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AN ASSESSMENT OF UNEQUAL MASSES OF ATOMS MOVING IN GEOMETRY OF TWO COMPONENT FERMION SYSTEM

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ABSTRACT

The goal of this paper is to obtain the unequal masses of the atoms moving in geometry of two component fermions system. The two component fermions systems have been considered. The fermion-dimer scattering length a_{fd} and dimer-dimer scattering length a_{dd} in the universal limit of large fermion-fermion scattering length a_{ff} have been computed. The scattering properties of the fermion-dimer, dimer-dimer and the mass ratio dependence of the universal fermion-dimer and universal dimer-dimer scattering length have been analyzed. Scattering properties of composed system indicate a deep understanding of the dynamics in many-body of an atom system.

Key word: Atoms, fermion-fermion, fermion-dimer, dimer-dimer and Hamiltonian

INTRODUCTION

In particle physics, a fermion is a particle that follows Fermi–Dirac statistics. These particles obey the Pauli Exclusion Principle. Fermions include all quarks and leptons, as well as all composite particles made of an odd number of these, such as all baryons and many atoms and nuclei. Fermions differ from bosons, which obey Bose–Einstein statistics. A fermion can be an elementary particle, such as the electron, or it can be a composite particle, such as the proton. According to the spin- statistics theorem in any reasonable relativistic quantum field theory, particles with integer spin are bosons, while particles with half-integer spin are fermions. In addition to the spin characteristic, fermions have another specific property: they possess conserved baryon or lepton quantum numbers. Therefore, what is usually referred to as the spin statistics relation is in fact a spin statistics-quantum number relation. [1] As a consequence of the Pauli Exclusion Principle, only one fermion can occupy a particular quantum state at any given time. If multiple fermions have the same spatial probability distribution, then at least one property of each fermion, such as its spin, must be different, Fermions are usually associated with matter, whereas bosons are generally force carrier particles, although in the current state of particle physics the distinction between the two concepts is unclear. Weakly interacting fermions can also display bosonic behavior under extreme conditions. At low temperature fermions show super fluidity for uncharged particles and superconductivity for charged particles. Composite fermions, such as protons and neutrons are the key building blocks of everyday matter. The name fermion was coined by English theoretical physicist Paul Dirac from the surname of Italian physicist Enrico Fermi. [2] The Standard Model recognizes two types of elementary fermions: quarks and leptons. In all, the model distinguishes 24 different fermions. There are six quarks (up , down, strange , charm, bottom and top quarks), and six leptons (electron , electron neutrino , muon , muon neutrino , tau particle and tau neutrino), along with the corresponding antiparticle of each of these. Mathematically, fermions come in three types: Weyl fermions (massless), Dirac fermions (massive), and Majorana fermions (each its own antiparticle). Most Standard Model fermions are believed to be Dirac fermions, although it is unknown at this time whether the neutrinos are Dirac or Majorana fermions (or both). Dirac fermions can be treated as a combination of two Weyl fermions. [3] The atom helium-3 (3He) is made of two protons, one neutron, and two electrons, and therefore it is a fermion. The number of bosons within a composite particle made up of simple particles bound with a potential has no effect on whether it is a boson or a fermion. Fermionic or bosonic behavior of a composite particle (or system) is only seen at large (compared to size of the system) distances. At proximity, where spatial structure begins to be important, a composite particle (or system) behaves according to its constituent makeup. Fermions can exhibit bosonic behavior when they become loosely bound in pairs. This is the origin of superconductivity and the super fluidity of helium-3: in superconducting materials, electrons interact through the exchange of phonons, forming Cooper pairs, while in helium-3, Cooper pairs are formed via spin fluctuations. The quasiparticles of the fractional quantum Hall effect are also known as composite fermions, which are electrons with an even number of quantized vortices attached to them.

In a quantum field theory, there can be field configurations of bosons which are topologically twisted. These are coherent states (or solitons) which behave like a particle, and they can be fermionic even if all the constituent particles are bosons. This was discovered by Tony Skyrme in the early 1960s, so fermions made of bosons are named skyrmions after him. Skyrme's original example involved fields which take values on a three-dimensional sphere, the original nonlinear sigma model which describes the large distance behavior of peons. In Skyrme's model, reproduced in the large N or string approximation to quantum chromodynamics (QCD), the proton and neutron are fermionic topological solitons of the pion field. Whereas Skyrme's example involved pion physics, there is a much more familiar example in quantum electrodynamics with a magnetic monopole. A bosonic monopole with the smallest possible magnetic charge and a bosonic version of the electron will form a fermionic dyon. The analogy between the Skyrme field and the Higgs field of the electroweak sector has been used to postulate that all fermions are skyrmions. [4] This could explain why all known fermions have baryon or lepton quantum numbers and provide a physical mechanism for the Pauli Exclusion Principle.

It consists of a partially ordered set in which every two elements have a unique supremum (also called a least upper bound or join) and a unique infimum (also called a greatest lower bound or meet). An

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example is given by the natural numbers, partially ordered by divisibility, for which the unique supremum is the least common multiple and the unique infimum is the greatest common divisor .[5] Atoms moving in geometry can also be characterized as algebraic structures satisfying certain axiomatic identities. Since the two definitions are equivalent, lattice theory draws on both order theory and universal algebra. [6] Atoms moving in geometry have some connections to the family of group-like algebraic structures. Because meet and join both commute and associate, an atoms moving in geometry can be viewed as consisting of two commutative semigroups having the same domain. For a bounded atoms moving in geometry, these semigroups are in fact commutative monoids. The absorption law is the only defining identity that is peculiar to atoms moving in geometry theory. By commutativity and associativity one can think of join and meet as binary operations that are defined on non-empty finite sets, rather than on elements. In a bounded atoms moving in geometry the empty join and the empty meet can also be defined (as 0 and 1, respectively). [7] This makes bounded atoms moving in geometry somewhat more natural than general atoms moving in geometry and many authors require all atoms moving in geometry to be bounded.

MATERIALS AND METHOD

For the system of particles interacting with a finite range potential, at low energies the scattering phase shift $\delta(\rho)$ is parameterized by effective range expansion,

$$p\ \tan\delta(\rho) = \frac{1}{a_{ff}} + \frac{1}{2} r_{ff} p^2 + \cdots$$

1

Where p is the relative momentum between two fermions, a_{ff} is the scattering length, and r_{ff} is the effective range. [8] In the zero range limit, the scattering length is related to dimer binding energy by formula

$$B_d = \frac{1}{\left(2\mu\ \ a_{ff}^2\right)}$$

2

Where μ is the reduced mass the non-relativistic Hamiltonian in the continuum is

$$\widehat{H} = \sum_s \frac{1}{2\,m_s} \int dr\ \ \nabla b_s^{+}(r)\,\nabla b_s(r) + C_0 \int dr b_{\uparrow}^{+}(r) b_{\uparrow}(r) b_{\downarrow}^{+}(r) b_{\downarrow}(r)$$

3

Where s labels the particle species, C_0 is the zero range interaction strength, b_s and b_s^{+} are the annihilation and creation operators, respectively.

Therefore, the non-relativistic Hamiltonian with $O(a^4)$ improved action is

$$H = \sum_s \frac{1}{2\,m_s} \sum_n \left[\sum_{k=-3}^{3} \omega_{|k|}\ \ b_s^{+}(n)\, b_s(n+k)\right] + C_0\ \sum_n b_{\uparrow}^{+}(r) b_{\uparrow}(r) b_{\downarrow}^{+}(r) b_{\downarrow}(r) \quad 4$$

Where n labels the atoms moving in geometry and $\omega_0, \omega_1, \omega_2, \omega_3$.

Let us consider a two body system with zero total momentum and a potential of a finite range R on a periodic. Then the wave function at distances $r > R$ takes the asymptotic from $\Psi(r) \sim \cos[pr + \delta(\rho)]$, due to the periodicity it satisfies the condition $\Psi\left(\frac{La}{2}\right) = \Psi\left(-\frac{La}{2}\right)$ and $\partial_r \Psi(r)|_{\frac{La}{2}} = \partial_r \Psi(r)|_{-\frac{La}{2}}$, which yield

$$pLa + 2\delta(\rho) = 2n\pi \qquad n = 0, 1, 2, \dots$$

5

This relation gives us direct access to the scattering.

First, at non zero atom moving in geometry spacing the effective mass of the dimer is not equal to $m_\uparrow + m_\downarrow$. Therefore, we compute the dimer effective mass by

$$D(p, m_d) = c_0 \frac{p^2}{2\, m_d} + c_1 p^4 + \cdots$$

6

Where c_i are the coefficients to be determined by the fit, p is the total momentum of the moving dimer and m_d is the physical dimer mass. The relative momentum of the fermion-dimer is determined by

$$E_{fd}^L = \frac{p^2}{2\mu_{fd}^*} - B_d - \Delta B_d^L \cos(p \;\; a \;\; L \;\; \alpha)$$

7

Where E_{fd}^L is the fermion-dimer energy at atom size L, ΔB_d is the finite volume correction of the dimer binding energy $\Delta B_d = B_d^L - B_d$, $\quad \alpha = \frac{m_\uparrow}{(m_\uparrow + m_\downarrow)}$ and μ_{fd}^* is the fermion-dimer reduced mass. In each calculation make a fit using the phase shifts and the relative momentum in the truncated effective range expansion

$$a_{ff}\, p \;\tan\delta(\rho) = \frac{1}{\frac{a_{fd}}{a_{ff}}} + \frac{1}{2}\left(\frac{r_{fd}}{a_{ff}}\right)\left(a_{ff}p\right)^2 + \cdots$$

8

Where a_{fd} and r_{fd} are the fermion-dimer scattering length and effective range. [8] Therefore, the relative momentum between two dimer are given by

$$E_{dd}^L = \frac{p^2}{2\mu_{dd}^*} - 2B_d - 2\Delta B_d^L \cos(p \;\; a \;\; L \;\; \alpha)$$

9

Where E_{dd}^L is the dimer-dimer energy at atom size L and μ_{dd}^* is the dimer-dimer reduced mass. The computed scattering phase shifts using the data of atom moving in geometry are used in the following truncated effective range expansion to extract the scattering length

$$a_{ff}\, p\, \tan\delta(\rho) = \frac{1}{\frac{a_{dd}}{a_{ff}}} + \frac{1}{2}\left(\frac{r_{dd}}{a_{ff}}\right)\left(a_{ff}p\right)^2 + \cdots$$

10

Where a_{dd} is the dimer-dimer scattering length and r_{dd} is the dimer-dimer effective range.

Fermion-dimer

Let consider two spin ↑ and one spin ↓ particles interacting via delta function potential. The Hamiltonian of the system is,

$$H_{fd} = -\frac{1}{2m_1}\partial_{x_1}^2 - \frac{1}{2m_2}\partial_{x_2}^2 - \frac{1}{2m_3}\partial_{x_3}^2 + C_0[\delta(x_3 - x_1) + \delta(x_3 - x_2)]$$

11

Where $\partial_x^2 = \frac{\partial^2}{\partial x^2}$ m_1, m_2 and m_3 are the masses and x_1, x_2 and x_3 are the coordinates of the spin ↑, spin ↑ and spin ↓ particles respectively. Equation (11) can be rewritten as

$$H_{fd} = -\frac{1}{2m_2}\partial_x^2 - \frac{1}{2m_3}\partial_y^2 + C_0\left[\delta\left(y - \frac{x}{2}\right) + \delta\left(y - \frac{x}{2}\right)\right]$$

12

Where

$$m_1 = m_2 = m_\uparrow$$
$$m_3 = m_\uparrow$$
$$\mu_2 = \frac{m_\uparrow}{2}$$
$$\mu_3 = \frac{2m_\uparrow m_\downarrow}{(2m_\uparrow + m_\downarrow)}$$
$$x = x_2 - x_1$$
$$y = \frac{m_1x_1 + m_2x_2}{(m_1 + m_2)} - x_3$$

13

The Schrödinger equation in the limit $m_\downarrow \to \infty$ is $-\frac{1}{2\mu_3}\partial_y^2\phi(y;x) + C_0\left[\delta\left(y - \frac{x}{2}\right) + \delta\left(y - \frac{x}{2}\right)\right]\phi(y;x) = u(x)\phi(y;x)$ 14

Where $\phi(y;x)$ is the solution of equation (14) for a fixed value of x. Using boundary conditions, the continuity of the wave functions and the discontinuity of their first derivative $y = \pm\frac{x}{2}$, we obtain the energy as a function of x,

$$u_l(x) = \frac{1}{2\mu_3}\left[-\beta + \frac{1}{x}W\left((-1)^{l+1}x\beta e^{x\beta}\right)\right]^2$$

15

Where $\beta = C_0\mu_3$, $W(r)$ is the Lambert W function and $l = 0$ $(l = 1)$ gives the even (odd) solution. [8] Now, using $u_l(x)$, the solutions of equation (14) in equation (12) and we can solve equation (12)

$$-\frac{1}{2\mu_3}\partial_y^2\Psi(x) + u_l(x)\Psi(x) = E\Psi(x) \tag{16}$$

The total wave function of the fermion-dimer system, equation (12), $\Psi(x,y) = \Psi(x)\,\phi(y;x)$ is antisymmetric under exchange of $x_1 \leftrightarrow x_2$.

Dimer-dimer

Let consider two spin ↑ and one spin ↓ particles. The particles with different species are interacting via delta function potential. The Hamiltonian of the system is,

$$H_{fd} = -\frac{1}{2m_1}\partial_{x_1}^2 - \frac{1}{2m_2}\partial_{x_2}^2 - \frac{1}{2m_3}\partial_{x_3}^2 - \frac{1}{2m_3}\partial_{x_4}^2 + c_0[\delta(x_3 - x_1) + \delta(x_3 - x_2) + \delta(x_4 - x_1) + \delta(x_4 - x_2)] \tag{17}$$

Where m_1, m_2, m_3 and m_4 are the masses and x_1, x_2, x_3 and x_4 are the coordinates of the spin ↑, spin ↑ and spin ↓ particles respectively. Equation (17) can be rewritten as

$$H_{fd} = -\frac{1}{2\mu_2}\partial_x^2 - \frac{1}{2\mu_3}\partial_y^2 - \frac{1}{2\mu_4}\partial_y^2 + c_0\left[\delta\left(y - \frac{x}{2}\right) + \delta\left(y - \frac{x}{2}\right) + \delta\left(z - \frac{x}{2} + \frac{m_\uparrow y}{2m_\uparrow + m_\downarrow}\right) + \delta\left(z - \frac{x}{2} + \frac{m_\uparrow y}{2m_\uparrow + m_\downarrow}\right)\right] \tag{18}$$

Where

$$m_1 = m_2 = m_\uparrow$$
$$m_3 = m_4 = m_\downarrow$$
$$\mu_2 = \frac{m_\uparrow}{2}$$
$$\mu_3 = \frac{2m_\uparrow m_\downarrow}{(2m_\uparrow + m_\downarrow)}$$
$$\mu_4 = \frac{(2m_\uparrow + m_\downarrow)m_\downarrow}{(2m_\uparrow + m_\downarrow)}$$
$$x = x_2 - x_1$$
$$y = \frac{m_1x_1 + m_2x_2}{(m_1 + m_2)} - x_3$$
$$z = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} - x_4 \tag{19}$$

The Schrödinger equation in the limit $m_\downarrow \to \infty$ is

$$\left[-\frac{1}{2\mu_3}\partial_y^2 + C_0\delta\left(y-\frac{x}{2}\right) + C_0\,\delta\left(y+\frac{x}{2}\right) - u_3(x)\right]\phi(y,z;x) + \left[-\frac{1}{2\mu_3}\partial_z^2 + C_0\delta\left(z-\frac{x}{2}\right) + C_0\,\delta\left(z+\frac{x}{2}\right) - u_4(x)\right]\phi(y,z;x) = 0$$
20

Where $\phi(y,z;x)$ is the solution of equation (20) for a fixed value of x and $u_3(x) + u_4(x)$ is the energy of the system of equation (20) for a fixed value of x. Using boundary conditions, the continuity of the wave functions and the discontinuity of their first derivative $y = \pm\frac{x}{2}$, and $z = \pm\frac{x}{2}$ we obtain the following solutions

$$u_{3,l}(x) = \frac{1}{2\mu_3}\left[-\beta_3 + \frac{1}{x}W\left((-1)^{l+1}x\beta_3 e^{x\beta_3}\right)\right]^2$$
21

$$u_{4,l}(x) = \frac{1}{2\mu_4}\left[-\beta_4 + \frac{1}{x}W\left((-1)^{l+1}x\beta_4 e^{x\beta_4}\right)\right]^2$$
22

Where $\beta_3 = C_0\mu_3, \beta_4 = C_0\mu_4,$ $W(r)$ is the Lambert W function and $l = 0$ $(l = 1)$ gives the even (odd) solution. [8] Now, using $u_{3,l}(x)$ and $u_{4,l}(x)$ in equation (18)

$$-\frac{1}{2\mu_2}\partial_x^2\Psi(x) + u_{3,l}(x) + u_{4,l}(x)\Psi(x) = E\Psi(x) \qquad 23$$

The dimer-dimer system, the total wave function of equation (12), $\Psi(x,y,z) = \Psi(x)\,\phi(y,z;x)$ is antisymmetric under exchange of $x_1 \leftrightarrow x_2$.

Hamiltonian Movement of Atom in Geometry

The non-relativistic Hamiltonian with $O(a^n)$ improved action and we show the derivation of the hopping coefficients in equation (4). Let us consider a single particle non-relativistic free Hamiltonian,

$$\hat{H}_{free} = \frac{1}{2m}\int dr\,\nabla b_s^+\,\nabla b_s(r)$$
24

And it's the movement of atom in geometry can be written as

$$H_{free} = \frac{1}{2m}\sum_n \sum_{k=0}^{k_{max}} w_k\,[b_s^+\,(n+k) + b_s(n)\,b_s^+(n)b_s(n+k)]$$
25

Using the Schrödinger equation for the single particle, we can find the expression for the energy dispersion relation

$$H_{free}|p\rangle = Q(p)|p\rangle$$
26

Where $|p\rangle$ is defined in the momentum space as in the following

$|p\rangle = \frac{1}{\sqrt{L}}\sum_n e^{ipn}\, b_s^+(n)|p\rangle$

27

And $Q(p)$is the movement of atom dispersion relation

$Q(p) = \frac{1}{m}\sum_{k=0}^{k_{max}} w_k\ \cos(k\,p) = \sum_{k=0}^{k_{max}} w_k \sum_{v=0}^{\infty} \frac{(-1)^v}{(2v)!} k^{2v} p^{2v}$

28

The final expression can be solved up to desired order in momentum such that the dispersion relation is $Q(p) = \frac{p^2}{2m} + O(p^{2l})$. [8] For instance, if we solve equation (28) for w_k hopping coefficients which gives the dispersion relation $Q(p) = \frac{p^2}{2m} + O(p^8)$, then we find $k_{max} = 3$ and we obtain the following set of equations,

$w_0 + w_1 + w_2 + w_3 = 0$

29

$-\frac{w_1}{2} - 2w_2 - \frac{9w_3}{2} = \frac{1}{2}$

30

$\frac{w_1}{24} + \frac{2w_2}{3} + \frac{27w_3}{8} = 0$

31

$-\frac{w_1}{720} - \frac{4w_2}{45} - \frac{81w_3}{80} = 0$

32

Where $w_0 = \frac{49}{18}, w_1 = \frac{-3}{2}, w_2 = \frac{3}{20}, w_3 = \frac{-1}{90}$

Results

The following are the results obtain in the tables below:

Table 1a: Two fermion Scattering length of $\frac{m_\uparrow}{m_\downarrow} = 5.0$ and $\frac{m_\uparrow}{m_\downarrow} = 10.0$

S/N	$\frac{a_{ff}^*}{a_{ff}}$	$\frac{m_\uparrow}{m_\downarrow} = 5.0$ $\frac{a}{a_{ff}}$	$\frac{m_\uparrow}{m_\downarrow} = 10.0$ $\frac{a}{a_{ff}}$

1	1	0.40	0.42
2	1	0.50	0.54
3	1	0.57	0.60
4	1	0.65	0.67
5	1	0.70	0.74
6	1	0.75	0.78
7	1	0.80	0.83
8	1	0.85	0.90
9	1	0.90	0.96
10	1	0.95	1.00
11	1	1.00	-

Table 1b: Two fermion Scattering length of $\frac{m_\uparrow}{m_\downarrow} = 0.5$ and $\frac{m_\uparrow}{m_\downarrow} = 1.0$

S/N	$\frac{a^*_{ff}}{a_{ff}}$	$\frac{m_\uparrow}{m_\downarrow} = 5.0$ $\frac{a}{a_{ff}}$	$\frac{m_\uparrow}{m_\downarrow} = 10.0$ $\frac{a}{a_{ff}}$
1	1	0.25	0.30
2	1	0.31	0.37
3	1	0.38	0.45
4	1	0.43	0.50
5	1	0.45	0.55
6	1	0.50	0.58
7	1	0.53	0.61
8	1	0.55	0.66
9	1	0.57	0.70
10	1	0.60	0.74
11	1	0.63	0.77
12	1	0.65	0.80
13	1	0.67	0.83
14	1	0.70	0.85
15	1	0.73	0.88
16	1	0.75	0.92
17	1	0.78	0.94
18	1	0.80	0.96
19	1	0.82	0.99

Table 2: The fermion-dimer scattering length extrapolation with values of the mass ratio

S/N	$\frac{m_\uparrow}{m_\downarrow} = 0.1$		$\frac{m_\uparrow}{m_\downarrow} = 1.0$		$\frac{m_\uparrow}{m_\downarrow} = 2.0$		$\frac{m_\uparrow}{m_\downarrow} = 4.0$		$\frac{m_\uparrow}{m_\downarrow} = 7.0$		$\frac{m_\uparrow}{m_\downarrow} = 10.0$	
	$\frac{a_{fd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{fd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{fd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{fd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{fd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{fd}}{a_{ff}}$	$\frac{a}{a_{ff}}$
1	1.02	0.06	1.50	0.12	1.98	0.15	2.59	0.16	3.20	0.17	3.22	0.18
2	1.02	0.07	1.50	0.16	1.98	0.19	2.59	0.20	3.20	0.22	3.22	0.22
3	1.02	0.09	1.50	0.20	1.98	0.23	2.59	0.24	3.20	0.25	3.22	0.26
4	1.02	0.10	1.50	0.21	1.98	0.25	2.59	0.26	3.20	0.27	3.22	0.29
5	1.02	0.13	1.51	0.30	1.99	0.36	2.59	0.37	3.21	0.38	3.23	0.40
6	1.02	0.18	1.52	0.37	1.99	0.45	2.59	0.47	3.21	0.48	3.34	0.50
7	1.02	0.19	1.55	0.43	1.99	0.52	2.59	0.54	3.22	0.52	3.35	0.60
8	1.02	0.22	1.57	0.49	1.99	0.57	2.59	0.62	3.23	0.65	3.38	0.66
9	1.02	0.24	1.60	0.53	1.99	0.64	2.59	0.69	3.24	0.71	3.46	0.74
10	1.02	0.26	1.63	0.56	1.99	0.68	2.59	0.74	3.25	0.76	3.47	0.78
11	1.02	0.27	1.66	0.62	1.99	0.71	2.59	0.78	3.26	0.83	3.48	0.84
12	1.02	0.28	1.70	0.66	2.00	0.77	2.60	0.83	3.27	0.84	3.49	0.90
13	1.02	0.30	1.70	0.70	2.00	0.80	2.60	0.88	3.28	0.72	3.49	0.95
14	1.02	0.31	1.70	0.72	2.00	0.84	2.60	0.92	3.30	0.97	3.50	0.99
15	1.02	0.33	1.70	0.76	2.00	0.88	2.60	0.96	-	-	-	-
16	1.02	0.35	1.70	0.79	2.00	0.92	-	-	-	-	-	-
17	1.02	0.36	1.70	0.83	2.00	0.94	-	-	-	-	-	-
18	1.02	0.37	1.70	0.84	2.00	0.98	-	-	-	-	-	-
19	1.02	0.38	1.70	0.88	-	-	-	-	-	-	-	-
20	1.02	0.39	1.70	0.90	-	-	-	-	-	-	-	-
21	1.02	0.40	1.70	0.93	-	-	-	-	-	-	-	-
22	1.02	0.42	1.70	0.95	-	-	-	-	-	-	-	-
23	1.02	0.43	1.70	0.99	-	-	-	-	-	-	-	-

Table 3: The Dimer-dimer scattering length extrapolation with values of the mass ratio

S/N	$\frac{m_\uparrow}{m_\downarrow} = 1.0$		$\frac{m_\uparrow}{m_\downarrow} = 2.0$		$\frac{m_\uparrow}{m_\downarrow} = 5.0$		$\frac{m_\uparrow}{m_\downarrow} = 8.0$		$\frac{m_\uparrow}{m_\downarrow} = 11.0$		$\frac{m_\uparrow}{m_\downarrow} = 15.0$	
	$\frac{a_{dd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{dd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{dd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{dd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{dd}}{a_{ff}}$	$\frac{a}{a_{ff}}$	$\frac{a_{dd}}{a_{ff}}$	$\frac{a}{a_{ff}}$
1	1.50	0.30	1.50	0.35	1.20	0.40	1.55	0.41	1.75	0.41	1.97	0.42
2	1. 50	0.35	1.50	0.43	1.21	0.48	1.55	0.50	1.77	0.52	1.98	0.53
3	1. 50	0.43	1.60	0.50	1.	0.56	1.56	0.57	1.78	0.59	1.98	0.60

					21							
4	1. 50	0.47	1.60	0.56	1. 21	0.64	1.56	0.66	1.78	0.67	1.99	0.69
5	1. 50	0.54	1.60	0.62	1. 21	0.70	1.57	0.72	1.79	0.73	1.99	0.75
6	1. 50	0.59	1.60	0.67	1. 21	0.76	1.58	0.78	1.80	0.80	2.00	0.80
7	1. 50	0.61	1.60	0.72	1. 22	0.80	1.59	0.84	1.81	0.86	2.00	0.86
8	1. 50	0.66	1.70	0.76	1. 22	0.86	1.60	0.88	1.81	0.91	2.01	0.91
9	1. 50	0.69	1.70	0.80	1. 22	0.91	1. 60	0.92	1.81	0.93	2.01	0.96

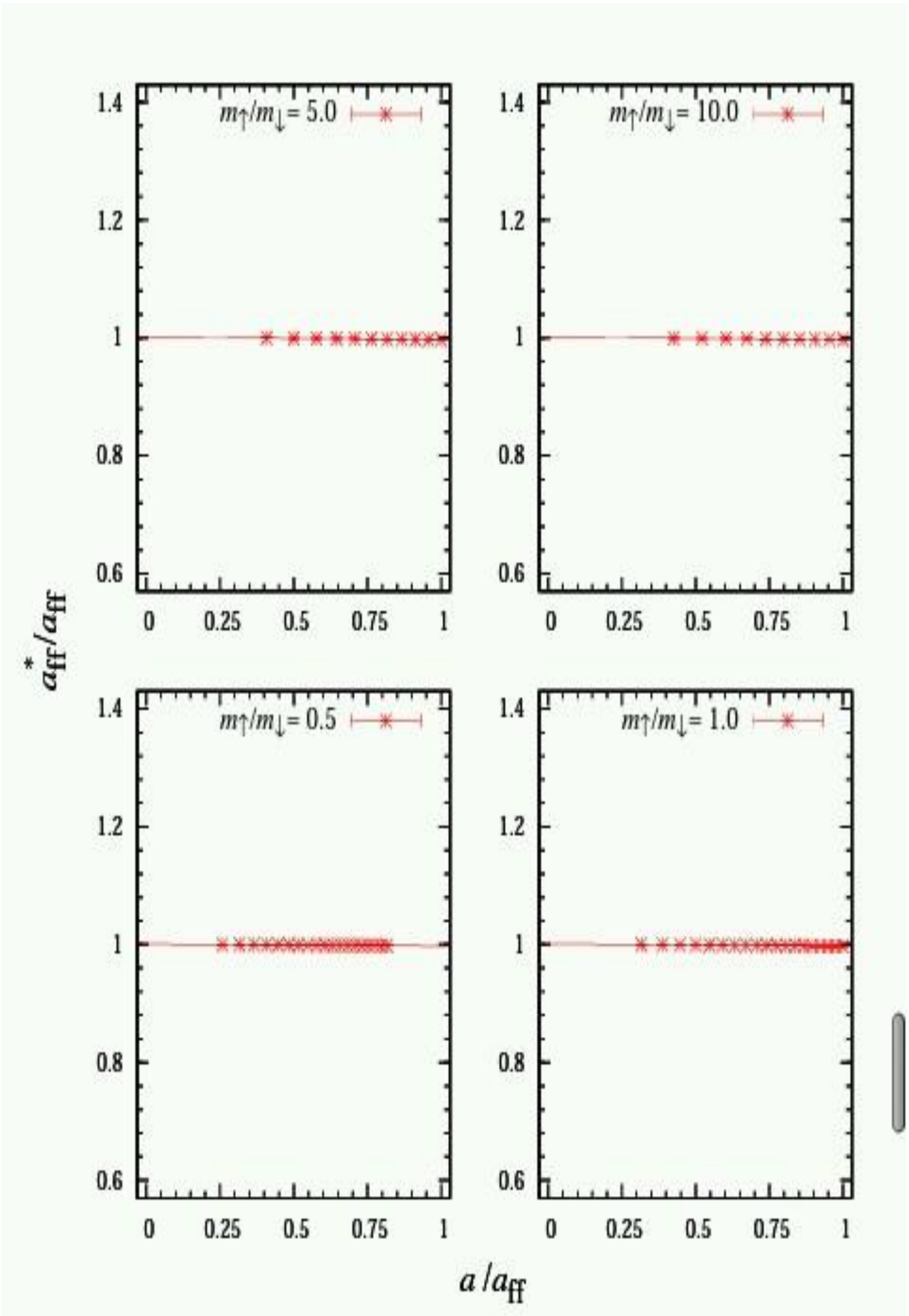

Fig. 1: The ratio of the two-fermion scattering length.

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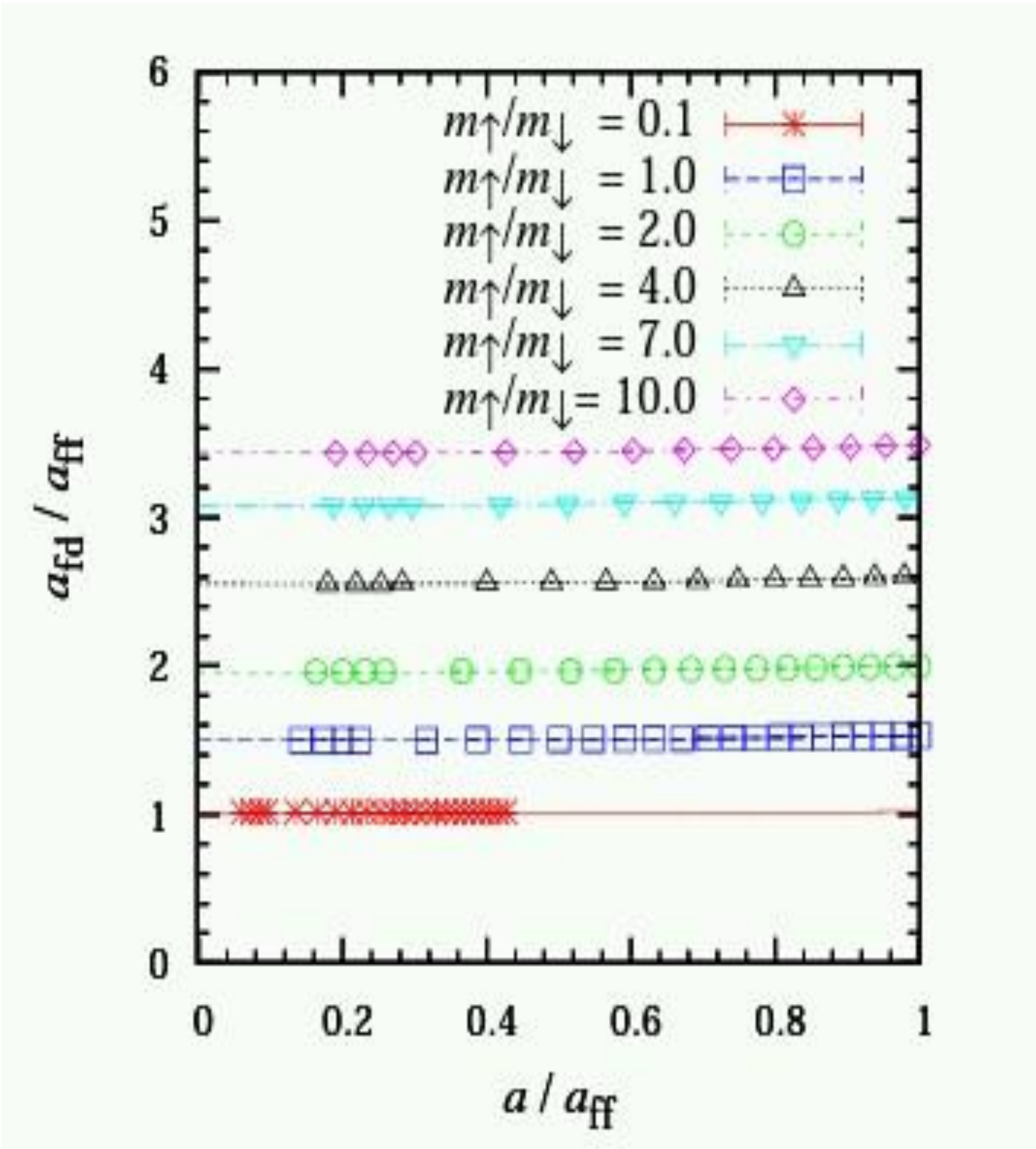

Fig. 2: Plot of $\frac{a_{fd}}{a_{ff}}$ versus $\frac{a}{a_{ff}}$ for fermion-dimer scattering length.

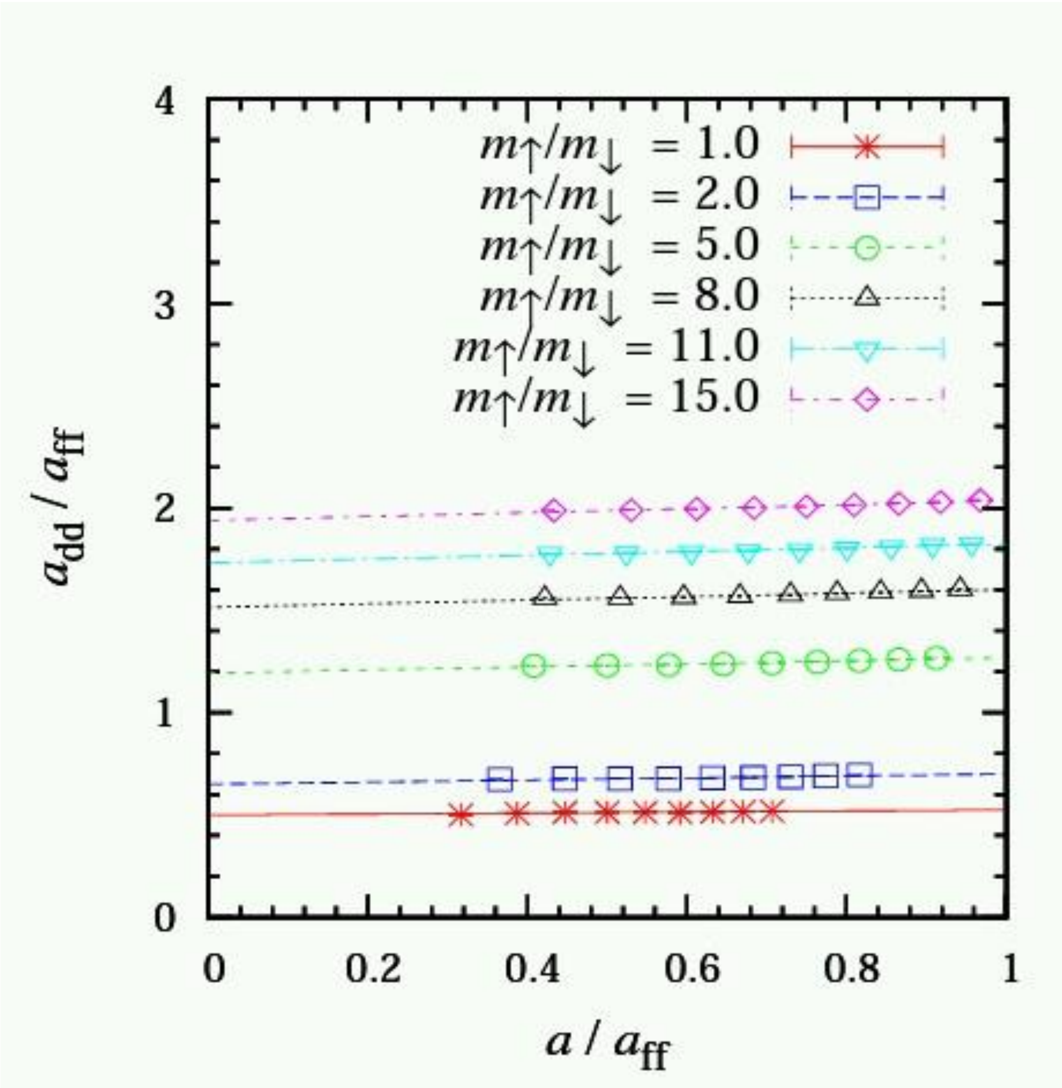

Fig. 3: Plot of $\frac{a_{dd}}{a_{ff}}$ versus $\frac{a}{a_{ff}}$ for Dimer-dimer scattering length.

DISCUSSIONS

The two-fermion scattering lengths from the atoms moving in geometry calculations for a different mass ratio $\frac{m_\uparrow}{m_\downarrow}$ and for having a broad range of space where the effective range expansion is extracted a^*_{ff} using the calculation scattering phase shift $\delta(p)$ and the relative momentum p from the atom movement in the geometry. The scattering length a_{ff} is calculated through the use of binding energy. Fig. 1 demonstrates the extract the scattering length from the atoms movement geometry calculations

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with negligible atoms artifacts. The continuum limits extrapolation of the fermion-dimer scattering length a_{fd} as a fraction of the fermion-fermion scattering lengtha_{ff}. The ratio $\frac{a_{fd}}{a_{ff}}$ is universal and it is known as the universal fermion-dimer scattering length. Fig. 2 display the atom movement in geometry discretization errors are negligible for smaller mass ratio $\frac{m_\uparrow}{m_\downarrow}$ while the continuum limit extrapolation is necessary as $m_\uparrow \to \infty$. Fig. 3 indicate the continuum limit extrapolation of the dimer-dimer scattering length a_{dd} as a fraction of the fermion-fermion scattering length a_{ff}. The ratio $\frac{a_{dd}}{a_{ff}}$ is universal and it is called the universal dimer-dimer scattering length.

CONCLUSION

All properties of the atom moving in geometry system scale is proportionally with fermion-fermion scattering length while the case of the two component fermions with different masses, the ratio is a new parameter and it changes some of the properties of the system. Fig. 2 and 3 shows the final results of the universal fermion-dimer scattering length and universal dimer-dimer scattering length.

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