. Figure 10, for the heavy gamma particle emitter that the shape of upside down v, w and zigzag in an ascending order at a high mass number (A) for heavy gamma particle. The reasons for the shapes of upside down v is as the result of the order nuclide had slit change of logarithms of calculated half-life than the order points, for the w shape is as a result of fluctuation in the logarithms of calculated half-life and also for the zigzag is at a closed range distance in an ascending order.

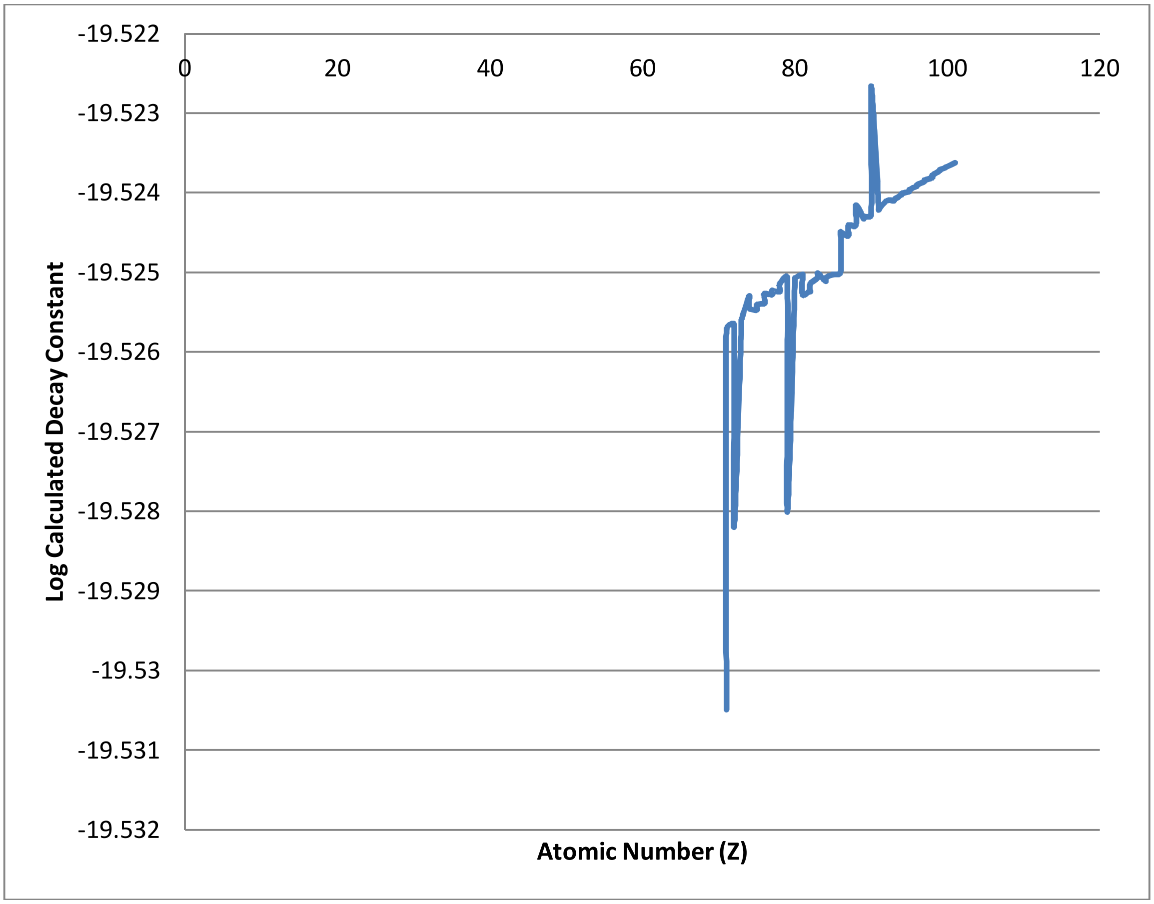

Figure 11: Logarithmic plot of Calculated Decay Constant versus Atomic Number (Z) of Gamma Particle for Heavy mass nuclei.

Figure 11 represents the logarithms calculated Decay constant versus atomic number Z for heavy mass Gamma particle emitters respectively. Figure 11, for the heavy gamma particle emitter shows that the shape of 2v, upside down v and zigzag at a low atomic number Z for heavy gamma particle emitting nuclei. The reasons that the atomic number Z that have the shape of v is as a result of low in one of the logarithm calculated decay constant, also for the one that have upside down v is as a result of high in one of the logarithm calculated than the order and for the zigzag is as result of fluctuation value of the logarithms calculated decay constant for the atomic number Z after the tunneling probability.

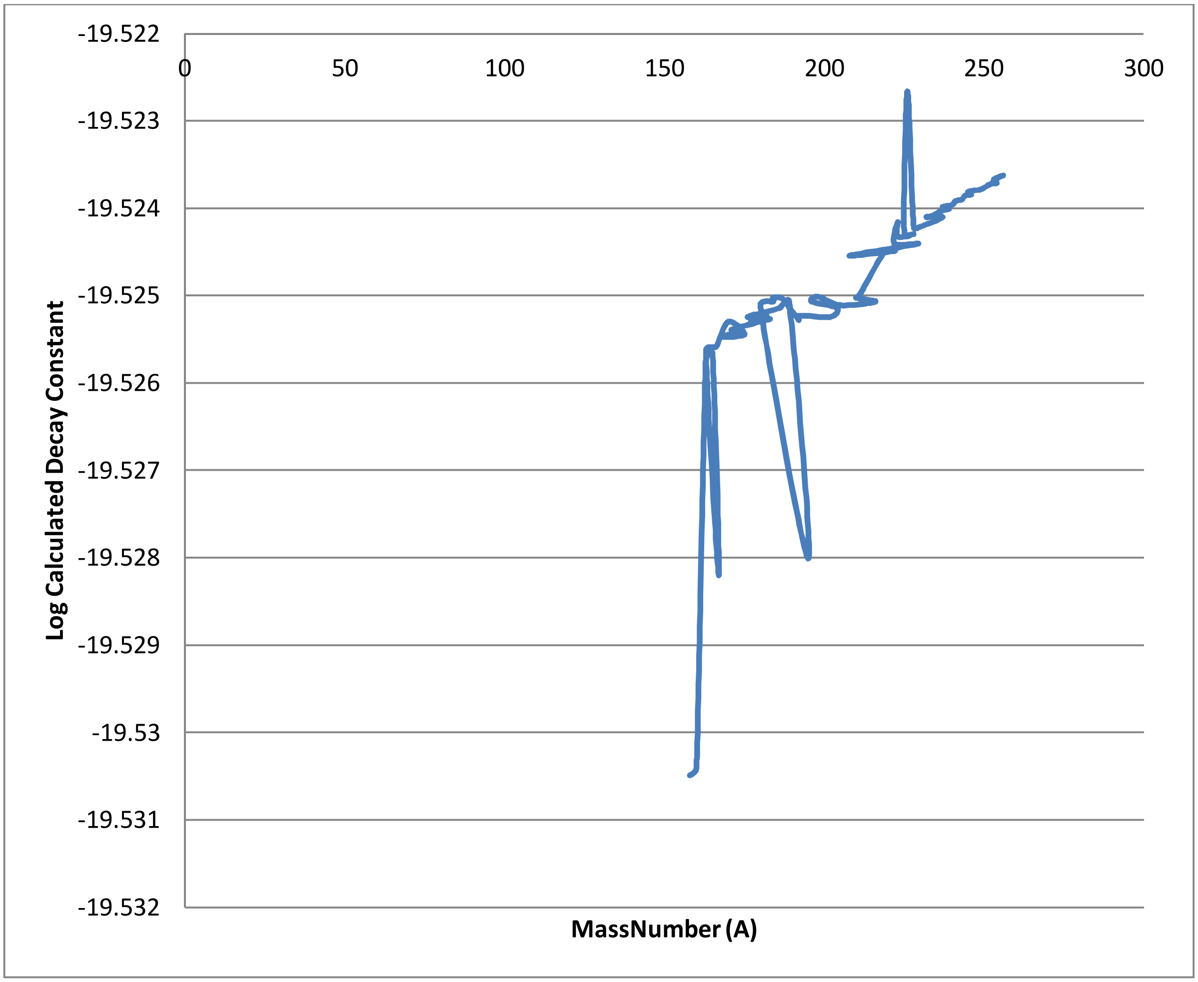

Figure 12: Logarithmic plot of Calculated Decay Constant versus Mass Number (A) of Gamma Particle for Heavy mass nuclei.

Figure 12 represents the logarithm decay constant versus mass number (A) for heavy mass Gamma particle emitters respectively. Figure 12, for the heavy gamma particle emitter shows that the shape of 2 cones, an upside down v and zigzag at a low mass number (A) for heavy gamma particle emitting nuclei. The reason for the shape of a cone is as a result of the middle value of mass number (A) is having a lower value of logarithm decay constant which the two high point are covered, for the upside down v is as a result of the middle value of mass number (A) is having a higher value of logarithm decay constant than the orders and also for the shape of zigzag is as a result of fluctuation of values of logarithm decay constant after the tunneling probability.

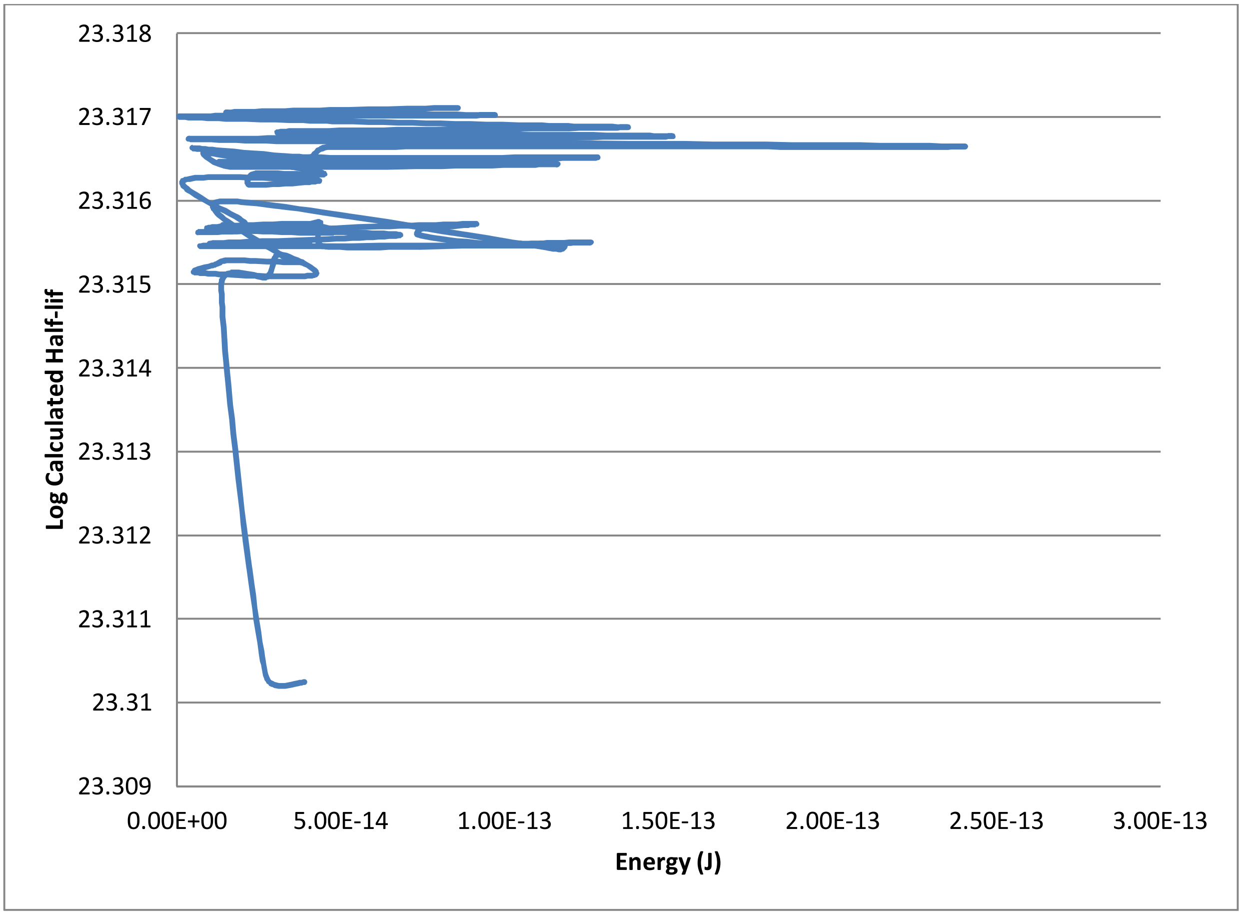

Figure 13: Logarithmic plot of Calculated Half-life versus Energy (J) of Gamma Particle for Heavy mass nuclei.

Figure 13 represents the logarithm calculated Half-life versus Energy (J) for heavy mass Gamma particle emitters respectively. Figure 13, for the heavy gamma particle emitters that it has a closed distance zigzag, shape of letter S and also a shape of letter L for heavy gamma particle emitting nuclei. The reason for the shape of zigzag is as a result of different value of Energy (J) that fluctuate, for the shape letter S is as a result of fluctuation value of logarithm calculated half-life at a distance after the tunneling probability and for the letter L shape is as a result of the Energy (J) that have lower logarithm calculated half-life that increase high from lower to a high point.

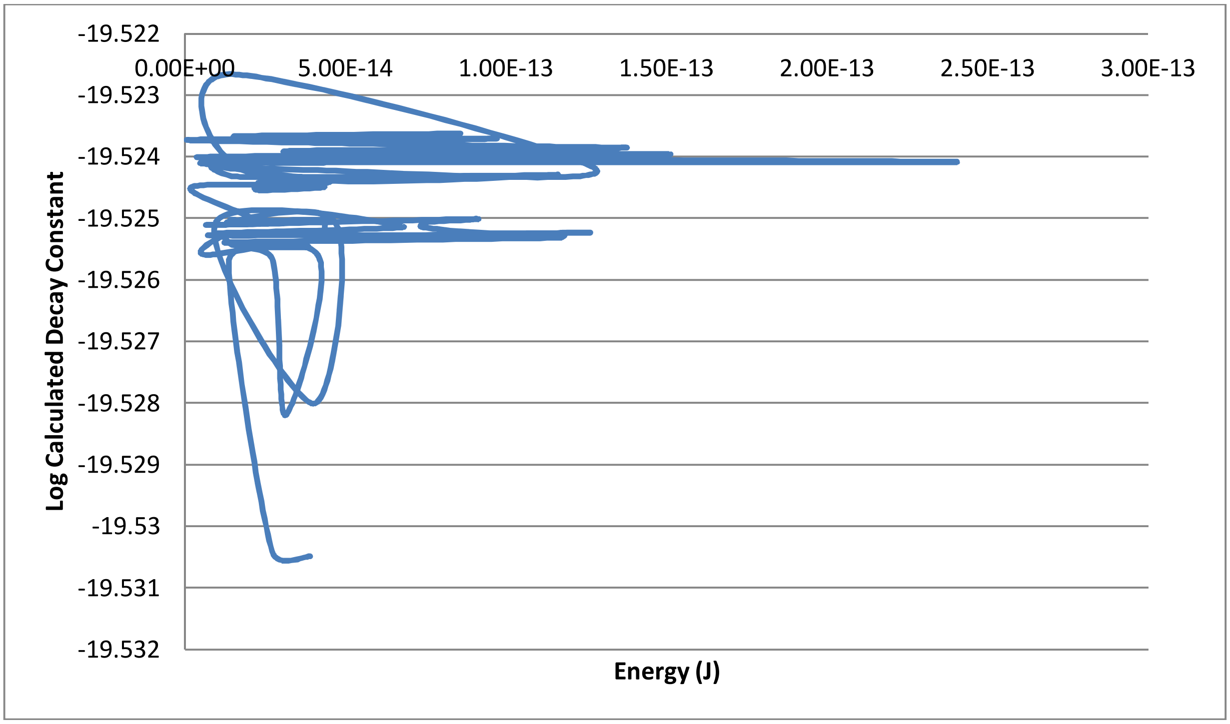

Figure 14: Logarithmic plot of Calculated Decay Constant versus Energy (J) of Gamma Particle for Heavy mass nuclei.

Figure 14 represents the logarithm calculated decay versus Energy (J) for heavy mass Gamma particle emitters respectively. Figure 14, for the heavy gamma particle emitters that it has a closed distance zigzag, 2 shape of cone in the position of unstable equilibrium, a shape of a cone in the position of neutral equilibrium and also a shape of letter L for heavy gamma particle emitting nuclei. The reason for the shape of zigzag is as a result of different value of Energy (J) that fluctuation, for the cones that are in the position of unstable equilibrium is as a result of the middle one is having a less value of decay constant than the orders, the cone on the neutral equilibrium position lies on the high Energy (J) than the order and for the letter L shape is as a result of the Energy (J) that have lower logarithm calculated decay constant that increase from lower point to high point.

CONCLUSION

This paper has revealed that emission of Heavy gamma particle. There is an increase of emission with increase in atomic number when there is an increase in number of nuclear mass. The result of the calculated and experimental values agrees favorably. Gamma particle half-lives of the heavy nuclei are in a good agreement with decay constant. Gamma particle emissions of the natural logarithmic tunneling probability versus atomic number are in the form of zigzag.

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