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# THEORETICAL EVALUATION OF STEPS APPROACHING ZERO EMISSION ON A DOUBLE THICK BARRIER OF A GAMMA PARTICLE

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## ABSTRACT

The goal of this work is to obtain tunneling probability of a gamma particle. The application of Schrödinger's equation in barrier penetration has been applied to gamma particle decay for light, medium and heavy nuclei. Gamma particle tunneling probability has been calculated analytically. Decay probability computed for each gamma particle emitting nucleus shows interesting variations. Log plot of calculated Decay constant plotted against atomic number (Z), mass number (A) and Energy for gamma particle emitting nucleus shows the variations interesting. Half-life which is a function of decay probability plotted against gamma particle energy or against atomic number of gamma particle emitting nucleus shows the variations of decay probabilities. Log plot of Calculated Half-life plotted against atomic number (Z), mass number (A) and Energy for gamma particle emitting nucleus shows interesting variations of decay probabilities. Calculated half-lives compared with experimental half-lives for each gamma particle emitting nucleus shows results which are in good agreement.

**Key word**: Schrödinger's equation, Emission, Half-life, Gamma and Decay constant.

## INTRODUCTION

Gamma decay is a type of radioactive decay in which gamma rays are emitted. Gamma decay occurs when a nuclide is produced in an excited state, gamma emission occurring by transition to a lower energy state. It can occur in association with alpha decay and beta decay (Raju et al., 2006). A gamma ray or gamma radiation (symbol γ), is a penetrating electromagnetic radiation arising from the radioactive decay of atomic nuclei. It consists of the shortest wavelength electromagnetic waves and so imparts the highest photon energy. Paul Villard, a French chemist and physicist, discovered gamma radiation in 1900 while studying radiation emitted by radium (Villard, 1900a). In 1903, Ernest Rutherford named this radiation gamma rays based on their relatively strong penetration of matter; he had previously discovered two less penetrating types of decay radiation, which he named alpha rays and beta rays in ascending order of penetrating power (Rutherford, 1903). Gamma rays from radioactive decay are in the energy range from a few kilo electron volts ( keV ) to approximately 8 Mega electron volts (~8 MeV), corresponding to the typical energy levels in nuclei with reasonably long lifetimes. The energy spectrum of gamma rays can be used to identify the decaying radionuclides using gamma spectroscopy. Very-high-energy gamma rays in the 100–1000 tera electron volt ( TeV ) range have been observed from sources such as the Cygnus X-3 micro quasar. Natural sources of gamma rays originating on Earth are mostly as a result of radioactive decay and secondary radiation from atmospheric interactions with cosmic ray particles (Villard, 1900b. However, there are other rare natural sources, such as terrestrial gamma-ray flashes, which produce gamma rays from electron action upon the nucleus. Notable artificial sources of gamma rays include fission, such as that which occurs in nuclear reactors, and high energy physics experiments, such as neutral pion decay and nuclear fusion. Gamma rays and X-rays are both electromagnetic radiation, and since they overlap in the electromagnetic spectrum, the terminology varies between scientific disciplines. In some fields of physics, they are distinguished by their origin: Gamma rays are created by nuclear decay, while in the case of X-rays; the origin is outside the nucleus. In astrophysics, gamma rays are conventionally defined as having photon energies above 100 keV and are the subject of gamma ray astronomy, while radiation below 100 keV is classified as X-rays and is the subject of X- ray astronomy. This convention stems from the early man-made X-rays, which had energies only up to 100 keV, whereas many gamma rays could go to higher energies. A large fraction of astronomical gamma rays are screened by Earth's atmosphere.

## MATERIALS AND METHOD

### Materials

The materials used are the*Schrödinger*'s equation.

### Method

We now consider the beam of a particle incident upon a square potential barrier of height $V_0$ presumed positive for now and width a. As mentioned above, this geometry is particularly important as it includes the simplest example of scattering phenomenon in which a beam of particles is 'deflected' by a local potential. Moreover, this one-dimensional geometry also provides a plat form to explore a phenomenon peculiar to quantum mechanics quantum tunneling (Dyson, 1951).

The potential energy variation in the case of a rectangular potential barrier shown in figure 1 is given by

$$V(x) = \begin{matrix} 0, & x < 0 \\ V_o, & 0 < x < L \end{matrix}\Bigg\}$$

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(1)

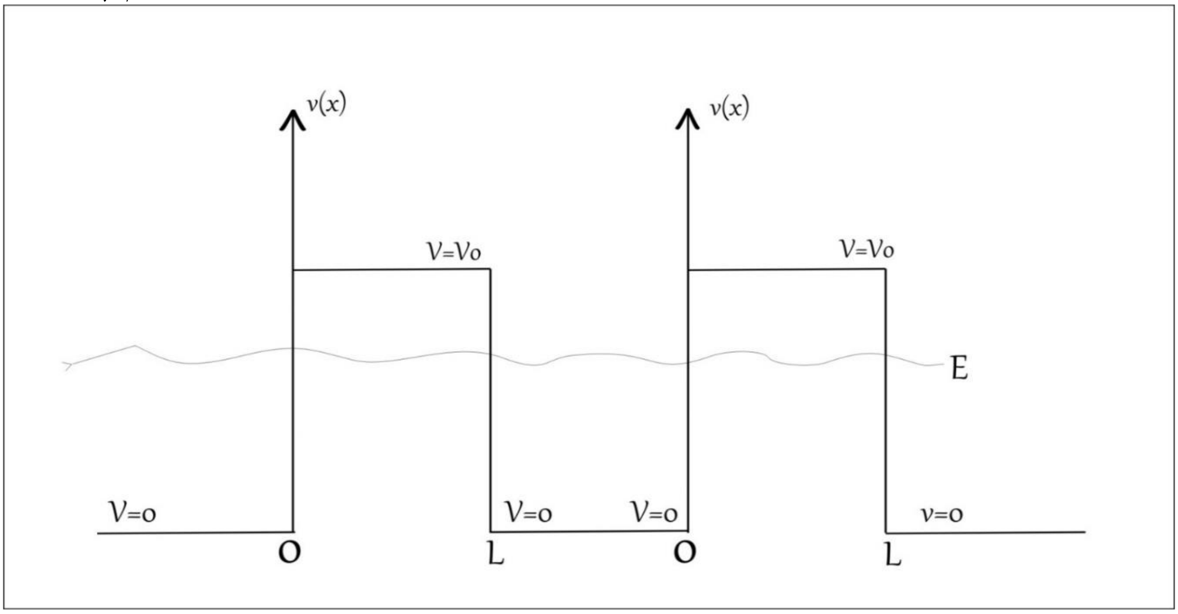


**Fig. 1**: a rectangular double thick potential barrier of width L and height$V_0$.

Let us consider two cases

(i) $0 < E < V_0$ Classically a particle of energy E if incident from the left would be reflected at the double thick barriers as it cannot enter $(0 < x < L)$ in which its K.E is negative. To describe the behavior of particle quantum mechanically, we will have to solve the $Schrödinger$ equation,

$$\left(\frac{d^2\varphi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\varphi(x)\right)\left(\frac{d^2\varphi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\varphi(x)\right) = 0$$

Or

$$\left(\frac{d^2\varphi(x)}{dx^2} + k^2\varphi(x)\right)\left(\frac{d^2\varphi(x)}{dx^2} + k^2\varphi(x)\right) = 0, k^2 = \frac{2m\,E}{\hbar^2}, x < 0 \; and \; x > L \qquad (2)$$

And

$$\left(\frac{d^2\varphi(x)}{dx^2} + \gamma^2\varphi(x)\right)\left(\frac{d^2\varphi(x)}{dx^2} + \gamma^2\varphi(x)\right) = 0, \gamma^2 = \frac{2m(V_0 - E)}{\hbar^2}, 0 < x < L \qquad (3)$$

The general solutions of these equations are given by

$\varphi^2(x) = (A\,e^{ikx} + Be^{-ikx})(A\,e^{ikx} + Be^{-ikx})\,, x < 0$ (4)
$\varphi^2(x) = (Ce^{\alpha x} + De^{-\alpha x})(Ce^{\alpha x} + De^{-\alpha x}), 0 < x < L$ (5)
$\varphi^2(x) = (F\,e^{ikx} + Ge^{-ikx})(F\,e^{ikx} + Ge^{-ikx}), x < L$ (6)

Notice that we allow for waves traveling in both the directions for $x < 0$ representing the incident and reflected waves. We must also allow for $e^{\gamma x}$ and $e^{-\gamma x}$term in the region $0 < x < L$ because x is finite and there is no danger of $\varphi$ becoming infinite. We have only a wave traveling from left to right of $x > L$ as there cannot be any wave travelling from right to left (reflected wave) since there is no discontinuity in the potential. Hence we must set G=0. The solution, therefore would be

$\varphi^2(x) = (F\,e^{ikx})(F\,e^{ikx}), x > L$ (7)

The continuity conditions (that is, $\varphi$ *and* $d\varphi/dx$ *be* continuous ) at $\mathrm{x} = 0$ and at $\mathrm{x} = L$ yield

At $\mathrm{x} = 0, A + B = C + D$ *and* $ik(A - B) = \alpha(C + D)$ (8)

At $x > L$,

$(Ce^{\gamma L} + De^{-\gamma L})(Ce^{\gamma L} + De^{-\gamma L}) = (F\,e^{ikL})^2$ *and* $\gamma(Ce^{\gamma L} + De^{-\gamma L})\gamma(Ce^{\gamma L} + De^{-\gamma L}) = (ikF\,e^{ikL})^2$ (9)

There are number of ways of solving these equations. If solution leads to

$$\left.\begin{aligned} C^2 &= \left(\frac{[(\gamma+ik)A+(\gamma-ik)B]}{2\gamma}\right)\left(\frac{[(\gamma+ik)A+(\gamma-ik)B]}{2\gamma}\right) \\ D^2 &= \left(\frac{[(\gamma-ik)A+(\gamma+ik)B]}{2\gamma}\right)\left(\frac{[(\gamma-ik)A+(\gamma+ik)B]}{2\gamma}\right)\end{aligned}\right\} x = 0$$

(10)

Similarly

$$\left.\begin{aligned} C^2 &= \left(\frac{[(\gamma+ik)Ae^{-(\gamma-ik)L}F]}{2\gamma}\right)\left(\frac{[(\gamma+ik)Ae^{-(\gamma-ik)L}F]}{2\gamma}\right) \\ D^2 &= \left(\frac{[(\gamma-ik)Ae^{(\gamma+ik)L}F]}{2\gamma}\right)\left(\frac{[(\gamma-ik)Ae^{(\gamma+ik)L}F]}{2\gamma}\right)\end{aligned}\right\} x = L$$

(11)

Equating the values of $C^2$ and $D^2$ to each other yield

$$\left((\gamma + ik)A + (\gamma - ik)B\right)^2 = \left((\gamma + ik)Ae^{-(\gamma-ik)L}F\right)\left((\gamma + ik)Ae^{-(\gamma-ik)L}F\right)$$

(12)

And

$$\left((\gamma - ik)A + (\gamma + ik)B\right)^2 = \left((\gamma - ik)Ae^{(\gamma+ik)L}F\right)\left((\gamma - ik)Ae^{(\gamma+ik)L}F\right)$$

(13)

And so

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$$(B/A)^2 = \left(\frac{(\gamma-ik)}{(\gamma+ik)}\left[e^{(\gamma+ik)L} F/A - 1\right]\right)\left(\frac{(\gamma-ik)}{(\gamma+ik)}\left[e^{(\gamma+ik)L} F/A - 1\right]\right) \quad (14)$$

Putting the above value of $(B/A)^2$ in to (3.14) yields

$$\left(\frac{(\gamma + ik)+(\gamma - ik)^2}{(\gamma + ik)}\left[e^{(\gamma+ik)L}\frac{F}{A} - 1\right]\right)^2 = \left((\gamma + ik)e^{(\gamma+ik)}\frac{F}{A}\right)\left((\gamma + ik)e^{(\gamma+ik)}\frac{F}{A}\right)$$

Or

$$\left(\frac{(\gamma + ik)^2 + (\gamma - ik)^2}{(\gamma + ik)}\left[e^{(\gamma+ik)L}\frac{F}{A} - 1\right]\right)^2 = \left((\gamma + ik)^2 e^{(\gamma+ik)}\frac{F}{A}\right)\left((\gamma + ik)^2 e^{(\gamma+ik)}\frac{F}{A}\right)$$

Or

$$((\gamma + ik)^2 - (\gamma - ik)^2)^2 = \left(\frac{F}{A}\left[(\gamma + ik)^2 e^{(\gamma+ik)L} - (\gamma - ik)^2 e^{(\gamma+ik)L}\right]\right)\left(\frac{F}{A}\left[(\gamma + ik)^2 e^{(\gamma+ik)L} - (\gamma - ik)^2 e^{(\gamma+ik)L}\right]\right)$$

$$\left(\frac{F}{A}\right)^2 = \left(\frac{4ik\gamma}{[(\gamma + ik)^2 e^{(\gamma+ik)L} - (\gamma - ik)^2 e^{(\gamma+ik)L}]}\right)\left(\frac{4ik\gamma}{[(\gamma + ik)^2 e^{(\gamma+ik)L} - (\gamma - ik)^2 e^{(\gamma+ik)L}]}\right)$$

After multiplying the numerator and denominator $e^{(\gamma-ik)L}$

$$\left(\frac{F}{A}\right)^2 = \left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma + ik)^2 - (\gamma - ik)^2 e^{2\gamma L}]}\right)\left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma + ik)^2 - (\gamma - ik)^2 e^{2\gamma L}]}\right)$$

$$= \left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma + ik)^2 - (\gamma - ik)^2 e^{2\gamma L}]}\right)\left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma + ik)^2 - (\gamma - ik)^2 e^{2\gamma L}]}\right)$$

$$= \left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\,(1+e^{2\gamma L})]}\right)\left(\frac{4ik\gamma e^{(\gamma-ik)L}}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\,(1+e^{2\gamma L})]}\right) \quad (15)$$

Putting the value of $\left(\frac{F}{A}\right)^2$ from above into equation (5), we get

$$\left(\frac{F}{A}\right)^2 = \left(\frac{(\gamma^2-k^2)(e^{2\gamma L}-1)}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\,(1+e^{2\gamma L})]}\right)\left(\frac{(\gamma^2-k^2)(e^{2\gamma L}-1)}{[(\gamma^2-k^2)(1-e^{2\gamma L})+2ik\gamma\,(1+e^{2\gamma L})]}\right) \quad (16)$$

It may be mentioned here that in case one is interest in finding $C/A$and$D/A$, this can be achieved by substituting the value of $\left(\frac{F}{A}\right)^2$ from (15) into equations (11).

From (7), the reflection coefficient (or the probability of reflection) is given by

$$R = \frac{j_{ref}}{j_{inc}} = \left(\frac{\hbar k/m\,|B|^2}{\hbar k/m\,|A|^2}\right)^2 = (|B/A|^2)^2 = \left[\left(\frac{B}{A}\right) * \left(\frac{B}{A}\right)\right]^2$$

$$= \left(\frac{(\gamma^2 - k^2)^2(e^{2\gamma L} - 1)^2}{[(\gamma^2 - k^2)(1 - e^{2\gamma L})^2 + 4k^2\gamma^2(1 + e^{2\gamma L})^2]}\right)\left(\frac{(\gamma^2 - k^2)^2(e^{2\gamma L} - 1)^2}{[(\gamma^2 - k^2)(1 - e^{2\gamma L})^2 + 4k^2\gamma^2(1 + e^{2\gamma L})^2]}\right)$$

After dividing the numerator and denominator by $(1 - e^{2\gamma L})^2$ one gets

$$R^2 = \left(\frac{(\gamma^2 - k^2)^2}{\left[(\gamma^2 - k^2)^2 + 4k^2\gamma^2\left\{\frac{(1 + e^{2\gamma L})}{(1 - e^{2\gamma L})}\right\}^2\right]}\right)\left(\frac{(\gamma^2 - k^2)^2}{\left[(\gamma^2 - k^2)^2 + 4k^2\gamma^2\left\{\frac{(1 + e^{2\gamma L})}{(1 - e^{2\gamma L})}\right\}^2\right]}\right)$$

Or

$$= \frac{(\gamma^2 - k^2)^2}{(\gamma^2 - k^2) + 4k^2\gamma^2\left(\frac{1 + e^{4\gamma L} + 2e^{2\gamma L}}{1 + e^{4\gamma L} - 2e^{2\gamma L}} - 1\right) + 4k^2\gamma^2}$$

$$= \left(\frac{(\gamma^2 - k^2)^2}{(\gamma^2 - k^2) + 4k^2\gamma^2\left\{\frac{4}{(e^{2\gamma L} + e^{-2\gamma L} - 2)}\right\}}\right)\left(\frac{(\gamma^2 - k^2)^2}{(\gamma^2 - k^2) + 4k^2\gamma^2\left\{\frac{4}{(e^{2\gamma L} + e^{-2\gamma L} - 2)}\right\}}\right)$$

$$R^2 = \frac{(\gamma^2 - k^2)^2}{(\gamma^2 - k^2) + \frac{4k^2\gamma^2\ 1}{\left(\frac{e^{\gamma L} - e^{-\gamma L}}{2}\right)^2}}$$

$$= \frac{(\gamma^2 - k^2)^2}{\left[(\gamma^2 - k^2) + \frac{4k^2\gamma^2\gamma^2}{\sin\hbar\gamma L}\right]} \qquad (17)$$

After substituting the values of $\gamma^2$and$k^2$, one gets

$$R^2 = \left(\frac{V_O^2}{\left[V_O^2 + \frac{4E(V_O - E)}{\sin\hbar\,\alpha L}\right]}\right)\left(\frac{V_O^2}{\left[V_O^2 + \frac{4E(V_O - E)}{\sin\hbar\,\alpha L}\right]}\right)$$

$$= \left[1 + \frac{4E(V_O - E)}{V_O^2\sin\hbar\alpha L}\right]^{-1} \times \left[1 + \frac{4E(V_O - E)}{V_O^2\sin\hbar\alpha L}\right]^{-1} \qquad (18)$$

The probability of finding the particle in a region$X > 0$, is given the name transmission coefficient T and using equation (15) we have

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$$T^2 = \frac{j_{ref}}{j_{inc}} = \left(\frac{\hbar k/m\,|F|^2}{\hbar k/m\,|A|^2}\right)^2 = \left(\left|\frac{F}{A}\right|^2\right)^2 = \left[\left(\frac{F}{A}\right) * \left(\frac{F}{A}\right)\right]^2$$

$$= \left(\frac{16k^2\gamma^2 e^{2\gamma L}}{[(\gamma^2 - k^2)(1 - e^{2\gamma L})^2 + 4k^2\gamma^2(1 + e^{2\gamma L})^2]}\right)\left(\frac{16k^2\gamma^2 e^{2\gamma L}}{[(\gamma^2 - k^2)(1 - e^{2\gamma L})^2 + 4k^2\gamma^2(1 + e^{2\gamma L})^2]}\right)$$

$$= \left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 4k^2\gamma^2(e^{2\gamma L} + e^{2\gamma L} + 2)}\right)\left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 4k^2\gamma^2(e^{2\gamma L} + e^{2\gamma L} + 2)}\right)$$

Adding and subtracting $4k^2\gamma^2(e^{2\gamma L} + e^{-2\gamma L} - 2)$ from the denominator, one get

$$= \left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 16k^2\gamma^2}\right)\left(\frac{16k^2\gamma^2}{(\gamma^2 - k^2)^2(e^{2\gamma L} + e^{2\gamma L} - 2) + 16k^2\gamma^2}\right)$$

$$= \left(\frac{4k^2\gamma^2}{\left[(\gamma^2 - k^2)^2\left(\frac{e^{\gamma L} - e^{\gamma L}}{2}\right)^2 + 4k^2\gamma^2\right]}\right)\left(\frac{4k^2\gamma^2}{\left[(\gamma^2 - k^2)^2\left(\frac{e^{\gamma L} - e^{\gamma L}}{2}\right)^2 + 4k^2\gamma^2\right]}\right)$$

$$= \left(\frac{4k^2\gamma^2}{(\gamma^2 - k^2)^2 \sin\hbar^2\gamma L + 4k^2\gamma^2}\right)\left(\frac{4k^2\gamma^2}{(\gamma^2 - k^2)^2 \sin\hbar^2\gamma L + 4k^2\gamma^2}\right) \quad (19)$$

Putting the value of $\gamma^2$ and $k^2$ one gets

$$T^2 = \left[1 + \frac{V_O^2 \sin\hbar^2\alpha L}{4E(V_O - E)}\right]^{-1} \times \left[1 + \frac{V_O^2 \sin\hbar^2\alpha L}{4E(V_O - E)}\right]^{-1} \quad (20)$$

One may, however check that$R + T = 1$. There are two interesting situations in which equations (17) to (20) become simpler considering the purely formal limit in which$\hbar \to 0$. The quantity $\hbar$ is a physical constant, but we can consider as a mathematical variable in order to examine the classical limit of our formulas. As$\hbar \to 0$, k and $\gamma$ approach infinity and hence$T \to 0$, $R \to 1$, which is of course, the proper behavior of a classical particle with $E < V_O$. The other interesting limit occurs for high and wide barrier, that is, when $\gamma \gg 1$. In that case $\sin\hbar^2 \gamma L \approx \frac{1}{2}e^{\gamma L}$, hence form (3.20) after neglecting 1 in comparison to the other which is very large, one gets

$$T^2 = \left(\frac{4E(V_O - E)}{V_O^2\left[\frac{1}{2}e^{-2\left\{\frac{2m(V_O - E)}{\hbar^2}\right\}^{\frac{1}{2}}L}\right]^2}\right)\left(\frac{4E(V_O - E)}{V_O^2\left[\frac{1}{2}e^{-2\left\{\frac{2m(V_O - E)}{\hbar^2}\right\}^{\frac{1}{2}}L}\right]^2}\right)$$

$$= \left(16\,\frac{E}{V_O}\left(1 - \frac{E}{V_O}\right)e^{-2\left\{\frac{2m(V_O - E)}{\hbar^2}\right\}^{\frac{1}{2}}L}\right)\left(16\,\frac{E}{V_O}\left(1 - \frac{E}{V_O}\right)e^{-2\left\{\frac{2m(V_O - E)}{\hbar^2}\right\}^{\frac{1}{2}}L}\right) \quad (21)$$

From equation (21) transmission coefficient would be given by

$$T^2 = \left(16\,\frac{E}{V(r_0)}\left[1-\frac{E}{V(r_0)}\right]e^{-2\gamma L}\right)\left(16\,\frac{E}{V(r_0)}\left[1-\frac{E}{V(r_0)}\right]e^{-2\gamma L}\right) \quad 22)$$

$\gamma L \gg 1$, the most important factor in the above equation is the exponential. The factor in front of the exponential which is of the order of 2 is not significant since its variation with V and E is negligible as compared to the variation in exponential itself(Chaddha, 1983). Hence we can write

$$InT^2 \simeq -4\gamma L \quad (23)$$

For a rectangular double thick potential barrier of thickness dx, we can write

$$InT^2 \simeq -4\gamma dx \quad (24)$$

Where

$$\gamma^4 = \left(\frac{2m}{\hbar^2}[V(x)-E]\right)\left(\frac{2m}{\hbar^2}[V(x)-E]\right)$$

$$= \left(\frac{2m}{\hbar^2}\left[\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right]\right)\left(\frac{2m}{\hbar^2}\left[\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right]\right) \quad (25)$$

making$\gamma$ a function of x

Equation (25) expression for the transmission coefficient or tunneling probability of a rectangular barrier. The actual barrier encountered by gamma particle has an exponential tail. We can approximate it as consisting of many rectangular barrier of decreasing height and obtain the total probability by summing the tunneling probability of each barrier the region between $r_0$and$r_1$. In this entire region, of course $E < V$. Hence taking the summation over all the rectangular potential barriers, we gets

$$In\,T^2 = \left(-2\int_{r_0}^{r_1}\gamma(x)dx\right)\left(-2\int_{r_0}^{r_1}\gamma(x)dx\right) \quad (26)$$

From equation (3.25) that $\gamma$ can be while is a function of x

$$\gamma = \left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right)^{\frac{1}{2}}\right)\left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right)^{\frac{1}{2}}\right) \quad (27)$$

Substituting equation (27) in to equation (26)

$$In\,T^2 = \left(-2\int_{r_0}^{r_1}\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right)^{\frac{1}{2}}dx\right)\left(-2\int_{r_0}^{r_1}\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\left(\left(\frac{2Ze^2}{4\pi\epsilon_0 x}\right)-E\right)^{\frac{1}{2}}dx\right) \quad (28)$$

Making use of equation (21), leads to

$$In\,T^2 = \left(-2\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\int_{r_0}^{r_1}\left(\frac{r_0}{x}-1\right)^{\frac{1}{2}}dx\right)\left(-2\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\int_{r_0}^{r_1}\left(\frac{r_0}{x}-1\right)^{\frac{1}{2}}dx\right) \quad (29)$$

Putting $x = r_1 \cos^2\theta, dx = r_1 2\cos\theta\ (-\sin\theta\ d\theta)$ and also changing the limits to $\theta($ $at\ x = r_0, \theta_0 = \cos^{-1}\left(\frac{r_0}{x}\right)^{\frac{1}{2}}$ $and\ at\ x = r_0, \theta_0 = 0)$, one gets

$$In\,T^2 = \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\left(\frac{1}{\cos^2\theta} - 1\right)^{\frac{1}{2}} \sin\theta\cos\theta\,dx\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\left(\frac{1}{\cos^2\theta} - 1\right)^{\frac{1}{2}} \sin\theta\cos\theta\,dx\right) \quad (30)$$

Since

$$\left(\left(\frac{1}{\cos^2\theta} - 1\right)^{\frac{1}{2}}\right)^2 = \left(\frac{(1-\cos^2\theta)^{\frac{1}{2}}}{\cos\theta}\right)^2 = \left(\frac{\sin\theta}{\cos\theta}\right)^2$$

The double thick potential barrier is on the x coordinate

$$In\,T^2 = \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\sin^2\theta\,d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\sin^2\theta\,d\theta\right) \quad (31)$$

Using trigonometric rule and integrating

$$In\,T^2 = \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\frac{1-\cos 2\theta}{2}\,d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\frac{1-\cos 2\theta}{2}\,d\theta\right) \quad (32)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right)d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right)d\theta\right) \quad (33)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\frac{1}{2}d\theta - \frac{1}{2}\int_0^{\theta_0}\cos 2\theta\ d\theta\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \int_0^{\theta_0}\frac{1}{2}d\theta - \frac{1}{2}\int_0^{\theta_0}\cos 2\theta\ d\theta\right) \quad (34)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \left(\frac{1}{2}\int_0^{\theta_0}d\theta - \frac{1}{2}\int_0^{\theta_0}\cos 2\theta\ d\theta\right)\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1 \left(\frac{1}{2}\int_0^{\theta_0}d\theta - \frac{1}{2}\int_0^{\theta_0}\cos 2\theta\ d\theta\right)\right) \quad (35)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1(\theta_0 - (\cos^2\theta - \sin^2\theta))\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1(\theta_0 - (\cos^2\theta - \sin^2\theta))\right) \quad (36)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\theta_0 - \left(\cos^2\theta - (1-\cos^2\theta)\right)\right)\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1\left(\theta_0 - \left(\cos^2\theta - (1-\cos^2\theta)\right)\right)\right) \quad (37)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1(\theta_0 - (\cos^2\theta + \cos^2\theta - 1))\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1(\theta_0 - (\cos^2\theta + \cos^2\theta - 1))\right) \quad (38)$$

$$= \left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1(\theta_0 - (2\cos^2\theta - 1))\right)\left(-2\left(\frac{2m\,E}{\hbar}\right)^{\frac{1}{2}} r_1(\theta_0 - (2\cos^2\theta - 1))\right) \quad (39)$$

After putting the value of E

$$In\,T^2 = \left(-2\left[\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right]^{1/2}\left[\cos^{-1}\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}-\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\left(1-\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right)\left(-2\left[\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right]^{1/2}\left[\cos^{-1}\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}-\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\left(1-\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right) \quad (40)$$

Because of the fact that the potential barrier is relatively wide, $r_1 \gg r_0$,

$$\cos^{-1}\left(\frac{r_0}{x}\right)^{\frac{1}{2}} \approx \frac{\pi}{2}-\left(\frac{r_0}{x}\right)^{\frac{1}{2}}$$

As $\cos\left\{\frac{\pi}{2}-\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right\} = \sin\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} \simeq \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}$

If $\left(\frac{r_0}{r_1}\right) \ll 1$

Also

$$\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} \approx \left(\frac{r_0}{r_1}\right)^{\frac{1}{2}} \approx 1$$

Hence from equation (39)

$$In\,T^2 = \left(-2\left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right)^{\frac{1}{2}}\left[\pi/2-2\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right)\left(-2\left(\left(\frac{2m}{\hbar}\right)^{\frac{1}{2}}\frac{2Ze^2}{4\pi\epsilon_0 r_1}\right)^{\frac{1}{2}}\left[\pi/2-2\left(\frac{r_0}{r_1}\right)^{\frac{1}{2}}\right]\right) \quad (41)$$

Replacing $r_1$ by $r_1 = \frac{2Ze^2}{4\pi\epsilon_0}$ and simplifying

$$InT^2 = \left(4\,\frac{e}{\hbar}\left(\frac{m}{\pi\epsilon_0}\right)^{\frac{1}{2}} Z^{\frac{1}{2}} r_0^{\frac{1}{2}} - \frac{e^2}{\hbar\epsilon_0}\left(\frac{m}{2}\right)^{\frac{1}{2}} Z\,E^{-\frac{1}{2}}\right)\left(4\,\frac{e}{\hbar}\left(\frac{m}{\pi\epsilon_0}\right)^{\frac{1}{2}} Z^{\frac{1}{2}} r_0^{\frac{1}{2}} - \frac{e^2}{\hbar\epsilon_0}\left(\frac{m}{2}\right)^{\frac{1}{2}} Z\,E^{-\frac{1}{2}}\right) \quad (42)$$

$$InT^2 = 4^2\left(\frac{e}{\hbar}\right)^2\left(\frac{m}{\pi\epsilon_0}\right) Z^{\frac{1}{4}} r_0^{\frac{1}{4}} - \frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right) Z^2 E^{-\frac{1}{4}} \quad (43)$$

Equation (43) gives the natural logarithm of the tunneling probability of the gamma particle.

## Results

We assess the ability of gamma particle in tunneling through a barrier, its relationship with decay constant and half-life using equation (43)

$$\underbrace{In\,T^2}_{K_1} = \underbrace{\underbrace{4^2\frac{e^2}{\hbar^2}\left(\frac{m}{\pi\epsilon_0}\right)}_{I_1} Z^{\frac{1}{4}} r_0^{\frac{1}{4}}}_{J_1} - \underbrace{\underbrace{\frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right)}_{I_2} Z^2 E^{-\frac{1}{4}}}_{J_2} \qquad 43$$

The constant $I_1$ and $I_2$ are to be calculated while:

$Z$ = atomic number of the daughter nucleus (the gamma emitting nucleus)

$$r_o = 1.1\left(A_d^{\frac{1}{2}} + A_\gamma^{\frac{1}{2}}\right) \times 10^{-15} m \text{ (for each nucleus)} \qquad 44$$

$E$ = Potential energy of the emitted gamma particle

= or energy of decay for each nucleus

$m$ = mass of gamma particle

$1$ atomic mass unit $= 1.66 \times 10^{-27} kg$

$$\left.\begin{aligned} e &= 1.6 \times 10^{-19} C \\ \hbar &= 1.05477 \times 10^{-34} Js \\ \epsilon_o &= 8.85 \times 10^{-12} Farad/m \end{aligned}\right\} \text{all are in S.I unit}$$

To keep equation (3.64) as simple as possible we calculate the constant $I_1$ and $I_2$

$$I_1 = 4^2\frac{e^2}{\hbar^2}\left(\frac{m}{\pi\epsilon_0}\right) \qquad 45$$

$$I_1 = 8.792420946 \times 10^{15} \qquad 46$$

$$I_2 = \frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right) \qquad 47$$

$$I_2 = 2.496984634 \times 10^{-12} \qquad 48$$

$$K_1 = T^2 \qquad 49$$

Let $T^2$ be $DT$

$$K_1 = DT \qquad 50$$

$$InDT = 8.792420946 \times 10^{15} Z^{\frac{1}{4}} r_0^{\frac{1}{4}} - 2.496984634 \times 10^{-12} Z^2 E^{-\frac{1}{4}} \qquad 51$$

Equation (51) is used to get the result for tunneling for every $\gamma$ emitting nucleus as show in Table 4.1

The decay probability per unit time or constant we write

$$\lambda = \Gamma T \qquad 52$$

Where $\Gamma$ = number of time per second gamma particle within a nucleus strikes the potential barrier

$T$ = the probability of transmission through the barrier.

Assume only one gamma particle exists within a nucleus moving to and fro in the nuclear diameter

$$\Gamma = \frac{v}{2r_0}$$ 53

Where $v = \gamma$ particle velocity when it finally leaves the nucleus

$$\lambda = \frac{v}{2r_0} DT$$ 54

$$v = 10^7 ms^{-1}, r_0 = 10^{-14} m$$

$$\lambda = \frac{10^7}{2\times10^{-14}} DT \simeq 10^{-21} DT$$ 55

Equation (55) can be used to get the result for decay probability per unit time. The half life $t_{\frac{1}{2}}$ is the time taken for half the original number of atom present to decay. Mathematically half-life $t_{\frac{1}{2}}$ can written as

$$t_{\frac{1}{2}} = \frac{In\ 2}{\lambda}$$ 56

Substitute equation (56) into (55) gives

Table 1: ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ gamma particle emitting nuclei and their decay probability

| S/N | Nucleus (name) | Mass No. (A) | Z | Mass Excess A(KeV) | $r_0$ | E $\gamma(J)$ | In DT(E12) | DT | Decay constant (E-20) | | Half-life (E23) $t_{\frac{1}{2}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Kr | 75 | 36 | 132.4 | 9.526279442 | 1.081469678E-14 | 6.728293536 | 29.53734267 | 2.953734267 | 2.046937849 | |
| 2 | Rb | 76 | 37 | 2571.1 | 9.589577676 | 3.97566408E-13 | 6.785920598 | 29.54587108 | 2.954587108 | 2.047528866 | |
| 3 | Sr | 80 | 38 | 589.0 | 9.838699101 | 2.146917582E-14 | 6.82001297 | 29.55088249 | 2.956088249 | 2.047846157 | |
| 4 | Y | 80 | 39 | 385.9 | 9.838699101 | 4.524548695E-14 | 6.920046991 | 29.56544368 | 2.956544368 | 2.048885247 | |
| 5 | Y | 81 | 39 | 124.2 | 9.90 | 1.357044173E-14 | 6.930800892 | 29.56699649 | 2.956699649 | 2.046937847 | |
| 6 | Y | 87 | 39 | 484.5 | 10.26011696 | 5.34005694E-14 | 6.992986692 | 29.5792886 | 2.95792886 | 2.04961187 | |
| 7 | Zr | 80 | 40 | 311.0 | 9.838699101 | 5.12696736E-15 | 6.963896005 | 29.57177313 | 2.957177313 | 2.049323878 | |
| 8 | Zr | 85 | 40 | 416.5 | 10.1414989 | 2.552670702E-14 | 7.016960099 | 29.57935121 | 2.957935121 | 2.049849036 | |
| 9 | Zr | 89 | 40 | 909.1 | 10.37737925 | 6.802844816E-14 | 7.056410711 | 29.58509935 | 2.958509935 | 2.050247385 | |
| 10 | Zr | 90 | 40 | 2186.2 | 10.43551628 | 5.609222727E-14 | 7.067274417 | 29.58649601 | 2.958649601 | 2.050344173 | |
| 11 | Nb | 84 | 41 | 540.0 | 10.08166653 | 6.584948703E-14 | 6.995280444 | 29.57625182 | 2.957265682 | 2.049634598 | |
| 12 | Nb | 86 | 41 | 751.7 | 10.20098035 | 1.02411173E-13 | 7.070740754 | 29.58698636 | 2.988698636 | 2.050378155 | |
| 13 | Nb | 88 | 41 | 1057.1 | 10.31891467 | 1.026354778E-13 | 7.09112895 | 29.58986568 | 2.958986568 | 2.050577692 | |
| 14 | Nb | 89 | 41 | 1627.7 | 10.37737952 | 2.285505918E-13 | 7.101111936 | 29.5912725 | 2.95912725 | 2.050695184 | |
| 15 | Mo | 106 | 42 | 465.7 | 11.32519316 | 4.381954916E-14 | 7.30182856 | 29.51915492 | 2.961914592 | 2.052606812 | |
| 16 | Mo | 107 | 42 | 400.3 | 11.37848848 | 4.897856006E-14 | 7.310413426 | 29.62032094 | 2.962032094 | 2.052688241 | |
| 17 | Tc | 88 | 43 | 741.0 | 10.31891467 | 1.10742495E-13 | 7.176027451 | 29.60176707 | 2.960176707 | 2.051402458 | |
| 18 | Tc | 90 | 43 | 948.1 | 10.43551628 | 1.24525198E-13 | 7.19621402 | 29.6045747 | 2.960457617 | 2.051596817 | |
| 19 | Tc | 91 | 43 | 653.0 | 10.94486181 | 8.558831137E-14 | 7.282460819 | 29.61648995 | 2.961468995 | 2.052422754 | |
| 20 | Ru | 91 | 44 | 393.7 | 10.49333122 | 4.401181043E-14 | 7.247696298 | 29.61170478 | 2.961170478 | 2.052091141 | |
| 21 | Ru | 97 | 44 | 215.7 | 10.83374358 | 1.858525668E-15 | 7.305774774 | 29.61968622 | 2.961968622 | 2.052644255 | |
| 22 | Ru | 105 | 44 | 724.3 | 11.27164584 | 6.054628017E-14 | 7.378506638 | 29.62959239 | 2.962959239 | 2.053330753 | |
| 23 | Rh | 92 | 45 | 893.0 | 10.55082935 | 1.183047718E-13 | 7.298493917 | 29.61868913 | 2.961868913 | 2.052575157 | |

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|---|---|---|---|---|---|---|---|---|---|---|
| 24 | Rh | 94 | 45 | 756.2 | 10.66489569 | 8.690209675E-14 | 7.318140675 | 29.62137741 | 2.962137741 | 2.052761454 |
| 25 | Rh | 96 | 45 | 832.6 | 10.77775487 | 7.459737509E-14 | 7.337425015 | 29.62400908 | 2.962400908 | 2.052943829 |
| 26 | Rh | 99 | 45 | 341.0 | 10.94486181 | 2.007528157E-14 | 7.365702458 | 29.62785554 | 2.962785554 | 2.053210389 |
| 27 | Pd | 115 | 46 | 749.0 | 11.79618582 | 6.410311377E-14 | 7.542677371 | 29.6207549 | 2.965207549 | 2.054888832 |
| 28 | Pd | 117 | 46 | 247.3 | 11.898831921 | 1.912999696E-14 | 7.56256872 | 29.65423203 | 2.965423203 | 2.055038279 |
| 29 | Ag | 95 | 47 | 1261.2 | 10.72147378 | 1.636634412E-13 | 7.407924692 | 29.63357145 | 2.963357145 | 2.053606501 |
| 30 | Ag | 99 | 47 | 342.6 | 10.94486181 | 1.238483053E-14 | 7.446213897 | 29.63872682 | 2.963472682 | 2.053963768 |
| 31 | Cd | 100 | 48 | 936.6 | 11.0 | 1.177600316E-14 | 7.494919158 | 29.64524646 | 2.964524646 | 2.054415558 |
| 32 | Cd | 105 | 48 | 961.8 | 11.27164584 | 1.350795682E-13 | 7.540768622 | 29.65134513 | 2.965134523 | 2.054838224 |
| 33 | In | 104 | 49 | 658.0 | 11.21784293 | 2.834251644E-14 | 7.570678971 | 29.65530387 | 2.965530387 | 2.055112558 |
| 34 | In | 106 | 49 | 632.6 | 11.32519316 | 8.797555554E-14 | 7.588726425 | 29.6576849 | 2.96576849 | 2.055277564 |
| 35 | Sn | 105 | 50 | 1281.7 | 11.27164584 | 1.752461531E-13 | 7.618119947 | 29.66155073 | 2.966155073 | 2.055545466 |
| 36 | Sn | 107 | 50 | 678.6 | 11.37848848 | 8.765512008E-14 | 7.63610896 | 29.66390929 | 2.966390929 | 2.055708914 |
| 37 | Sb | 108 | 51 | 1205.8 | 11.43153533 | 1.312663862E-13 | 7.682934965 | 29.67002275 | 2.967002275 | 2.056132577 |
| 38 | Sb | 112 | 51 | 1257.1 | 11.64130577 | 1.56548744E-13 | 7.717940754 | 29.6745687 | 2.96745687 | 2.056447611 |
| 39 | Te | 113 | 52 | 814.0 | 11.69316039 | 1.056956365E13 | 7.72651097 | 29.67567851 | 2.967567851 | 2.056524521 |
| 40 | Te | 115 | 52 | 770.4 | 11.79618582 | 4.364330965E-14 | 7.781166519 | 29.68272734 | 2.968272734 | 2.057013005 |
| 41 | I | 112 | 53 | 689.0 | 11.64130577 | 9.308650113E-14 | 7.79251889 | 29.68418527 | 2.968418527 | 2.057114039 |
| 42 | I | 114 | 53 | 708.8 | 11.74478608 | 5.380111373E-14 | 7.809778513 | 29.68639772 | 2.968639772 | 2.057267262 |
| 43 | Xe | 135 | 54 | 786.9 | 12.78084504 | 2.235037334E-14 | 8.013953247 | 29.71220529 | 2.971220529 | 2.059055827 |
| 44 | Xe | 140 | 54 | 805.6 | 13.01537552 | 3.471918209E-14 | 8.050467255 | 29.71675125 | 2.971675125 | 2.059370862 |
| 45 | Cs | 116 | 55 | 393.5 | 11.84736258 | 1.361850705E-14 | 7.8995902 | 29.697832 | 2.9697832 | 2.058059134 |
| 46 | Cs | 125 | 55 | 525.0 | 12.29837388 | 6.025788825E-14 | 7.973721585 | 29.70717245 | 2.970717245 | 2.05870705 |
| 47 | Ba | 126 | 56 | 233.6 | 12.34746938 | 3.52479006E-15 | 8.01770307 | 29.7126731 | 2.97126731 | 2.059088246 |
| 48 | Ba | 143 | 56 | 211.5 | 13.15405652 | 2.611548999E-14 | 8.145554522 | 29.72849344 | 2.972549344 | 2.060184595 |
| 49 | La | 126 | 57 | 256.0 | 12.34746938 | 1.385883365E-14 | 8.053259166 | 29.71709799 | 2.971709799 | 2.059394591 |
| 50 | La | 130 | 57 | 357.4 | 12.54192968 | 1.345828932E-14 | 7.533071049 | 29.65032392 | 2.965032392 | 2.054767448 |
| 51 | Ce | 127 | 58 | 120.4 | 12.39637044 | 8.395409052E-15 | 7.305774775 | 29.61928622 | 2.961968622 | 2.052644255 |
| 52 | Ce | 133 | 58 | 477.2 | 12.68581885 | 5.913636414E-14 | 7.378506684 | 29.62959239 | 2.962959239 | 2.053330752 |
| 53 | Pr | 129 | 59 | 203.8 | 12.49359836 | 1.336215868E-14 | 7.2984939176 | 29.61868913 | 2.961868913 | 2.052575157 |
| 54 | Pr | 137 | 59 | 836.9 | 12.8751699 | 9.989575466E-14 | 7.318140675 | 29.62137741 | 2.962137741 | 2.052761455 |
| 55 | Nd | 133 | 60 | 402.8 | 12.68581885 | 4.24256549E-14 | 7.337425015 | 29.62400708 | 2.962400908 | 2.05293829 |
| 56 | Nd | 152 | 60 | 278.6 | 13.56171081 | 2.377661113E-14 | 7.365702458 | 29.62785564 | 2.962785564 | 2.053210389 |
| 57 | Pm | 136 | 61 | 373.7 | 12.82809417 | 3.375787571E-14 | 7.546278371 | 29.65207549 | 2.965207549 | 2.04888832 |
| 58 | Sm | 137 | 62 | 380.5 | 12.8751699 | 2.911156154E-14 | 7.56256872 | 29.65423205 | 2.965423203 | 2.055638279 |
| 59 | Eu | 139 | 63 | 719.0 | 12.96880873 | 5.423370161E-14 | 7.407924692 | 29.63357145 | 2.963357145 | 2.053606501 |
| 60 | Gd | 159 | 64 | 363.0 | 13.87047223 | 4.792112304E-14 | 7.4462137979 | 29.63872782 | 2.963872728 | 2.053963758 |
| 61 | Tb | 144 | 65 | 284.0 | 13.20 | 1.683888342E-14 | 8.462141713 | 29.76662342 | 2.976662342 | 2.062827003 |
| 62 | Dy | 145 | 66 | 578.2 | 13.24575404 | 8.542809364E-14 | 8.501853707 | 29.77130541 | 2.977130534 | 2.06315146 |
| 63 | Ho | 146 | 67 | 682.7 | 13.29135057 | 7.786581678E-14 | 8.541211008 | 29.77592392 | 2.977592392 | 2.063471528 |
| 64 | Er | 151 | 68 | 1140.2 | 13.5770263 | 1.691899229E-14 | 8.609065064 | 29.78383684 | 2.978383684 | 2.064019893 |
| 65 | Tm | 152 | 69 | 808.2 | 13.56171081 | 5.322432991E-14 | 8.647675105 | 29.78831163 | 2.978831163 | 2.064329996 |
| 66 | Yb | 157 | 70 | 231.1 | 1379296049 | 2.291113539E-14 | 8.714021135 | 29.79595447 | 2.979595447 | 2.064859645 |

$$t_{\frac{1}{2}} = \frac{In\ 2}{10^{-21}DT} \quad 57$$

This equation gives the result for half-life of gamma emitting nucleus substitute equation (57) into (51)

$$t_{\frac{1}{2}} = \frac{In\ 2}{10^{-21}} e^{-\left[4^2\frac{e^2}{\hbar^2}\left(\frac{m}{\pi\epsilon_0}\right)Z^{\frac{1}{4}}r_0^{\frac{1}{4}} - \frac{e^4}{(\hbar\epsilon_0)^2}\left(\frac{m}{2}\right)Z^2E^{-\frac{1}{4}}\right]} \qquad 58$$

$$t_{\frac{1}{2}} = 6.93 \times 10^{21} \times e^{-\left[8.792420946\times10^{15}Z^{\frac{1}{4}}r_0^{\frac{1}{4}} - 2.496984634\times10^{-12}Z^2E^{-\frac{1}{4}}\right]} \qquad 59$$

Table 2: ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ gamma particle emitting nuclei and their calculated and experimental half lives

| S/N | Nucleus (name) | Mass No. (A) | Z | E $\gamma(J)$ | In DT (E12) | DT | Log Decay constant | Log Half-life $t_{\frac{1}{2}}$ | Log Half-life $t_{\frac{1}{2}}$ (from chart) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Kr | 75 | 36 | 1.081469678E-14 | 6.728293536 | 29.53734267 | -19.52962858 | 23.31110466 | 2.411619406 |
| 2 | Rb | 76 | 37 | 3.97566408E-13 | 6.785920598 | 29.54587108 | -19.529032 | 23.31123003 | 1.568201724 |
| 3 | Sr | 80 | 38 | 2.146917582E-14 | 6.82001297 | 29.55088249 | -19.52942955 | 23.31130369 | 3.804275767 |
| 4 | Y | 80 | 39 | 4.524548695E-14 | 6.920046991 | 29.56544368 | -19.5292156 | 23.31151764 | 0.6812412374 |
| 5 | Y | 81 | 39 | 1.357044173E-14 | 6.930800892 | 29.56699649 | -19.5219279 | 23.31110466 | 1.908485019 |
| 6 | Y | 87 | 39 | 5.34005694E-14 | 6.992986692 | 29.5792886 | -19.52901228 | 23.31167163 | 3.683407299 |
| 7 | Zr | 80 | 40 | 5.12696736E-15 | 6.963896005 | 29.57177313 | -19.52912263 | 23.3116106 | 0.6989700043 |
| 8 | Zr | 85 | 40 | 2.552670702E-14 | 7.016960099 | 29.57935121 | -19.52901136 | 23.31172188 | 1.037426498 |
| 9 | Zr | 89 | 40 | 6.802844816E-14 | 7.056410711 | 29.58509935 | -19.52892697 | 23.31180627 | 3.672910245 |
| 10 | Zr | 90 | 40 | 5.609222727E-14 | 7.067274417 | 29.58649601 | -19.58728819 | 23.31182677 | 2.907945522 |
| 11 | Nb | 84 | 41 | 6.584948703E-14 | 6.995280444 | 29.57625182 | -19.52905679 | 23.31167644 | 1.09181246 |
| 12 | Nb | 86 | 41 | 1.02411173E-13 | 7.070740754 | 29.58698636 | -19.52889927 | 23.31183397 | 1.942504106 |
| 13 | Nb | 88 | 41 | 1.026354778E-13 | 7.09112895 | 29.58986568 | -19.52885701 | 23.31187623 | 2.66464976 |
| 14 | Nb | 89 | 41 | 2.285505918E-13 | 7.101111936 | 29.5912725 | -19.52883836 | 23.31189688 | 3.857332496 |
| 15 | Mo | 106 | 42 | 4.381954916E-14 | 7.30182856 | 29.51915492 | -19.52842747 | 23.31230577 | 0.9395192526 |
| 16 | Mo | 107 | 42 | 4.897856006E-14 | 7.310413426 | 29.62032094 | -19.52841020 | 23.31232299 | 0.5440680444 |
| 17 | Tc | 88 | 43 | 1.10742495E-13 | 7.176027451 | 29.60176707 | -19.52868286 | 23.31205087 | 0806179974 |
| 18 | Tc | 90 | 43 | 1.24525198E-13 | 7.19621402 | 29.6045747 | -19.52864115 | 23.31209202 | 1.691965103 |
| 19 | Tc | 91 | 43 | 8.558831137E-14 | 7.282460819 | 29.61648995 | -19.528466 | 23.31226682 | 2.29666519 |
| 20 | Ru | 91 | 44 | 4.401181043E-14 | 7.247696298 | 29.61170478 | -19.52853659 | 23.31219665 | 2.346352974 |
| 21 | Ru | 97 | 44 | 1.858525668E-15 | 7.305774774 | 29.61968622 | -19.52841955 | 23.31231369 | 3.619260335 |
| 22 | Ru | 105 | 44 | 6.054628017E-14 | 7.378506638 | 29.62959239 | -19.52827432 | 23.31245891 | 4.203685471 |
| 23 | Rh | 92 | 45 | 1.183047718E-13 | 7.298493917 | 29.61868913 | -19.52843417 | 23.31229907 | 0.6989700043 |
| 24 | Rh | 94 | 45 | 8.690209675E-14 | 7.318140675 | 29.62137741 | -19.52839475 | 23.31233848 | 1.411619706 |
| 25 | Rh | 96 | 45 | 7.459737509E-14 | 7.337425015 | 29.62400908 | -19.52835617 | 23.31237707 | 1957128198 |
| 26 | Rh | 99 | 45 | 2.007528157E-14 | 7.365702458 | 29.62785554 | -19.52829978 | 23.31243345 | 4.228400359 |
| 27 | Pd | 115 | 46 | 6.410311377E-14 | 7.542677371 | 29.6207549 | -19.5279449 | 23.31278833 | 1.698970004 |
| 28 | Pd | 117 | 46 | 1.912999696E-14 | 7.56256872 | 29.65423203 | -19.5279132 | 23.31281992 | 0.6434526765 |
| 29 | Ag | 95 | 47 | 1.636634412E-13 | 7.407924692 | 29.63357145 | -19.528216 | 23.31251723 | 0.27875601 |
| 30 | Ag | 99 | 47 | 1.238483053E-14 | 7.446213897 | 29.63872682 | -19.52841047 | 23.31259278 | 1.041392685 |
| 31 | Cd | 100 | 48 | 1.177600316E-14 | 7.494919158 | 29.64524646 | -19.52804493 | 23.3126883 | 1691081492 |
| 32 | Cd | 105 | 48 | 1.350795682E-13 | 7.540768622 | 29.65134513 | -19.5279556 | 23.31277764 | 3.522444234 |
| 33 | In | 104 | 49 | 2.834251644E-14 | 7.570678971 | 29.65530387 | -19.52789762 | 23.31283561 | 2.035829825 |
| 34 | In | 106 | 49 | 8.797555554E-14 | 7.588726425 | 29.6576849 | -19.52786275 | 23.31287048 | 2.502427212 |
| 35 | Sn | 105 | 50 | 1.752461531E-13 | 7.618119947 | 29.66155073 | -19.52780615 | 23.31292709 | 1.531478197 |
| 36 | Sn | 107 | 50 | 8.765512008E-14 | 7.63610896 | 29.66390929 | -19.52777162 | 23.31296162 | 2.243534107 |
| 37 | Sb | 108 | 51 | 1.312663862E-13 | 7.682934965 | 29.67002275 | -19.5276212 | 23.31305111 | 0.8692317197 |
| 38 | B | 112 | 51 | 1.56548744E-13 | 7.717940754 | 29.6745687 | -19.52761558 | 23.31311765 | 1.1710963119 |
| 39 | Te | 113 | 52 | 1.056956365E13 | 7.72651097 | 29.67567851 | -19.52759934 | 23.31313389 | 2.008600172 |
| 40 | Te | 115 | 52 | 4.364330965E-14 | 7.781166519 | 29.68272734 | -19.5274962 | 23.31232704 | 2.604226053 |
| 41 | I | 112 | 53 | 9.308650113E-14 | 7.79251889 | 29.68418527 | -19.52747487 | 23.31325837 | 0.531478917 |
| 42 | I | 114 | 53 | 5.380111373E-14 | 7.809778513 | 29.68639772 | -19.5274425 | 23.31329074 | 0.7923916895 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 43 | Xe | 135 | 54 | 2.235037334E-14 | 8.013953247 | 29.71220529 | -19.52706511 | 23.31366812 | 2.962842681 |
| 44 | Xe | 140 | 54 | 3.471918209E-14 | 8.050467255 | 29.71675125 | -19.52699867 | 23.31373456 | 1.133538908 |
| 45 | Cs | 116 | 55 | 1.361850705E-14 | 7.8995902 | 29.697832 | -19.52727559 | 23.31345785 | 0.84509804 |
| 46 | Cs | 125 | 55 | 6.025788825E-14 | 7.973721585 | 29.70717245 | -19.52713868 | 23.31354455 | 3.431363764 |
| 47 | Ba | 126 | 56 | 3.52479006E-15 | 8.01770307 | 29.7126731 | -19.52705828 | 23.31367496 | 3.773786445 |
| 48 | Ba | 143 | 56 | 2.611548999E-14 | 8.145554522 | 29.72849344 | -19.5268271 | 23.31390614 | 1.155336037 |
| 49 | La | 126 | 57 | 1.385883365E-14 | 8.053259166 | 29.71709799 | -19.5269936 | 23.31373963 | 1.698970004 |
| 50 | La | 130 | 57 | 1.345828932E-14 | 7.533071049 | 29.65032392 | -19.52797056 | 23.31276268 | 2.717670503 |
| 51 | Ce | 127 | 58 | 8.395409052E-15 | 7.305774775 | 29.61928622 | -19.52841955 | 23.31231369 | 1.531478917 |
| 52 | Ce | 133 | 58 | 5.913636414E-14 | 7.378506684 | 29.62959239 | -19.52827432 | 23.31245891 | 2.51054501 |
| 53 | Pr | 129 | 59 | 1.336215868E-14 | 7.2984939176 | 29.61868913 | -19.52843417 | 23.31229907 | 1.477121255 |
| 54 | Pr | 137 | 59 | 9.989575466E-14 | 7.318140675 | 29.62137741 | -19.52839475 | 23.31233843 | 1.88536122 |
| 55 | Nd | 133 | 60 | 4.24256549E-14 | 7.337425015 | 29.62400708 | -19.52835617 | 23.31237589 | 1.84509804 |
| 56 | Nd | 152 | 60 | 2.377661113E-14 | 7.365702458 | 29.62785564 | -19.52829978 | 23.31243345 | 2.835056102 |
| 57 | Pm | 136 | 61 | 3.375787571E-14 | 7.546278371 | 29.65207549 | -19.5279449 | 23.31278833 | 1.672097858 |
| 58 | Sm | 137 | 62 | 2.911156154E-14 | 7.56256872 | 29.65423205 | -19.52791332 | 23.3129467 | 1.653212514 |
| 59 | Eu | 139 | 63 | 5.423370161E-14 | 7.407924692 | 29.63357145 | -19.528216 | 23.31251723 | 1.255272505 |
| 60 | Gd | 159 | 64 | 4.792112304E-14 | 7.4462137979 | 29.63872782 | -19.52814044 | 23.31251275 | 3.045322979 |
| 61 | Tb | 144 | 65 | 1.683888342E-14 | 8.462141713 | 29.76662342 | -19.52627043 | 23.31446281 | 0.6232492904 |
| 62 | dy | 145 | 66 | 8.542809364E-14 | 8.501853707 | 29.77130541 | -19.52620212 | 23.31453111 | 1.146128036 |
| 63 | Ho | 146 | 67 | 7.786581678E-14 | 8.541211008 | 29.77592392 | -19.526123475 | 23.31489848 | 0.5785139399 |
| 64 | Er | 151 | 68 | 1.691899229E-14 | 8.609065064 | 29.78383684 | -19.526019366 | 23.31471388 | 0.7781512504 |
| 65 | Tm | 152 | 69 | 5.322432991E-14 | 8.647675105 | 29.78831163 | -19.52595411 | 23.31477912 | 0.6989700043 |
| 66 | Yb | 157 | 70 | 2.291113539E-14 | 8.714021135 | 29.79595447 | -19.5258427 | 23.31489054 | 1.5910064607 |

## Discussion

The results of tunneling probabilities of gamma particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ nuclei are shown in Tables 1 and 2. Table 1 and 2 have atomic number $Z = 36$ to 70 for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ gamma nuclei. The tables that indicate the medium gamma particle has an appropriate result obtained which shows that gamma decay is possible. The calculated tunnel probability in equation (4.8) indicate input data in Table 2. The isotopes of gamma particle emitter with $Z = 36$ to 70 that is $^{75}_{36}Kr - ^{157}_{70}Yb$ for medium gamma particle and $Z = 71$ to 101 that is $^{158}_{71}Lu - ^{256}_{101}Md$ for heavy gamma particle are shown. The half-life varies from one nucleus to another which indicates that from Table 2 observes that the values of calculated half-lives are so small but also match with the experimental half-lives. In general, the gamma particle half-life $t_{\frac{1}{2}}$ presented in the Table 2 are in agreement with the experimental result (see chart of Nuclides Edwards et al., 2002).

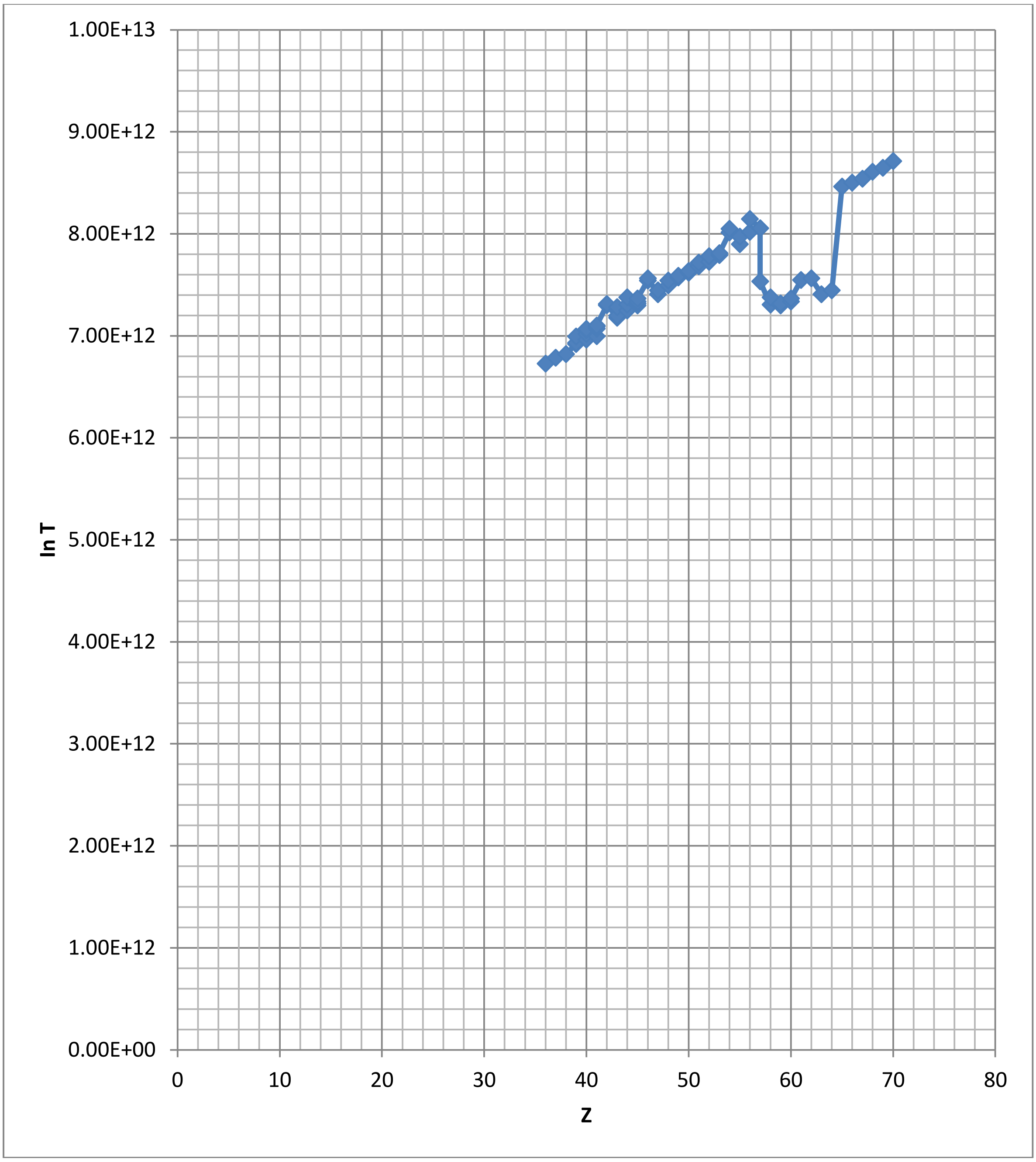


Figure 1: Natural logarithm of Tunneling probability versus Atomic number for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ Gamma Particle emitting nuclei.

Figure 1 represents the natural logarithm of tunneling probability versus atomic number $Z$ for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass gamma particle emitters respectively. Figure 1, the anomaly lies with high atomic number $Z$ values for the medium gamma particle nuclei. From atomic number $Z$= 42, 44, 46, 48, 52, 54 and 56 are slightly high than the orders also from atomic number $Z$= 57 to 65 makes a shape of " w" and from the atomic number $Z$= 65 diminishes with

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increasing value of natural logarithm of tunneling probability. The reason that anomaly lies at low atomic number Z is as result of different energy of gamma particle emitters with atomic number Z. The shape "w" is as a result from one nucleus to another, that the nuclei have either very small tunneling probability or the nuclei are stable and are depicted by points lying at the bottom for each isotope which even-even is with even-odd (even neutron and odd proton or even proton and odd neutron) the figure shows that the probability of gamma emission is higher than even-even nuclei. The atomic number Z= 42, 44, 46, 52, 54 and 56 it shows that even-even nuclei have the slightly high probability of gamma emission.

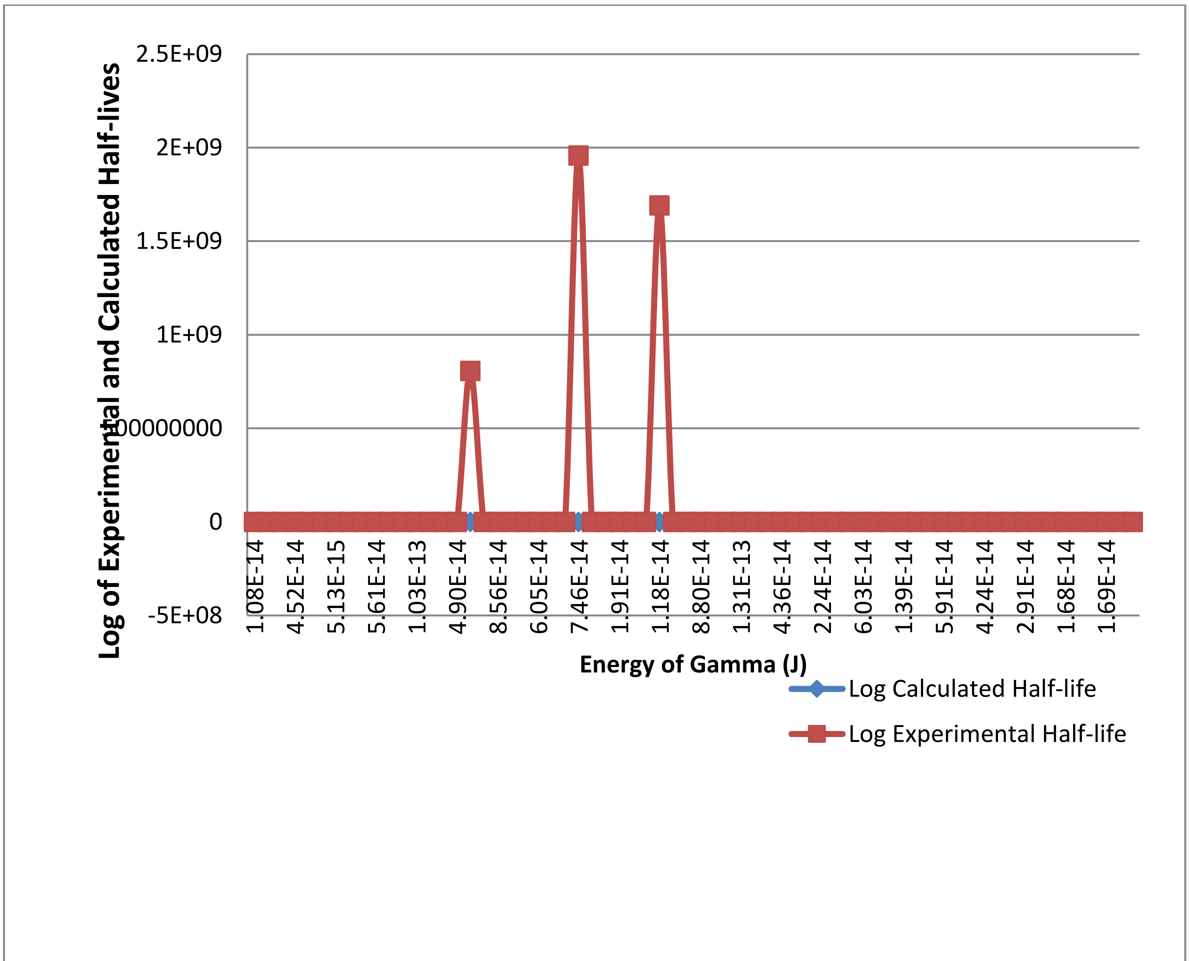


Figure 2: Logarithmic plot of Experimental and Calculated Half-lives versus Energy of Gamma Particle for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass nuclei.

Figure 2 shows the logarithm of experimental and calculated half-lives versus energy of gamma particle for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass nuclei. Figure 2, shows the anomaly lies with high energy of gamma particle values of experimental and calculated half-lives except for the three anomalies nuclei of the experimental half-lives that are high energy of gamma particle emitter prove that they have high half-lives experimentally as in the anomalies of the energy of gamma particle are 4.90 E-14 J, 7.46 E-14 J and 1.18 E-14 J.

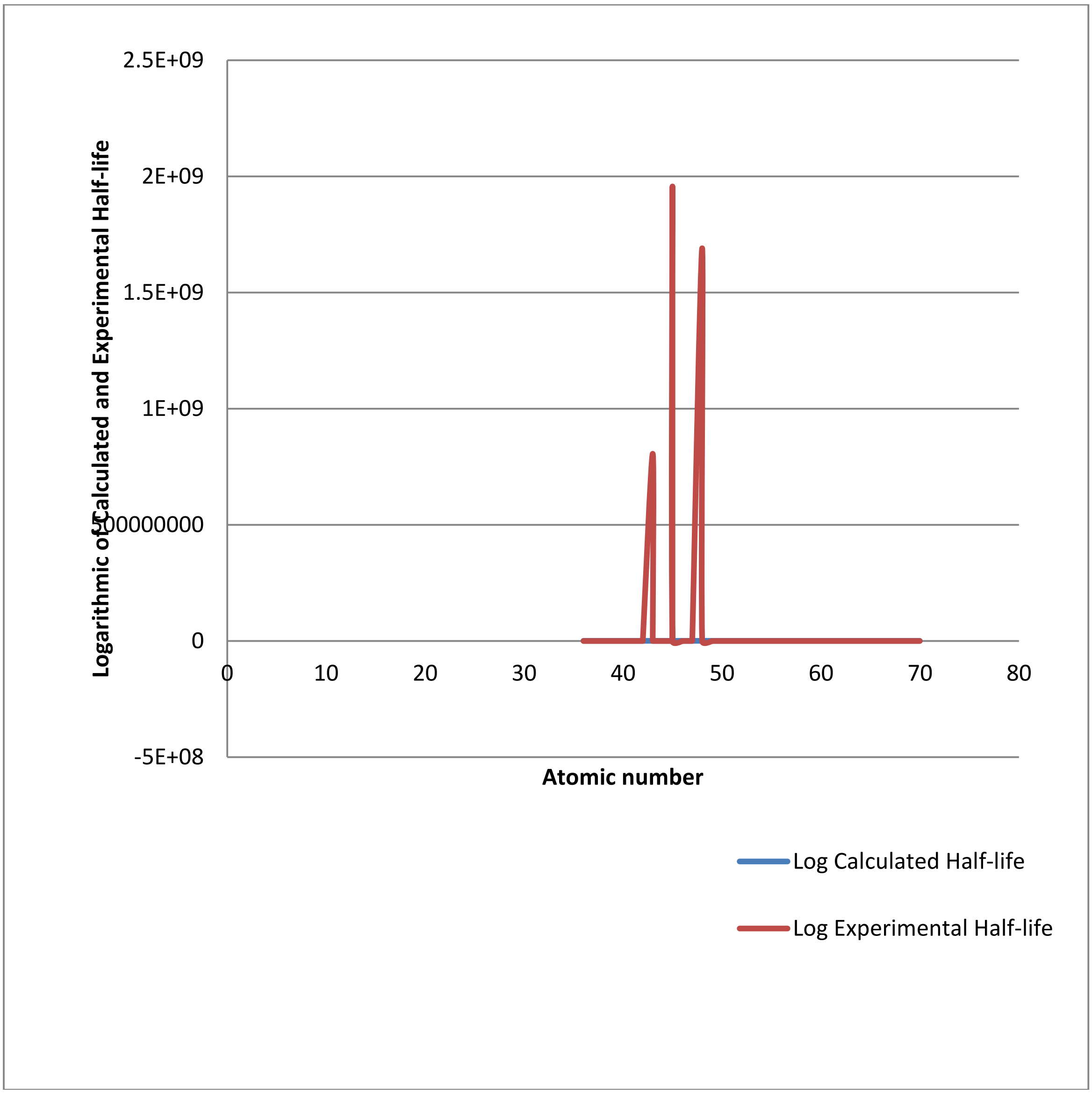


Figure 3: Logarithmic plot of Calculated and Experimental Half-lives versus Atomic Number for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass nuclei.

Figure 3 shows the logarithmic calculated and experimental half-lives versus Mass number A for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass nuclei respectively.
Figure 3 shows the anomaly lays with high mass number A values for the medium mass number A nuclei sustain a straight line of the value of calculated

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and experimental half-lives except for the three anomaly of the experimental half-lives that are high that is for the isotopes of the nuclei with mass number A = 94 to 99. These reveal that those low mass number A have a low rate of calculated and experimental half-lives while the three mass number A indicates that they have high experimental half-lives.

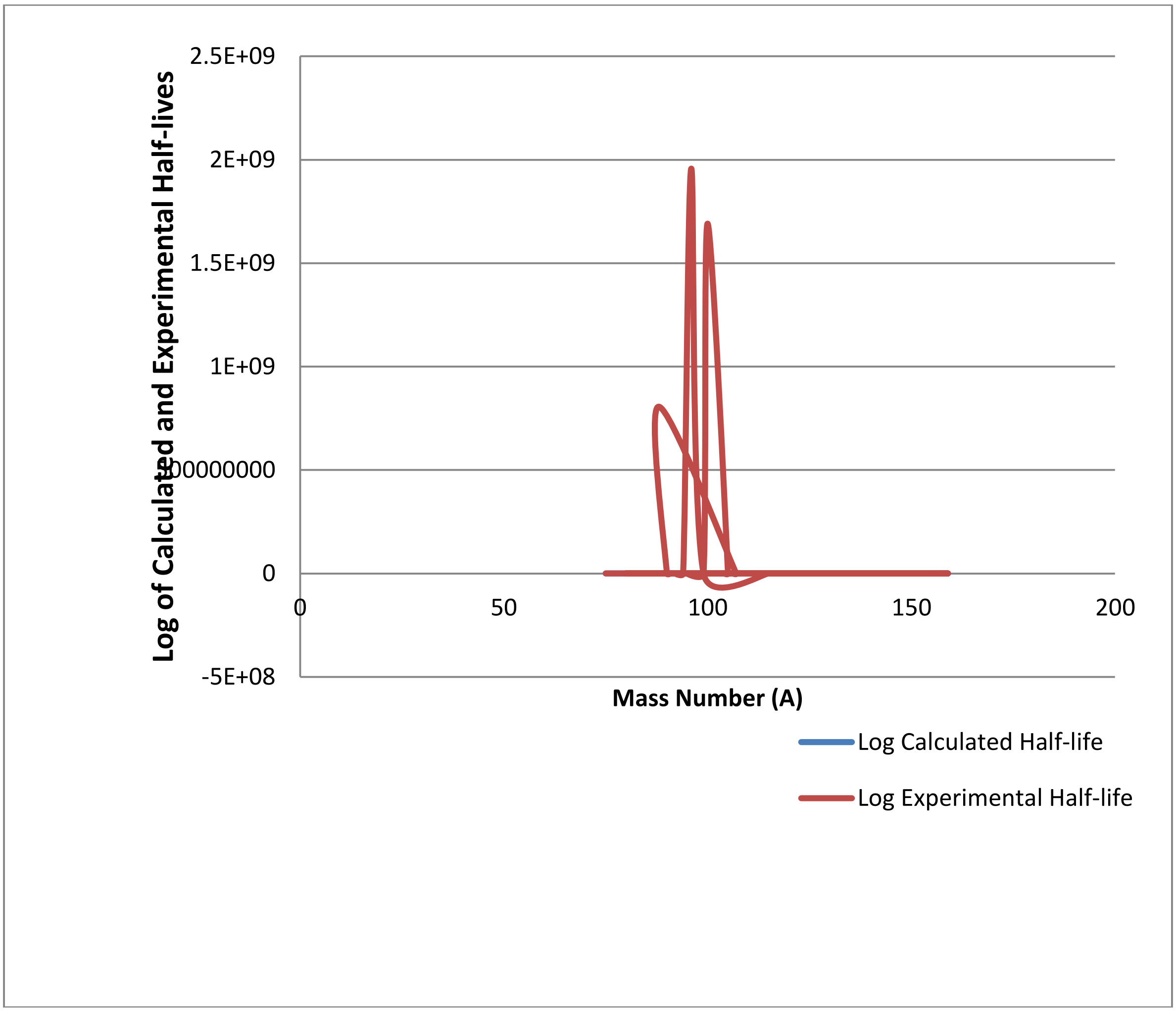


Figure 4: Logarithmic plot of Calculated and Experimental Half-lives versus Mass Number (A) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ Mass Nuclei.

Figure 4 shows the logarithm of experimental and calculated half-lives versus mass number of gamma particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei. Figure 4, indicates the anomaly lies with low mass number of gamma particle values of experimental and calculated half-lives except for the three anomalies nuclei of the experimental half-lives that are high mass number of gamma particle either prove that they have high half-lives experimental as in the anomalies of the mass number of gamma particle are 90, 99 and 101.

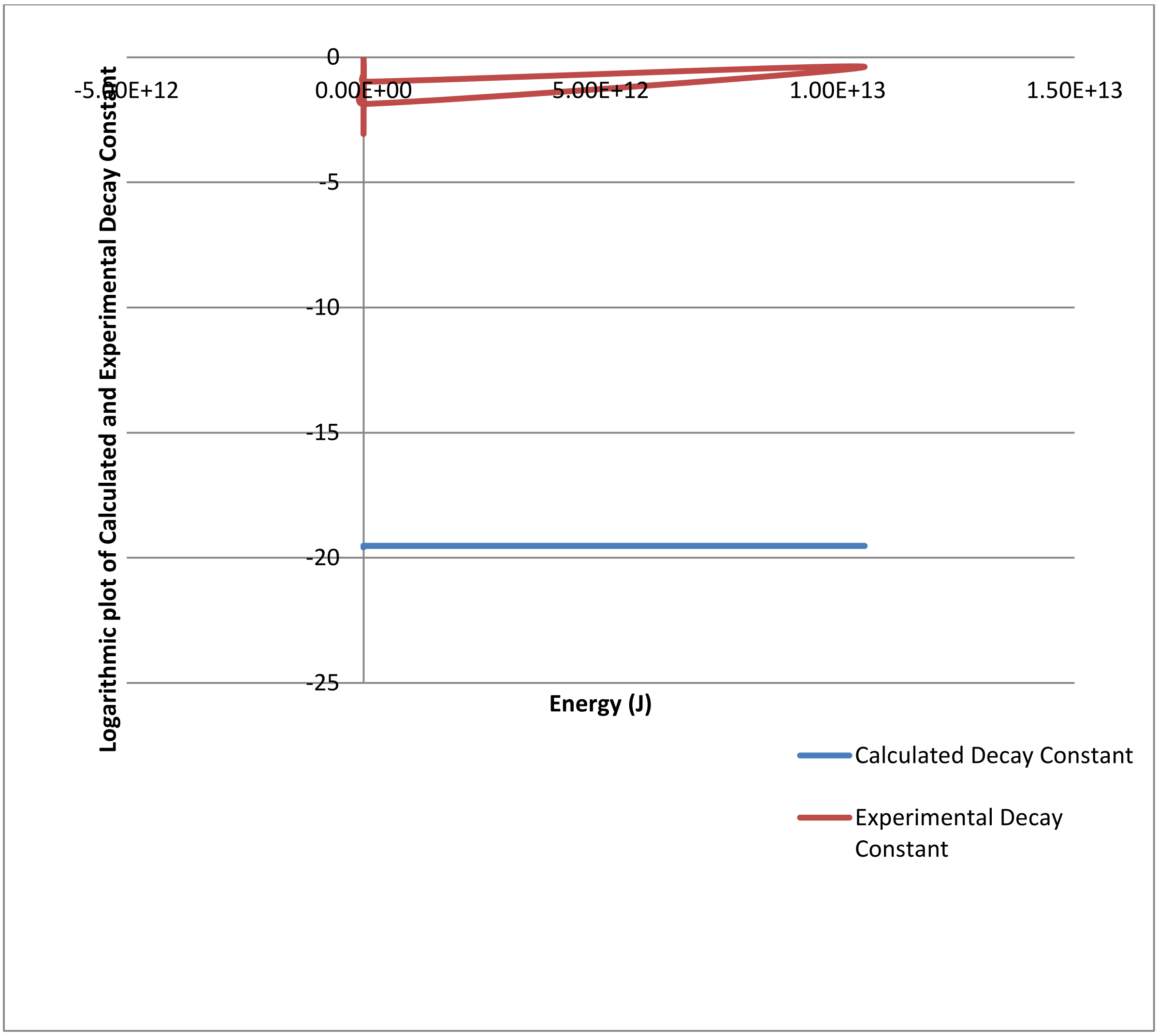


Figure 5: Logarithmic plot of Calculated and Experimental Decay constant versus Energy (J) of Gamma Particle for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ Mass Nuclei.

Figure 5 represents the logarithm calculated decay constant versus Energy (J) for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass gamma particle emitters respectively. Figure 5, the anomaly lays with low Energy (J) values for the medium gamma particles emitting nuclei. for the Energy (J) value of 0.00 is having a vertical line on the logarithm calculated decay constant from 0.00 to around -3.5 which also the figure it has a shape if cone on the position of neutral equilibrium. The cone

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neutral equilibrium position lies on the low Energy (J) than the order vertical line. The figure also shows a horizontal line on the Energy (J) from 0.00E+00 to 1.00E+13

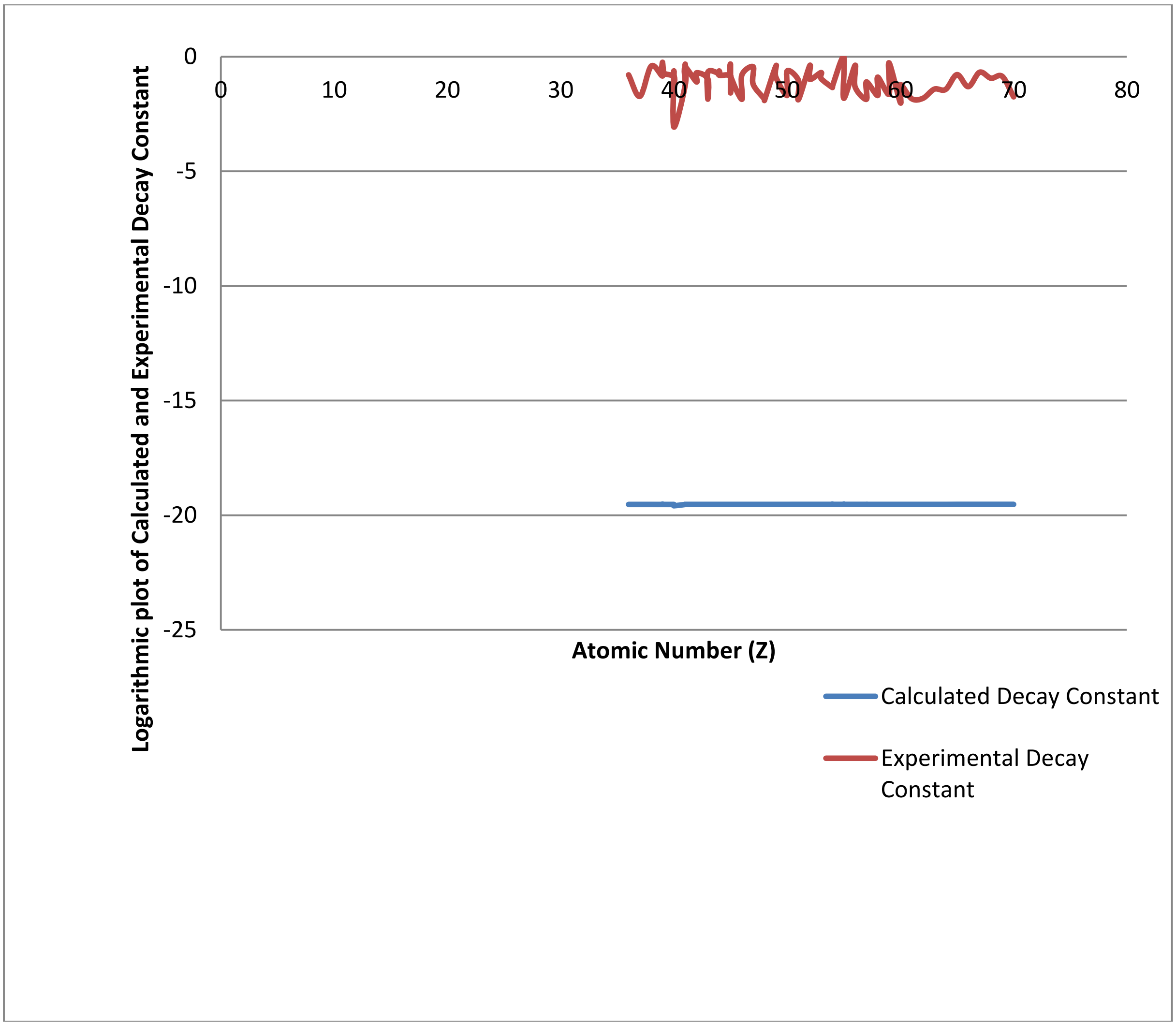


Figure 6: Logarithmic plot of Calculated and Experimental Decay constant versus Atomic Number (Z) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ Mass Nuclei.

Figure 6 shows the logarithm of calculated and experimental decay constant versus atomic number (z) of gamma particles for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei. Figure 6, shows the anomaly lies with low atomic number (Z) of gamma particle values of calculated and experimental decay constant for the medium gamma particle nuclei. The figure shows a zigzag and horizontal line. For the zigzag value on atomic number (Z) = 40 has the lowest value on the zigzag while from the atomic number (Z) = 60 to 70 it diminishes.

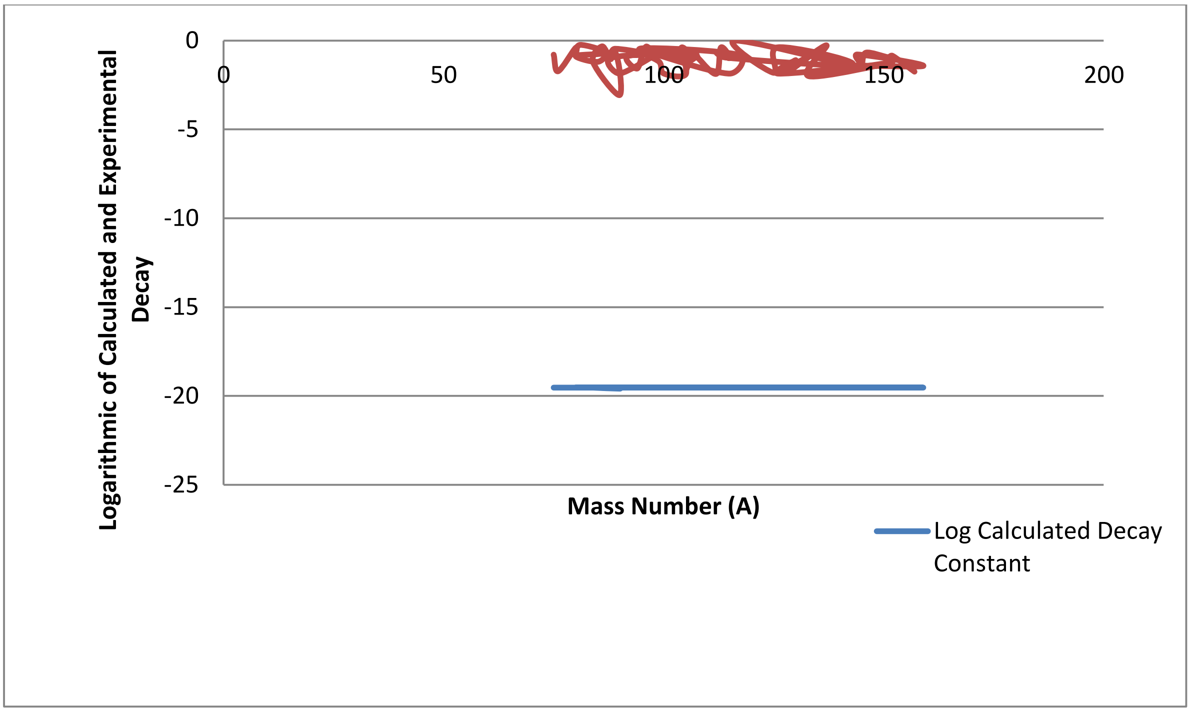


Figure 7: Logarithmic plot of Calculated and Experimental Decay constant versus Mass Number (A) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ Mass Nuclei.

Figure 7 shows the logarithm of calculated and experimental decay constant versus mass number (A) of gamma particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei. Figure 7, shows the anomaly lies with low mass number (A) values for the medium gamma particle nuclei. It shows the shapes of cones and a horizontal line. The shapes of cones are like in the position of neutral and also unstable equilibrium. The shapes lie in between mass number (A) = 71-160.

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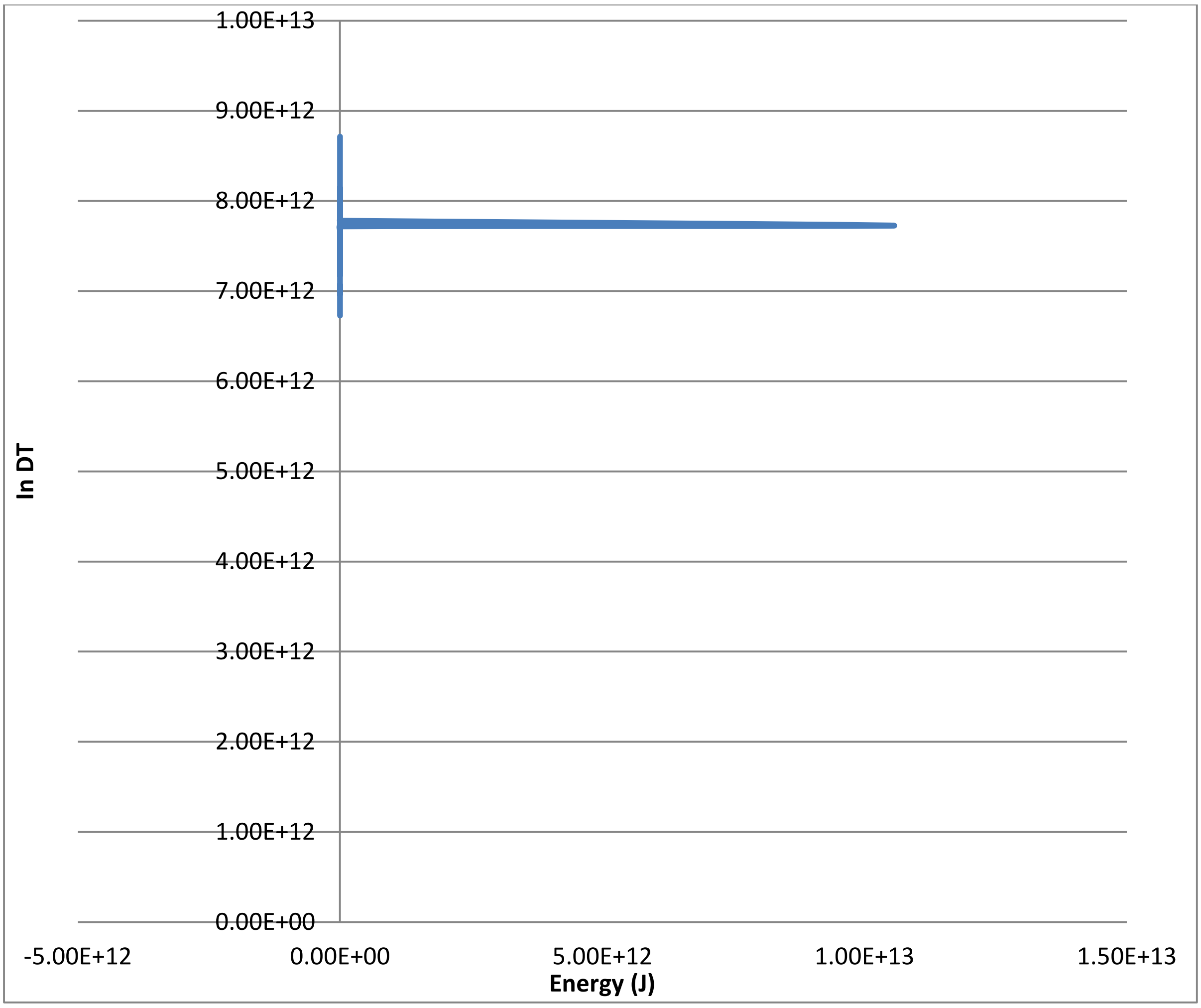


Figure 8: Natural logarithm of Tunneling probability versus Energy (J) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ Gamma Particle emitting nuclei.

Figure 8 represent the natural logarithm of tunneling probability versus Energy (J) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle emitters respectively.
Figure 8, the anomaly lie with high Energy (J) values for the medium Gamma particle emitting nuclei. From natural logarithms of tunneling probability axis that lies 0.00 E+00 on Energy (J) axis while from two different points that meet at a point that make a narrow space between the two point from the at a distance less than 5.00 E+12 Energy (J) it continuous up to a distance above 1.00 E+13. These means that the nuclides that Energy (J) to tunnel through the Double thick barrier, the Energy of the nuclides that have 0.00 E+00 lies in between a distance close to 7.00 E+12 to distance close to 9.00 E+12 and the two points are close 8.00 E+12 of the natural logarithm of the tunneling probability.

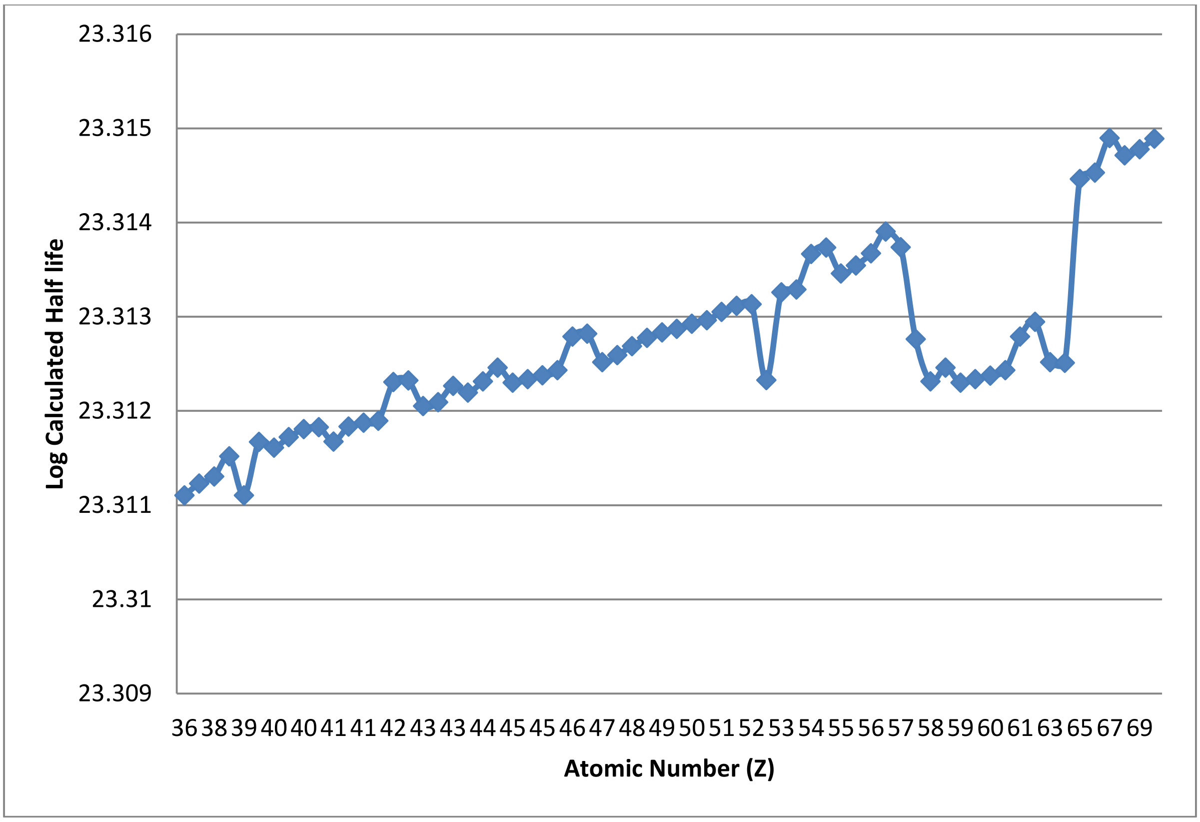


Figure 9: Logarithmic plot of Calculated Half-lives versus Atomic Number (Z) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 9 represents the logarithms of calculated half-life verses Atomic number Z for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle respectively. Figure 9, the anomaly lies with high atomic number Z value for the medium gamma particle emitting nuclei. From atomic number $Z = 42, 44, 46, 48, 52,54$ and 56 are slightly high than the orders also from atomic number $Z = 57$ to 65 makes a shape of w and from the atomic $Z = 65$ diminishes with increasing value of natural logarithm of tunneling probability. The reason that anomaly lies at low atomic number Z is as a result of different logarithms of calculated half-life. The shape w is as a result of a different time of tunneling to the other, that the nuclei have either very small time tunneling probability or the nuclei are stable are depicted by points lying at the bottom for each isotopes which even-even is with even-old (even number and odd proton or even proton and odd neutron) the figure shows that the time taken for the probability of gamma emission is high than even-even nuclei. The atomic number $Z = 42, 44, 46, 48, 52,54$ and 56 it shows that even-even nuclei have the slit high logarithms of calculated half-life of gamma emission.

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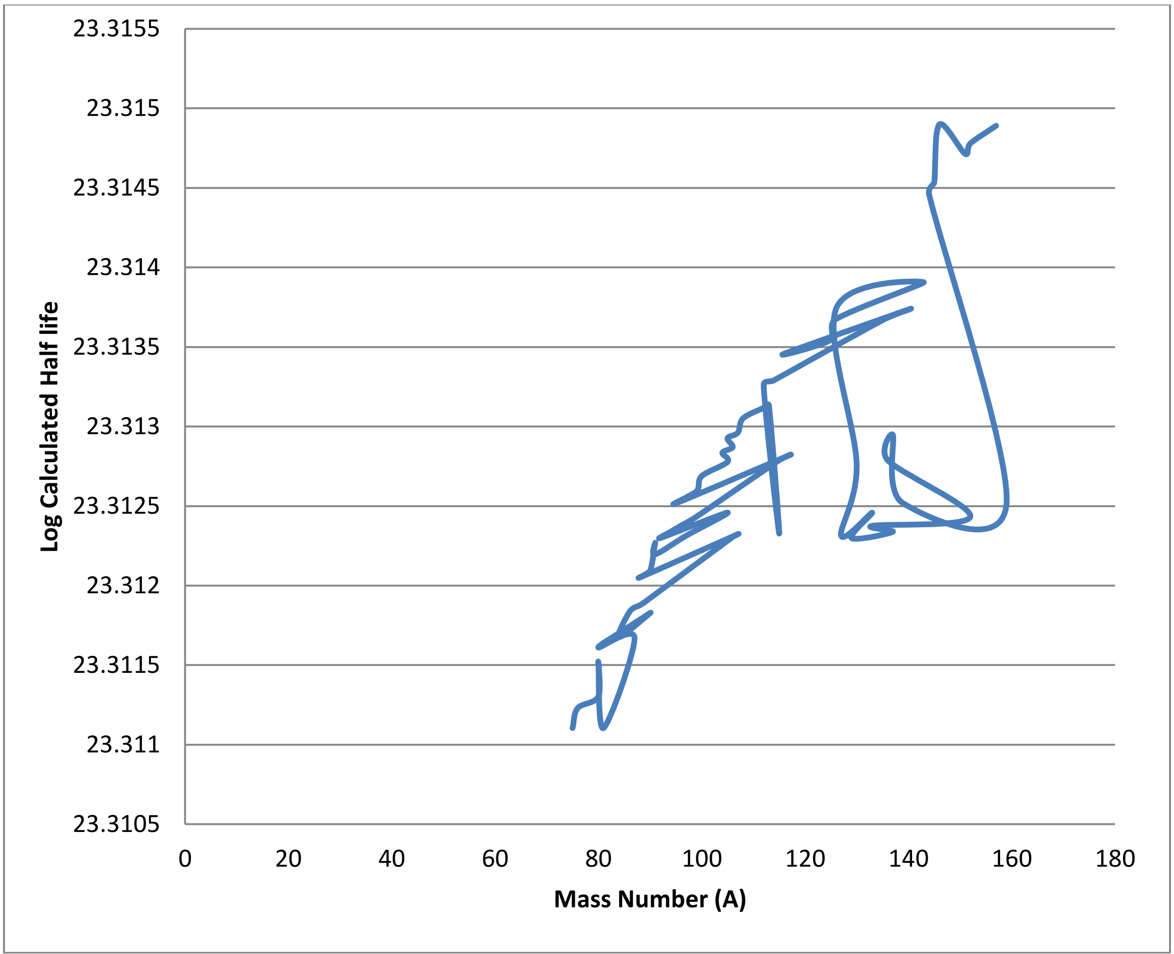


Figure 10: Logarithmic plot of Calculated Half-life versus Mass Number (J) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 10 represents the logarithms of calculated half-life versus mass number (A) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle emitters respectively.

Figure 10, the anomaly lies with high mass number (A) values for the medium gamma particle emitting nuclei. The figure shows the shape of v, zigzag shape in the ascending order and also a shape of w. The reason for the shape of v is as a result of low in logarithms of calculated half-life taken for tunneling probability, zigzag shape is as a result of different value of logarithm of calculated half-life which is not at a close distance and also shape w is as a result from one nucleus to another of the logarithm half-life for the tunneling probability.

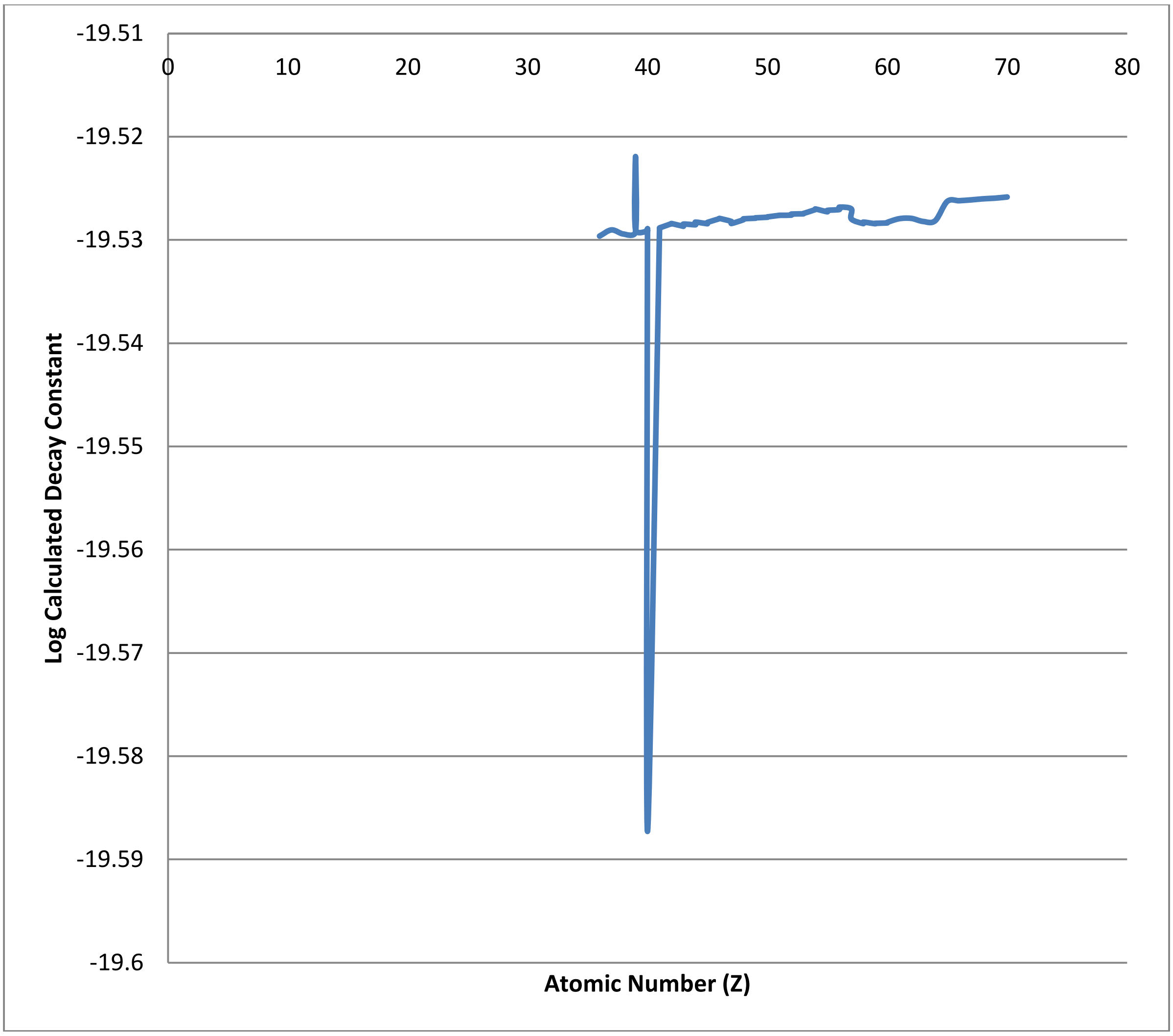


Figure 11: Logarithmic plot of Calculated Decay Constant versus Atomic Number (Z) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 11 represents the logarithms calculated Decay constant versus atomic number Z for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle emitters respectively.

Figure 11, the anomaly lies with low atomic number Z values for the medium gamma particle emitting nuclei. For the atomic number $Z = 39$ slightly high than the orders, also for atomic number $Z = 40$ is lower than the orders from atomic number $Z = 57$ to 65 makes a shape of w and also from atomic number $Z = 65$ diminishes with increasing value of logarithms of calculated decay constant. This shows that atomic number $Z = 39$ having a high logarithms calculated decay constant than order after the tunneling probability.

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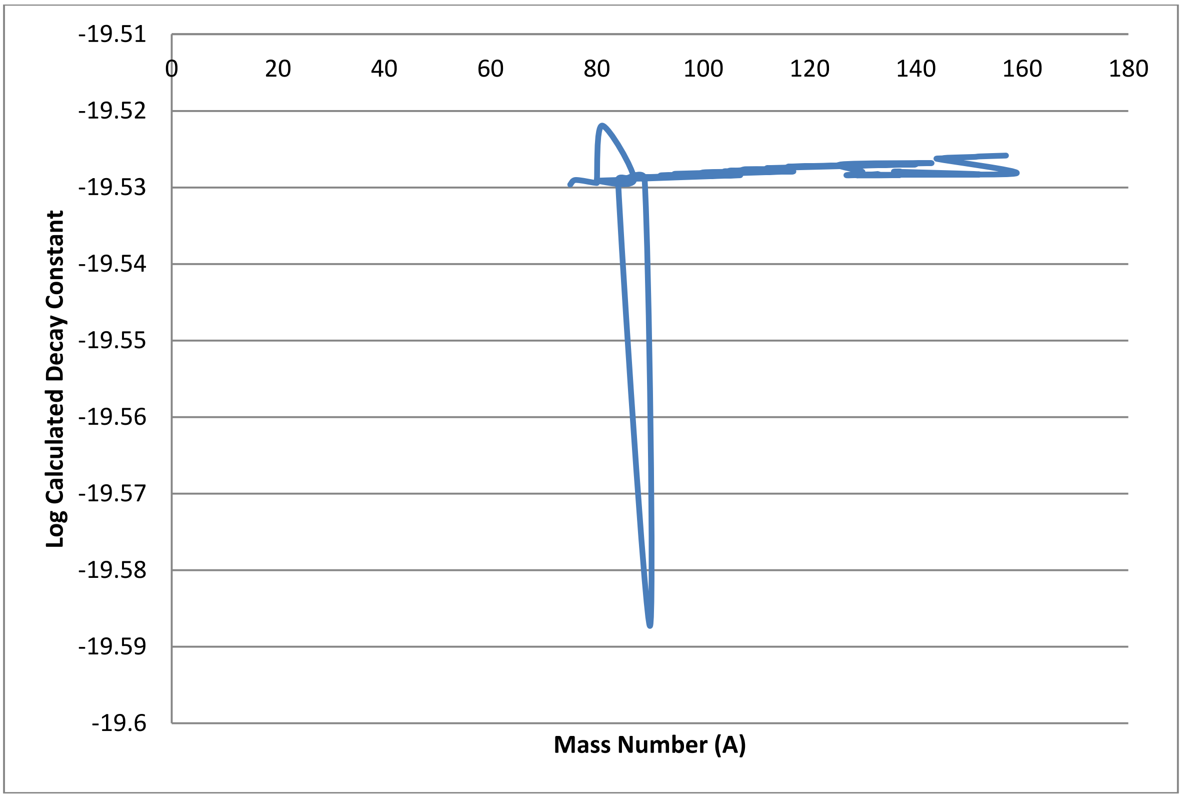


Figure 12: Logarithmic plot of Calculated Decay Constant versus Mass Number (A) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 12 represents the logarithm decay constant versus mass number (A) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle emitters respectively.
Figure 12, the anomaly lies with low mass number (A) values for the medium gamma particle nuclei. The figure shows a shape of a cone, shape of an upside down cone, closed distance zigzag and also a shape of letter S. The reason for the shape of a cone is one of the mass number (A) have lower logarithm decay constant value than the orders, for the upside down cone is as the result of the middle valve of mass number (A) is having a high value of logarithm decay constant than orders, closed distance zigzag shape is as a result of fluctuation of values of logarithm decay constant at a closed distance and also for the shape of letter "S" is as a result of fluctuation of values of logarithm decay constant at a distance after the tunneling probability.

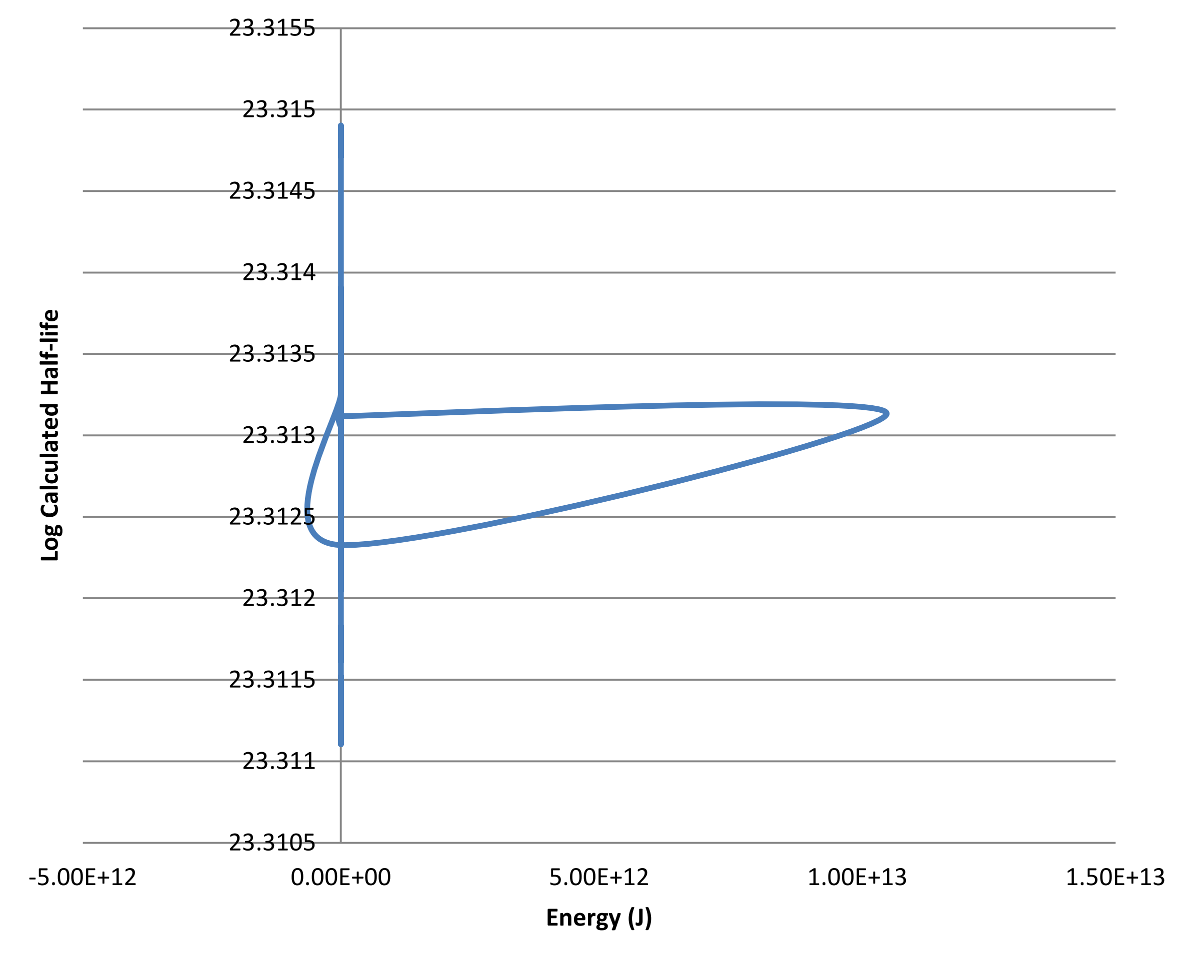


Figure 13: Logarithmic plot of Calculated Half-life versus Energy (J) of Gamma Particle for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass nuclei.

Figure 13 represents the logarithm calculated Half-life versus Energy (J) for $^{75}_{36}Kr$ to $^{157}_{70}Yb$ mass Gamma particle emitters respectively. Figure 13, the anomaly lies with high Energy (J) values for the medium gamma particle emitting nuclei. For the Energy (J) value 0.00 E+00 is having a vertical line on the logarithm calculated half-life from above 23.311 seconds to close to 23.315 seconds and also a shape of cone on the position of neutral equilibrium. The cone neutral equilibrium position lies on the higher Energy (J) than the order vertical line. The cone lies in between close to 23.3125 seconds to above 23.313 seconds of logarithm calculated half-life that is taken to tunneling probability.

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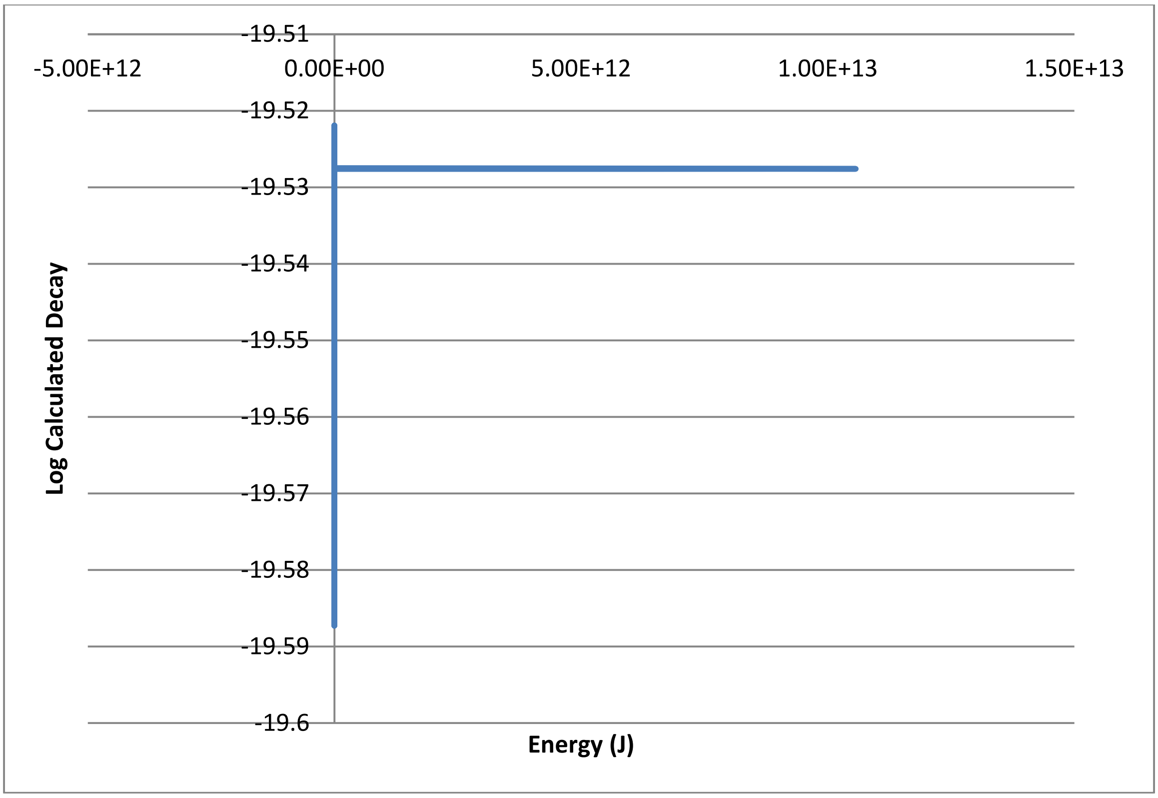


Figure 14: Logarithmic plot of Calculated Decay Constant versus Energy (J) of Gamma Particle for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass nuclei.

Figure 14 represents the logarithm calculated decay versus Energy (J) for ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ mass Gamma particle emitters respectively. Figure 14, the anomaly lies with low Energy (J) values for the medium gamma particle emitting nuclei. The figure shows the vertical and horizontal lines. The values of Energy (J) at 0.00 E+00 is having vertical line on the logarithm of calculated decay which lies below -19.52 to above -19.58, the horizontal lines is as a result of the Energy (J) that have 1.00 E+13 that is after the tunneling probability.

## CONCLUSION

It has been calculated analytically the quantum mechanical emission probability of barrier penetration ${}^{75}_{36}Kr$ to ${}^{157}_{70}Yb$ of the gamma particle decay of atomic nuclei. The Schrödinger's time-independent equation has been applied to a potential barrier whose height is greater than the gamma particle's energy. However, on application of barrier emission theory, the probability of the gamma particle crossing the barrier is in non-zero and this probability has been calculated.

## REFERENCES

Chaddha G. S (1983): Quantum Mechanics. Published by (India). Vol. 2nd. pp 68-72.

Dyson F. Y (1951): Advanced Quantum Mechanics. Addison Wesley Company. Vol. 2 pp 22-23.

Edward H. D, Baum M and Miller T. R (2002): Nuclide and Isotope. 16th edition, pp 35-29.

Raju A.R and Arrora B. L (2006): Dictionary of Physics. New Delhi. Academic (India) Publisher New Delhi; Vol.1, pp 22, 261 & 263.

Rutherford E. (1903): The magnetic and electric deviation of the easily absorbed rays from radium. Philosophical Magazine , Series 6, vol. 5, no. 26, pages 177–187.

Villard P (1900a): Very-High-Energy Gamma Rays from a Distant Quasar. Science. 92 (5884): 1752– 1754..

Villard P (1900b): Evidence for a lack of DNA double- strand break repair in human cells exposed to very low x-ray doses. Proceedings of the National Academy of Science,Vol.,100 (9): 5057–62.