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SERIAL CORRELATION OF A FINITE AND INFINITE LAG MODELS

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ABSTRACT
In this research, an evaluation of the relationship between finite and infinite leg models variable were considered using ARMA approach. The method made use of descriptive statistics, augmented dickey fuller test, histogram, and correlogram techniques to check the presence of least significant values in order to select and fit the best model. The statistical Software packages used are Microsoft Excel and Eviews 8 version. The assessment criteria for the analysis were based on the ARMA test approach with Akaike Information Criterions (AIC) and Schwarz Information Criterions (SIC) measures to select the least significant value and to make decision.

INTRODUCTION
A ***distributed-lag model*** is a dynamic model in which the effect of a regressor x on y occurs over time rather than all at once. In the simple case of one explanatory variable and a linear relationship, we can write the model where ut is a stationary error term.1 This form is very similar to the infinite-moving-average representation of an ARMA process, except that the lag polynomial on the right-hand side is applied to the explanatory variable x rather than to a white-noise process ε. The individual coefficients βs are called ***lag weights*** and the collectively comprise the ***lag distribution*** and they define the pattern of how x affects y over time.

THEORETICAL FRAME ANDLITERATURE REVIEW
We cannot, of course, estimate an infinite number of β coefficients without paring the lag to finite length q, which is appropriate if the lag distribution is effectively zero beyond q periods. Another approach is to use a functional form that allows the lag distribution to decay gradually to zero. This also explain that a stationary autoregressive process can be expressed as an infinite moving average with declining lag weights, so a form with one or more lags of y on the right-hand side will allow infinite-length lag distributions while requiring estimation of only a small number of parameters.

One difficulty that is common to all distributed-lag models is the choice of lag length, to check whether this can choose the point *q* at once in order to pair a finite lag distribution or choose how many lagged dependent variables to be included. From the following discussion above, we can presume that the estimated coefficients of the following Distributed Lag model is given as:

$Y_t = \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + ... + \beta_n x_{t-n} + u_t$. where $t = 1, 2, ..., T$. (1)
Hence we call this a "finite" Distributed Lag model if the value of n is finite. From the above assumption, an intercept can be added into the model or add other regressors to expand the equation. Since the giving model above appears to be simple regression, we'll presume that the error term, u_t, satisfies all of the usual assumption. If the maximum lag length in the model, n, is much less than T, then we could just apply OLS to estimate the regression coefficients. However, even if this is achievable, in the sense that there are positive degrees of freedom, this may not be the smartest way in which to proceed. For most economic time-series, where *x* happens to be the successive lags among variables with high correlation with each other. Predictably, this the reasons why *x* variables results are quite simple and useful in multicollinearity equations.

In response, Shirley Almon (1965) suggested a pretty neat way of re-formulating the model prior to its estimation. She made use of Weierstrass's Approximation Theorem, which tells us (roughly) that: "Every continuous function defined on a closed interval [a, b] can be uniformly approximated, arbitrarily closely, by a polynomial function of finite degree, P." Notice that the theorem *doesn't tell us* what the value of P will be. This presents a type of model-selection problem that we have to solve. The flip-side of this is that if we *select* a value for P, and get it wrong, then there will be model miss-specification issues that we have to face. In fact, we can re-cast these issues in terms of those associated with the incorrect imposition of linear restrictions on the parameters of our model.

Dynamic Effects of Temporary and Permanent Changes

In cross-sectional models, we often used econometric methods to estimate the ***marginal effect*** of an independent variable *x* on the dependent variable *y*, holding all of the other independent variables constant: . In time-series models, we must consider not only dy \ dx but also assume that *yt* does not depend on future values of *x*, thus we exclude negative values of *y* from the summation. However, it is theoretically possible to have negative lags from the right-hand side of the equation. For example, in real life situation, if law is given to people at a particular period of time, they may want to obey the law irrespective of the consequence in order to live in peace but with time, they might change their behavior if they know the law is going to change in the future.

METHODOLOGY

Determining lag length by statistical significance

An observable way to choose the length of a lag is to start with a long lag, test the statistical significance of the coefficient at the longest lag. Since the "irregular lag" shortens the lag by one period it cannot reject the null hypothesis with the effect at the longest lag when the lag is zero. If this happens, it will continue to shorten the lag until the irregular lag coefficient is statistically significant.

Although this method has applications, and there are dangers as well. We can simply recall that an insignificant t -statistic on the irregular lag only *fails to reject* the hypothesis of a zero coefficient; it does not prove that the coefficient is zero! It is therefore quite possible that this procedure will choose a lag length that is too short. An alternative that also relies on statistical tests of significance is to start with a very short lag and successively add lag terms, continuing to add lags that are statistically significant and discontinuing when the marginal lag coefficient is not.

Determining Lag Length by Information Criteria

Information criteria are considered to portion the amount of information about the dependent variable contained in a set of regressors. They are goodness-of-fit measures of the same type as R^2 , but without the appropriate explanation as share of variance explained that we give to $R2$ in an Ordinary Least Square Regression with an intercept term. The two most commonly used criteria are the ***Akaike information criterion (AIC)*** and the ***Schwartz/Bayesian information criterion (SBIC)***. They are usually calculated in log form by the formulas R^2

But the "main ingredient" in both information criteria is the sum of squared residuals, which we want to make as small as possible. Thus, we *minimize* the criteria and choose the model with the *smallest* AIC or SBIC value for the equation. Hence, when using the information criteria to choose lag length, we must be very careful to make sure that all candidate models among which we are choosing are estimated exactly over the same sample period. This also requires particular caution in lag models because there are usually more observations available for models with shorter lags (because with fewer lags we "lose" fewer observations at the beginning of the sample). The first term of the information criteria (common to both) is the log of the standard error of the estimate (SEE), uncorrected for degrees of freedom which measures the accurate Distributed-Lag Models (DLM).

Models with lagged dependent variables

The autoregressive-moving-average (ARMA) time-series is a process by which we model univariate time series. The autoregressive component of the ARMA model involves using one or more lagged values of *y* as determinants of the current value *yt*. We can apply the same method in a distributed-lag context by adding y_{t-1} and possibly additional lags to the right-hand side introduces us into the simplest model called "***Koyck lag***", which has one lag of *y* on the right-hand side with only the current value of *x* and the additional lagged values of *x* in addition to lagged values of *y* variables which leads to the ***rational lag*** model.

The first-order autoregressive lag model is often called the Koyck lag in recognition of the seminal application of the model to the macroeconomic investment function by L. M. Koyck (1954). With a single explanatory variable *x*, the model is written

$$Yt = \delta + \varphi_1 Y_{t-1} + \theta_0 x_t + u_t. \quad (2)$$

Estimation of equation presents challenges because *yt* – 1 is by definition not strictly exogenous and, unless *ut* is white noise, then ,is not even weakly exogenous. In terms of the lag operator, we can write equation (2) as

$$(1 - \phi_1 L)y_t = \delta + \theta_0 x_t + u_t \quad (3)$$

Where equation (3) suggests that solving the model for y_t gives

$$y_t \frac{\delta}{1-\phi_1 L} + \frac{\theta_0}{1-\phi_1 L} x_1 + \frac{1}{1-\phi_1 L} u_t, \quad (4)$$

which, by the methods of analogous used to examine autoregressive processes, can be written in an infinite-distributed-lag form as:

$$y_t = \frac{\delta}{1-\phi_1} + \theta_0 \sum_{s=0}^{\infty} \phi_1^s x_{t-s} + \sum_{s=0}^{\infty} \phi_1^s u_{t-s} \quad (5)$$

as long as . $|\phi_1|$<1

Equation (4) has the form of the infinite distributed lag (2), with

$$\alpha = \frac{\delta}{1-\phi_1} \quad (6)$$

$$\beta_s = \theta_0\phi_1^s. \quad (7)$$

and the disturbance term having an infinite-moving-average form.

Longer Autoregressive Lags

The Koyck lag treats y as a first-order autoregressive process. Although one lag of the dependent variable is often enough to capture the dynamic relationship between y and the regressors. However, the longer autoregressive lags can be included as well. Where the general auto-regressive lag model AR (p) would be written

$$\phi(L)y_t = \delta + \theta_0 x_t + u_t, \quad (8)$$

with $\phi(L)$, a p-order polynomial in the lag operator.

In order for the relationship between y and x to be dynamically stable, the roots of $\phi(L)$ must lie outside the unit circle. This generalizes the condition $|\phi_1| < 1$ from the Koyck lag model. If the stability condition does not hold on the Distributed-Lag Models then a one-time change in x will cause permanent or explosive changes in y, which suggests differencing y to make the order of integration the same on both sides of the equation.

DATA PRESENTATION AND RESULTS DESCRIPTION

These research emphases on the extents of which finite and infinite lag model can be regressed through the means of time series analysis which employed the use of a software program call Eviews 8 with a total data sets of 360 sample size. The values of the performance was calculated as to find:

i. The model that adequately describes the data
ii. The model that does not adequately describe the data
iii. The correlation relationship of four variables and select the best fitted model

The Procedure for Data Analysis

This section is divided into various parts;

Descriptive statistics of the Part, Comparison of the estimated models parameters, the graph and the criterion values consideration for model selection.

Descriptive statistics

Figure 1: Is the time plot of a finite and infinite lag models of monthly average exchange rate moments at various segments of the market for 2006 – 2015

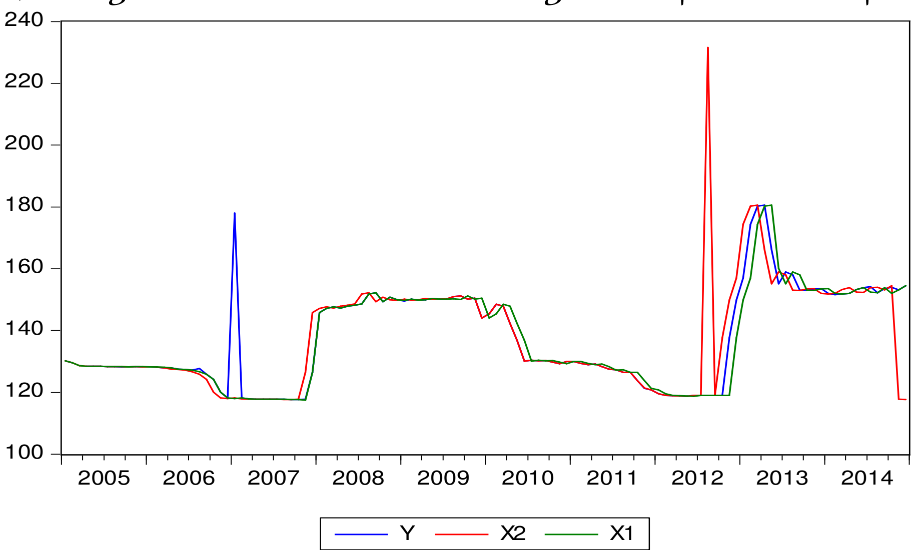

Table 1: Descriptive Statistics of dependent and independent variables

Dependent Variable: Y
Method: Least Squares
Date: 07/16/19 Time: 22:31
Sample: 2005M01 2014M12
Included observations: 119

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.286469	5.464118	1.150500	0.2523
X1	0.868757	0.053711	16.17477	0.0000
X2	0.089281	0.046399	1.924194	0.0568

R-squared	0.838191	Mean dependent var	137.8218
Adjusted R-squared	0.835401	S.D. dependent var	15.62209
S.E. of regression	6.338006	Akaike info criterion	6.555892
Sum squared resid	4659.757	Schwarz criterion	6.625954
Log likelihood	-387.0756	Hannan-Quinn criter.	6.584342
F-statistic	300.4469	Durbin-Watson stat	1.840166
Prob(F-statistic)	0.000000		

Table 2: Is the Estimation Command of the three variables:

======ss====================

LS Y C X1 X2

Estimation Equation:

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$Y = C(1) + C(2)*X1 + C(3)*X2$

Substituted Coefficients:

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$Y = 6.28646865379 + 0.868757350492*X1 + 0.0892805856209*X2$

Table 3: Augmented Dickey – Fuller Test Statistics

Null Hypothesis: Y has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.324300	0.0000
Test critical values:	1% level	-3.486551	
	5% level	-2.886074	
	10% level	-2.579931	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 07/16/19 Time: 22:23
Sample (adjusted): 2005M03 2014M12
Included observations: 118 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(-1)	-0.588953	0.110616	-5.324300	0.0000
D(Y(-1))	-0.189621	0.091675	-2.068410	0.0408
C	82.42700	15.57843	5.291100	0.0000
R-squared	0.385987	Mean dependent var		0.211695

Adjusted R-squared	0.375308	S.D. dependent var	30.07996
S.E. of regression	23.77444	Akaike info criterion	9.200194
Sum squared resid	65000.75	Schwarz criterion	9.270635
Log likelihood	-539.8114	Hannan-Quinn criter.	9.228795
F-statistic	36.14615	Durbin-Watson stat	1.306917
Prob(F-statistic)	0.000000		

Figure 2: Correlation of Finite and Infinite Lag Models

Date: 07/03/20 Time: 22:44
Sample: 2005M01 2014M12
Included observations: 119

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|******|</td><td>.|******|</td><td>1</td><td>0.834</td><td>0.834</td><td>84.786</td><td>0.000</td></tr>
<tr><td>.|******|</td><td>.|** |</td><td>2</td><td>0.766</td><td>0.232</td><td>156.94</td><td>0.000</td></tr>
<tr><td>.|***** |</td><td>.|. |</td><td>3</td><td>0.695</td><td>0.039</td><td>216.86</td><td>0.000</td></tr>
<tr><td>.|**** |</td><td>.|. |</td><td>4</td><td>0.611</td><td>-0.064</td><td>263.63</td><td>0.000</td></tr>
<tr><td>.|**** |</td><td>.|. |</td><td>5</td><td>0.531</td><td>-0.058</td><td>299.25</td><td>0.000</td></tr>
<tr><td>.|*** |</td><td>.|. |</td><td>6</td><td>0.450</td><td>-0.060</td><td>325.01</td><td>0.000</td></tr>
<tr><td>.|*** |</td><td>.|. |</td><td>7</td><td>0.382</td><td>-0.009</td><td>343.79</td><td>0.000</td></tr>
<tr><td>.|** |</td><td>.|. |</td><td>8</td><td>0.321</td><td>-0.002</td><td>357.16</td><td>0.000</td></tr>
<tr><td>.|** |</td><td>.|. |</td><td>9</td><td>0.258</td><td>-0.030</td><td>365.91</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>.|. |</td><td>10</td><td>0.200</td><td>-0.037</td><td>371.16</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>.|. |</td><td>11</td><td>0.164</td><td>0.035</td><td>374.75</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>12</td><td>0.152</td><td>0.092</td><td>377.86</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>*|. |</td><td>13</td><td>0.106</td><td>-0.077</td><td>379.39</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>*|. |</td><td>14</td><td>0.066</td><td>-0.072</td><td>379.99</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>15</td><td>0.028</td><td>-0.054</td><td>380.10</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>16</td><td>-0.004</td><td>-0.017</td><td>380.10</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>17</td><td>-0.037</td><td>-0.018</td><td>380.29</td><td>0.000</td></tr>
<tr><td>*|. |</td><td>.|. |</td><td>18</td><td>-0.072</td><td>-0.033</td><td>381.04</td><td>0.000</td></tr>
<tr><td>*|. |</td><td>.|. |</td><td>19</td><td>-0.101</td><td>-0.026</td><td>382.52</td><td>0.000</td></tr>
<tr><td>*|. |</td><td>*|. |</td><td>20</td><td>-0.147</td><td>-0.094</td><td>385.65</td><td>0.000</td></tr>
<tr><td>**|. |</td><td>*|. |</td><td>21</td><td>-0.207</td><td>-0.132</td><td>391.97</td><td>0.000</td></tr>
<tr><td>**|. |</td><td>.|. |</td><td>22</td><td>-0.236</td><td>0.020</td><td>400.21</td><td>0.000</td></tr>
<tr><td>**|. |</td><td>*|. |</td><td>23</td><td>-0.304</td><td>-0.146</td><td>414.06</td><td>0.000</td></tr>
<tr><td>**|. |</td><td>.|. |</td><td>24</td><td>-0.341</td><td>-0.045</td><td>431.64</td><td>0.000</td></tr>
<tr><td>***|. |</td><td>.|. |</td><td>25</td><td>-0.356</td><td>0.044</td><td>451.03</td><td>0.000</td></tr>
<tr><td>***|. |</td><td>.|. |</td><td>26</td><td>-0.385</td><td>-0.036</td><td>474.02</td><td>0.000</td></tr>
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***|.    |        .|.    |    27  -0.404 -0.036 499.52  0.000
***|.    |        .|.    |    28  -0.417 -0.036 527.03  0.000
***|.    |        .|.    |    29  -0.426 -0.040 556.07  0.000
***|.    |        .|.    |    30  -0.424 -0.022 585.15  0.000
***|.    |        .|.    |    31  -0.411  0.000 612.81  0.000
***|.    |        .|.    |    32  -0.393  0.020 638.35  0.000
***|.    |        .|.    |    33  -0.366  0.026 660.79  0.000
 **|.    |        .|.    |    34  -0.328  0.026 679.02  0.000
 **|.    |        .|.    |    35  -0.306 -0.007 695.07  0.000
 **|.    |        .|.    |    36  -0.264  0.039 707.20  0.000
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Figure 3: Is the differencing graph of the three variables

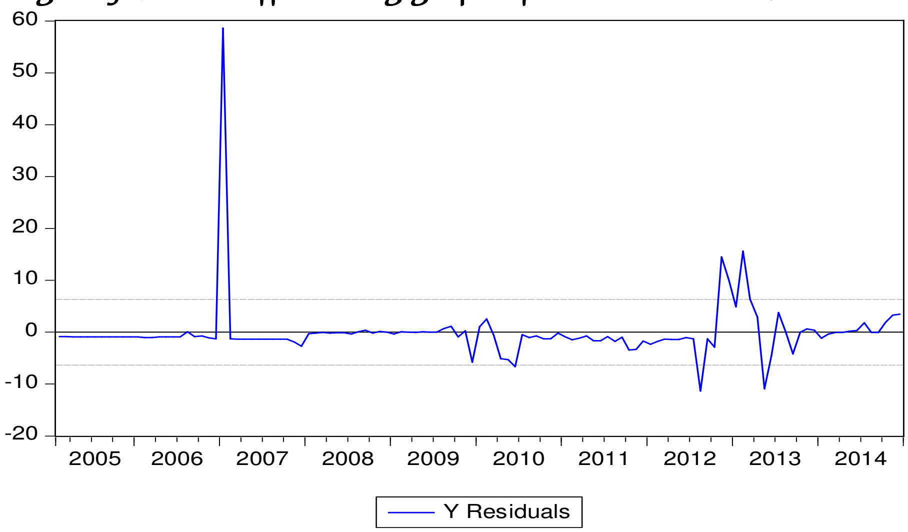


Table 5: Is the Gradients of the Three Variables:

Gradients of the Objective Function
Gradients evaluated at estimated parameters
Equation: UNTITLED
Method: Least Squares
Specification: Y C X1 X2

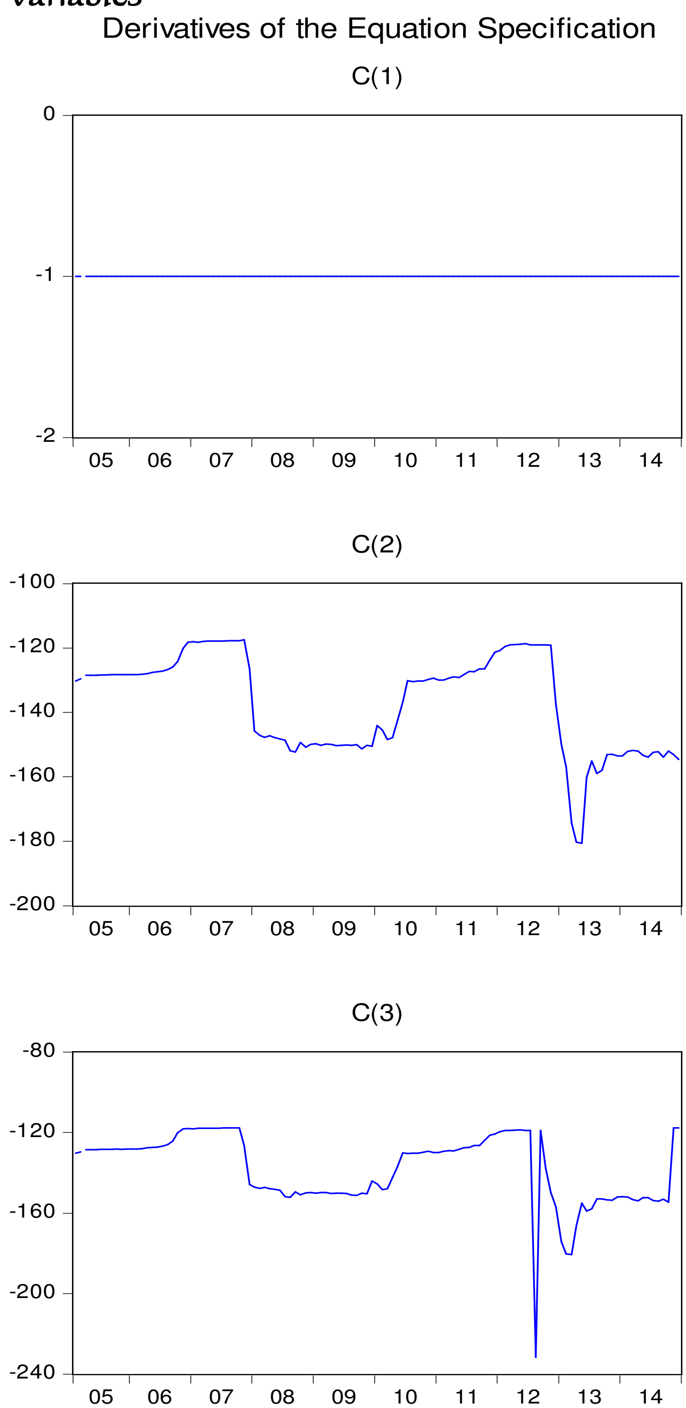
Variable	Sum	Mean	Weighted Grad.
C	3.42E-12	2.88E-14	2.42E-16
X1	-5.01E-10	-4.21E-12	-2.10E-24
X2	-1.49E-09	-1.25E-11	-1.86E-22

Table 6: Is the Derivatives of the Three Equation Specification

Derivatives of the Equation Specification
Equation: UNTITLED
Method: Least Squares
Specification: RESID = Y - (C(1) + C(2)*X1 + C(3)*X2)

Variable	Derivative of Specification
C(1)	-1
C(2)	-x1
C(3)	-x2

Figure 4: Is the derivatives of the equation specification graph of the three variables



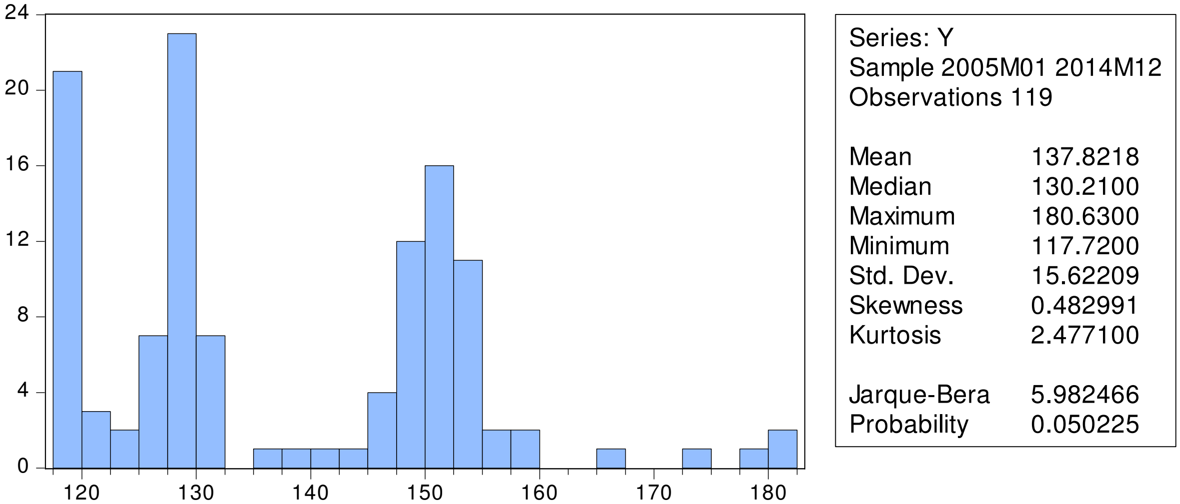


Figure 5: Is the serial residuals histogram of the three variables.

RESULTS AND DISCUSSION

The analysis of this research work was carried out with the use of three different samples in order to examine if there is a relationship or interception between the finite and infinite lag models also to check the decrease or increase exist between then with the aid of 320 sample variables. Finally, we further investigate the best suitable fitted model for the analysis.

Explanation of the Time Plot of the Finite and Infinite Lag Models

First of all, a time series plot of the finite and infinite were plotted to check the presence of trend if figure 1. This was important because it helps to detect if there is any traditional components of a time series such as trend, stationality, constant mean and constant variance exhibited by the data. From the graph, it displays non- stationary of the time plot of a finite and infinite lag models of monthly average exchange rate moments at various segments of the market for 2006 – 2015.

Test for Stationary

It is interesting to see from the figure that there is inconsistent variation of values between the three variables. We further difference the data for stationary purpose.

Figure 3: Shows the stationary time plot as a result of the first differencing graph of the three variables and it was notice that the first differencing enabled the time plot graph to have stationary at the different time lags.

Correlogram of the Residuals

In figure 3, is the correlogram of the three variable residuals. Appearance of these residuals shows that they are correlated.

Model Selection of Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC) Value

The models were estimated from the analysis of the various variables above and the following values of Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC) were estimated as:
Akaike Information Criterion (AIC) = 6.555892 and Schwarz Information Criterion (SIC) = 6.625954

The Correlogram of Model Selection of ACF and PACF Value

From the correlogram in figure 2, which indicate a spike at lag one, two and three of the ACF suggested a moving average model order one, two and three MA (1, 2 and 3) as it continued to diminished and latter begins again from the middle of the figure to infinity. In addition, there was another spike notice at lag **one of the PACF which suggest the presence of an autoregressive component of order one AR (1) and a tapering pattern to zero in correlogram.**

Fitted Model

Let the estimated coefficients of the following Distributed Lag model be given as:

$$Y_t = \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \ldots\ldots + \beta_n x_{t-n} + u_t \quad ; \quad t = 1, 2, \ldots, T.$$

Then we call this a "finite" Distributed Lag model if the value of n is finite.
Where

$Y_t = C(1) + C(2)^*X_1 + C(3)^*X_2$

Substituted Coefficients as:

$Y_t = 6.28646865379 + 0.868757350492^*X_1 + 0.0892805856209^*X_2 + \ldots\ldots + \beta_n x_{t-n}$

SUMMARY

The study estimated four models and selected the best fitted model with minimum Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC). Also, the Unit Root Test, Residual Plot of Correlogram and Histogram were check and the unit root test or Augmented Dickey-fuller test were observed to ascertain stationary of the time series while the residual plot of correlogram and histogram indicate or proved the adequate model.

CONCLUSSION

Significantly, the researcher was able to identify a suitable parameter for the finite and infinite leg model using Autoregressive model of order one AR (1) and Moving Average model of order one MA (1) as the best fitted model. It also showed that there exists a normal distribution in the residual plot which indicate that the model is adequate and affirm the existence of correlation of leg model.

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