International Journal of Science and Advanced Innovative Research
ISSN: 2536-7315 (Print) 2536-7323 (Online)
Volume 4, Number 3, September, 2019
http://www.casirmediapublishing.com

SENSITIVITY OF SOME STATIONARY ITERATIVE METHODS TO TOLERANCE PARAMETER

A. A. Ibrahim and R. O. Osinowo
Department of Mathematical Sciences,
Oduduwa University, Ipetumodu, Ile-Ife, Nigeria
Email: adebisiibrahim97@gmail.com

ABSTRACT

Stationary iterative solution of algebraic system of equations requires the input of the coefficient matrix A, the constant column vector b, the initial approximation x0, the tolerance parameter, tol and itmax, the intended maximum number of iterations.. While the coefficient matrix and the column vector are given, the initial approximation, usually taken to be the zero column vector of the same dimension as the constant column vectorb, the tolerance parameter and the intended maximum iterations are to be supplied by the one performing the operation. Again, the intended maximum iterations only sets the limit that must not be exceeded, the tolerance parameter plays a significant role in the arithmetic precision of the output. The Jacobi and the Gauss-Seidel iterations are used to solve some equations and they yielded useful results. The tolerance parameter was observed to have effect on the number of iterations as well as their minimization of the errors involved.
Keywords: Stationary iterative methods, algebraic system of equations, tolerance parameter, minimization of the errors.

INTRODUCTION:

The solution of linear algebraic systems of equations

$$Ax = b \tag{1}$$

appears in almost all aspects of human endeavor, especially the solution of discretized partial differential equations in modeling of engineering problems to economic problems [7, 8,10]. Direct methods of such equations are usually applied where the number of equations involved is small. But for large linear systems, iterative methods are preferable, especially where the coefficient matrix is sparse [3, 6, 9]. In this work the Jacobi and the Gauss-Seidel methods are used to solve algebraic linear systems with different tolerance parameters to test the degree of precision of the solution with the choice of the zero column vector as the initial approximation. While the n dimensional zero vector is usually chosen as the initial approximation, the choice of the tolerance factor varies. The degree of accuracy of solution is sensitive to the choice of these two factors. In fact the stability of the method depends on the tolerance [1,7].

MATERIALS AND METHODS

Iterative requires the normal splitting the coefficient matrix A into $A = M - N$ where M is invertible. This gives

$$x^{(k+1)} = M^{-1}Nx^{(k)} + M^{-1}b \tag{2}$$

as the iterative scheme [9,11].

Jacobi Iteration

The Jacobi method uses successive correction whereby the initial approximation $x^{(0)}$ is used to compute the next approximation $x^{(k)} = (x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \dots, x_n^{(k)})$ which is in turn

International Journal of Science and Advanced Innovative Research
ISSN: 2536-7315 (Print) 2536-7323 (Online)
Volume 4, Number 3, September, 2019
http://www.casirmediapublishing.com

used in the computation of the next approximation $x^{(k+1)} = (x_1^{(k+1)}, x_2^{(k+1)}, x_3^{(k+1)}, \ldots, x_n^{(k+1)})$.
The Jacobi method is therefore defined as

$$x^{(k+1)} = \frac{1}{a_{ii}}\left(-\sum_{j=1}^{i-1} a_{ij}x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)} + \mathrm{b}\right) \qquad (3)$$

and in vector form as $x^{(k+1)} = -D^{-1}Ex^{(k)} + D^{-1}b$ where $E = L + U$ [8]

Gauss-Seidel Iteration

The Gauss-Seidel iterative scheme on the other hand is called method of simultaneous correction improves the Jacobi method. In this case the initial approximation $x^{(0)}$ is used to compute the next approximation $x^{(k)} = (x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \ldots, x_n^{(k)})$ which is in turn used in the computation of the next approximation $x^{(k+1)} = (x_1^{(k+1)}, x_2^{(k)}, x_3^{(k)}, \ldots, x_n^{(k)})$ [10].
That is

$$x^{(k+1)} = \frac{1}{a_{ii}}\left(-\sum_{j=1}^{i-1} a_{ij}\, x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k)} + b\right) \qquad (4)$$

Thus we have

$$(L + D)x^{(k+1)} = -Ux^{(k)} + b \qquad (5)$$

$$Lx^{(k+1)} + Dx^{(k+1)} = -Ux^{(k)} + b \qquad (6)$$

$$x^{(k+1)} = -(L + D)^{-1}Ux^{(k)} + -(L + D)^{-1}b \qquad (7)$$

Convergence Criterion

Although, the convergence of any stationary iterative methods depends on the structure of the coefficient matrix A, that is the matrix A must be diagonally dominant [2], the speed of convergence is also related to the spectral radius $\rho(A)$ of the iteration matrix. That is (1) guaranteed to converge if $\rho(A) < 1$. This implies that given a matrix A, if there exist an eigenvalue λ and a non-zero vector x such that $(\mathrm{A} - \lambda \mathrm{I})\mathrm{x} = 0$, the largest eigenvalue λ is called the spectral radius of the matrix A [2 ,3].

The Tolerance Parameter:

The iteration process terminates when the convergence criterion is satisfied. One criterion that is usually used to terminate the iteration process is the relative error $\left|\frac{e}{x^{(k+1)}} < \epsilon\right|$ in the successive iterates. That is the relative error is less than

$$\left|\frac{x^{(k+1)} - x^{(k)}}{x^{(k+1)}}\right| < \epsilon$$

for some $\epsilon > 0$. The tolerance parameter tol is usually of the form $tol = 10^{-r}$ for some integer r[1, 7, 8].
When the difference between the successive iterates $x^{(k+1)}$ and $x^{(k)}$ is small.

RESULTS AND DISCUSSION

This section discusses the application of the Jacobi and Gauss-Seidel iterative schemes to four linear algebraic systems. For each problem different tolerance parameters are applied to test how sensitive the parameters are to the rate of convergence of the schemes.

The results obtained, the number of iterations and the computer execution time for each problem in Table 3.1 are shown in Table 3.2a -.2d

Table 3.1: The Problems

S/N	Matrix A	vector b	Dimension
1	$\begin{pmatrix} 8 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 6 \end{pmatrix}$	$\begin{pmatrix} 20 \\ 11 \\ 23 \end{pmatrix}$	$(3X3)$
2	$\begin{pmatrix} 7 & 1 & -3 & 1 \\ 2 & 10 & 1 & 4 \\ 3 & 1 & 8 & 2 \\ 1 & 2 & 4 & 9 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 16 \\ 36 \\ 27 \end{pmatrix}$	$(4X4)$
3	$\begin{pmatrix} 11 & 1 & -2 & 0 & 3 \\ 2 & 8 & 1 & -1 & 2 \\ 1 & 2 & 10 & 2 & 3 \\ 3 & 1 & 1 & 8 & 2 \\ 1 & -2 & 1 & 3 & 12 \end{pmatrix}$	$\begin{pmatrix} 48 \\ 19 \\ 11 \\ 33 \\ 54 \end{pmatrix}$	$(5X5)$
4	$\begin{pmatrix} 8 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 8 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 8 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 8 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 8 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 8 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 8 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 8 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 8 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 8 \end{pmatrix}$	$\begin{pmatrix} 10 \\ 12 \\ 12 \\ 12 \\ 12 \\ 12 \\ 12 \\ 12 \\ 12 \\ 10 \end{pmatrix}$	$(10X10)$

International Journal of Science and Advanced Innovative Research
ISSN: 2536-7315 (Print) 2536-7323 (Online)
Volume 4, Number 3, September, 2019
http://www.casirmediapublishing.com

Table 3.2a

		Jacobi			Gauss-Seidel		
	Exact Solution	Iterated Solution	$\|Error\|$	No. of Iterations	Iterated Solution	$\|Error\|$	No. of Iterations
$r=1$	$\begin{pmatrix}2.0000\\1.0000\\3.0000\end{pmatrix}$	$\begin{pmatrix}1.9757\\0.9522\\2.9418\end{pmatrix}$	$\begin{pmatrix}0.0243\\0.0478\\0.0582\end{pmatrix}$	6	$\begin{pmatrix}1.9937\\1.0140\\2.9941\end{pmatrix}$	$\begin{pmatrix}0.0063\\0.0140\\0.0059\end{pmatrix}$	3
$r=2$	$\begin{pmatrix}2.0000\\1.0000\\3.0000\end{pmatrix}$	$\begin{pmatrix}2.0036\\1.0073\\3.0079\end{pmatrix}$	$\begin{pmatrix}0.0036\\0.0073\\0.0079\end{pmatrix}$	9	$\begin{pmatrix}1.9990\\1.0020\\2.9992\end{pmatrix}$	$\begin{pmatrix}0.0010\\0.0020\\0.0008\end{pmatrix}$	4
$r=3$	$\begin{pmatrix}2.0000\\1.0000\\3.0000\end{pmatrix}$	$\begin{pmatrix}2.0003\\1.0006\\3.0006\end{pmatrix}$	$\begin{pmatrix}0.2796\\0.5593\\0.6183\end{pmatrix} * \mathrm{e}-03$	13	$\begin{pmatrix}1.9999\\1.0003\\2.9999\end{pmatrix}$	$\begin{pmatrix}0.1454\\0.2796\\0.1156\end{pmatrix} * 1.0\mathrm{e}-03$	5
$r=4$	$\begin{pmatrix}2.0000\\1.0000\\3.0000\end{pmatrix}$	$\begin{pmatrix}2.0000\\0.9999\\2.9999\end{pmatrix}$	$\begin{pmatrix}0.4086\\0.8173\\0.9052\end{pmatrix} * \mathrm{e}-04$	16	$\begin{pmatrix}2.0000\\1.0000\\3.0000\end{pmatrix}$	$\begin{pmatrix}0.2050\\0.3914\\0.1615\end{pmatrix} * 1.0\mathrm{e}-04$	6
$r=5$	$\begin{pmatrix}2.0000\\1.0000\\3.0000\end{pmatrix}$	$\begin{pmatrix}2.0000\\1.0000\\3.0000\end{pmatrix}$	$\begin{pmatrix}0.3148\\0.6296\\0.6971\end{pmatrix} * \mathrm{e}-05$	20	$\begin{pmatrix}2.0000\\1.0000\\3.0000\end{pmatrix}$	$\begin{pmatrix}0.4020\\0.7657\\0.3158\end{pmatrix} * 1.0\mathrm{e}-06$	8

Table 3.2b

		Jacobi	Gauss-Seidel				
	Exact Solution	Iterated Solution	$\lvert Error \rvert$	No. of Iterations	Iterated Solution	$\lvert Error \rvert$	No. of Iterations
$r=1$	$\begin{pmatrix} 1.0000 \\ -3.0000 \\ 4.0000 \\ 2.0000 \end{pmatrix}$	$\begin{pmatrix} 1.0788 \\ -3.0240 \\ 3.9051 \\ 1.8525 \end{pmatrix}$	$\begin{pmatrix} -0.0788 \\ 0.0240 \\ 0.0949 \\ 0.1475 \end{pmatrix} * 1.0e-00$	4	$\begin{pmatrix} 1.0137 \\ -2.9906 \\ 4.0070 \\ 1.9963 \end{pmatrix}$	$\begin{pmatrix} 0.0137 \\ 0.0094 \\ 0.0070 \\ 0.0037 \end{pmatrix} * 1.0e-00$	3
$r=2$	$\begin{pmatrix} 1.0000 \\ -3.0000 \\ 4.0000 \\ 2.0000 \end{pmatrix}$	$\begin{pmatrix} 0.9992 \\ -2.9891 \\ 4.0112 \\ 2.0098 \end{pmatrix}$	$\begin{pmatrix} 0.0008 \\ 0.0109 \\ 0.0112 \\ 0.0098 \end{pmatrix} * 1.0e-00$	7	$\begin{pmatrix} 1.0022 \\ -2.9997 \\ 4.0001 \\ 2.0001 \end{pmatrix}$	$\begin{pmatrix} 0.0022 \\ 0.0003 \\ 0.0001 \\ 0.0001 \end{pmatrix} * 1.0e-00$	4
$r=3$	$\begin{pmatrix} 1.0000 \\ -3.0000 \\ 4.0000 \\ 2.0000 \end{pmatrix}$	$\begin{pmatrix} 0.9999 \\ -2.9993 \\ 4.0006 \\ 2.0008 \end{pmatrix}$	$\begin{pmatrix} 0.1075 \\ 0.7033 \\ 0.5540 \\ 0.8238 \end{pmatrix} * 1.0e-03$	11	$\begin{pmatrix} 1.0000 \\ -3.0001 \\ 4.0000 \\ 2.0000 \end{pmatrix}$	$\begin{pmatrix} 0.4122 \\ 0.5333 \\ 0.1244 \\ 0.1280 \end{pmatrix} * 1.0e-04$	5
$r=4$	$\begin{pmatrix} 1.0000 \\ -3.0000 \\ 4.0000 \\ 2.0000 \end{pmatrix}$	$\begin{pmatrix} 1.0000 \\ -3.0001 \\ 3.9999 \\ 1.9999 \end{pmatrix}$	$\begin{pmatrix} 0.0061 \\ 0.0929 \\ 0.0732 \\ 0.1044 \end{pmatrix} * 1.0e-03$	14	$\begin{pmatrix} 1.0000 \\ -3.0001 \\ 4.0000 \\ 2.0000 \end{pmatrix}$	$\begin{pmatrix} 0.0461 \\ 0.3967 \\ 0.2876 \\ 0.2211 \end{pmatrix} * 1.0e-05$	6
$r=5$	$\begin{pmatrix} 1.0000 \\ -3.0000 \\ 4.0000 \\ 2.0000 \end{pmatrix}$	$\begin{pmatrix} 1.0000 \\ -3.0000 \\ 4.0000 \\ 2.0000 \end{pmatrix}$	$\begin{pmatrix} 0.0355 \\ 0.6294 \\ 0.4762 \\ 0.7030 \end{pmatrix} * 1.0e-05$	18	$\begin{pmatrix} 1.0000 \\ -3.0001 \\ 4.0000 \\ 2.0000 \end{pmatrix}$	$\begin{pmatrix} 0.9816 \\ 0.4005 \\ 0.1346 \\ 0.0397 \end{pmatrix} * 1.0e-07$	7

International Journal of Science and Advanced Innovative Research
ISSN: 2536-7315 (Print) 2536-7323 (Online)
Volume 4, Number 3, September, 2019
http://www.casirmediapublishing.com

Table 3.2c

		Jacobi	Gauss-Seidel				
	Exact Solution	Iterated Solution	$\lvert Error \rvert$	No. of Iterations	Iterated Solution	$\lvert Error \rvert$	No. of Iterations
$r=1$	$\begin{pmatrix} 3.0000 \\ 1.0000 \\ -1.0000 \\ 2.0000 \\ 4.0000 \end{pmatrix}$	$\begin{pmatrix} 3.0054 \\ 1.0240 \\ -0.9237 \\ 2.0538 \\ 4.0338 \end{pmatrix}$	$\begin{pmatrix} 0.0054 \\ 0.0240 \\ 0.0763 \\ 0.0538 \\ 0.0338 \end{pmatrix} * 1.0e - 00$	5	$\begin{pmatrix} 3.0297 \\ 0.9969 \\ -0.9827 \\ 1.9981 \\ 3.9960 \end{pmatrix}$	$\begin{pmatrix} 0.0297 \\ 0.0031 \\ 0.0173 \\ 0.0019 \\ 0.0040 \end{pmatrix} * 1.0e - 00$	3
$r=2$	$\begin{pmatrix} 3.0000 \\ 1.0000 \\ -1.0000 \\ 2.0000 \\ 4.0000 \end{pmatrix}$	$\begin{pmatrix} 3.0003 \\ 0.9979 \\ -1.0041 \\ 1.9963 \\ 3.9976 \end{pmatrix}$	$\begin{pmatrix} 0.0003 \\ 0.0021 \\ 0.0041 \\ 0.0037 \\ 0.0024 \end{pmatrix} * 1.0e - 00$	8	$\begin{pmatrix} 3.0045 \\ 0.9975 \\ -0.9984 \\ 1.9994 \\ 3.9992 \end{pmatrix}$	$\begin{pmatrix} 0.0045 \\ 0.0025 \\ -0.0016 \\ 0.0006 \\ 0.0008 \end{pmatrix} * 1.0e - 00$	4
$r=3$	$\begin{pmatrix} 3.0000 \\ 1.0000 \\ -1.0000 \\ 2.0000 \\ 4.0000 \end{pmatrix}$	$\begin{pmatrix} 3.0000 \\ 0.9997 \\ -1.0007 \\ 1.9994 \\ 3.9996 \end{pmatrix}$	$\begin{pmatrix} 0.0376 \\ 0.3162 \\ 0.6533 \\ 0.5630 \\ 0.3827 \end{pmatrix} * 1.0e - 03$	10	$\begin{pmatrix} 3.0007 \\ 0.9997 \\ -0.9997 \\ 1.9999 \\ 3.9999 \end{pmatrix}$	$\begin{pmatrix} 0.7429 \\ 0.2657 \\ -0.3333 \\ 0.0898 \\ 0.1115 \end{pmatrix} 1.0e - 04$	5
$r=4$	$\begin{pmatrix} 3.0000 \\ 1.0000 \\ -1.0000 \\ 2.0000 \\ 4.0000 \end{pmatrix}$	$\begin{pmatrix} 3.0000 \\ 1.0000 \\ -1.0001 \\ 1.9999 \\ 3.9999 \end{pmatrix}$	$\begin{pmatrix} 0.0053 \\ 0.0489 \\ 0.1033 \\ 0.0883 \\ 0.0595 \end{pmatrix} * 1.0e - 05$	12	$\begin{pmatrix} 3.0000 \\ 1.0000 \\ -1.0000 \\ 2.0000 \\ 4.0000 \end{pmatrix}$	$\begin{pmatrix} 0.1930 \\ 0.0826 \\ -0.0842 \\ 0.0249 \\ 0.0306 \end{pmatrix} 1.0e - 04$	7

$r = 5$	$\begin{pmatrix} 3.0000 \\ 1.0000 \\ -1.0000 \\ 2.0000 \\ 4.0000 \end{pmatrix}$	$\begin{pmatrix} 3.0000 \\ 1.0000 \\ -1.0000 \\ 2.0000 \\ 4.0000 \end{pmatrix}$	$\begin{pmatrix} 0.0266 \\ 0.2462 \\ 0.6991 \\ 0.5043 \\ 0.3471 \end{pmatrix} * 1.0e - 05$	15	$\begin{pmatrix} 3.0000 \\ 1.0000 \\ -1.0000 \\ 2.0000 \\ 4.0000 \end{pmatrix}$	$\begin{pmatrix} 0.3117 \\ 0.1378 \\ -0.1381 \\ 0.0404 \\ 0.0504 \end{pmatrix} 1.0e - 06$	8

International Journal of Science and Advanced Innovative Research
ISSN: 2536-7315 (Print) 2536-7323 (Online)
Volume 4, Number 3, September, 2019
http://www.casirmediapublishing.com

Table 3.2d

		Jacobi	Gauss-Seidel				
	Exact Solution	Iterated Solution	$\lvert Error \rvert$	No. of Iterations	Iterated Solution	$\lvert Error \rvert$	No. of Iterations
$r=1$	$\begin{pmatrix}1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}1.0098\\1.0195\\1.0244\\1.0293\\1.0303\\1.0303\\1.0293\\1.0244\\1.0195\\1.0098\end{pmatrix}$	$\begin{pmatrix}-0.0098\\-0.0195\\-0.0244\\-0.0293\\-0.0303\\-0.0303\\-0.0293\\-0.0244\\-0.0195\\-0.0098\end{pmatrix} * 1.0e - 00$	3	$\begin{pmatrix}1.0098\\1.0076\\1.0081\\1.0080\\1.0080\\1.0080\\1.0080\\0.9924\\1.0033\\0.9992\end{pmatrix}$	$\begin{pmatrix}-0.0098\\-0.0076\\-0.0081\\-0.0080\\-0.0080\\-0.0080\\-0.0080\\0.0076\\-0.0033\\0.0008\end{pmatrix} * 1.0e - 00$	3
$r=2$	$\begin{pmatrix}1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}0.9989\\0.9981\\0.9972\\0.9968\\0.9965\\0.9965\\0.9968\\0.9972\\0.9981\\0.9989\end{pmatrix}$	$\begin{pmatrix}0.0011\\0.0019\\0.0028\\0.0032\\0.0035\\0.0035\\0.0032\\0.0028\\0.0019\\0.0011\end{pmatrix} * 1.0e - 00$	8	$\begin{pmatrix}0.9981\\0.9984\\0.9984\\0.9984\\0.9984\\0.9984\\1.0023\\0.9986\\1.0006\\0.9999\end{pmatrix}$	$\begin{pmatrix}0.0019\\0.0016\\0.0016\\0.0016\\0.0016\\0.0016\\-0.0023\\0.0014\\-0.0006\\0.0001\end{pmatrix} * 1.0e - 00$	4

K=3	$\begin{pmatrix}1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}1.0001\\1.0002\\1.0003\\1.0004\\1.0004\\1.0004\\1.0004\\1.0003\\1.0002\\1.0001\end{pmatrix}$	$\begin{pmatrix}-0.1097\\-0.2172\\-0.2933\\-0.3588\\-0.3850\\-0.3850\\-0.3588\\-0.2933\\-0.2172\\-0.1097\end{pmatrix} * 1.0e-03$	11	$\begin{pmatrix}0.9999\\0.9999\\0.9999\\0.9999\\1.0002\\0.9998\\1.0001\\0.9999\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}0.0758\\0.0622\\0.0641\\0.0641\\-0.1802\\0.1739\\-0.1106\\0.0531\\-0.0197\\0.0049\end{pmatrix} * 1.0e-03$	6
$r=4$	$\begin{pmatrix}1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}0.1239\\0.2319\\0.3302\\0.3912\\0.4284\\0.4284\\0.3912\\0.3302\\0.2319\\0.1239\end{pmatrix} * 1.0e-04$	14	$\begin{pmatrix}1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}-0.1556\\-0.1213\\-0.1300\\0.4830\\-0.5555\\0.4153\\-0.2367\\0.1083\\-0.0394\\0.0098\end{pmatrix} * 1.0e-04$	7

International Journal of Science and Advanced Innovative Research
ISSN: 2536-7315 (Print) 2536-7323 (Online)
Volume 4, Number 3, September, 2019
http://www.casirmediapublishing.com

$r=5$	$\begin{pmatrix}1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}0.0651\\0.1231\\0.1741\\0.2078\\0.2270\\0.2270\\0.2078\\0.1741\\0.1231\\0.0651\end{pmatrix} * 1.0e - 05$	18	$\begin{pmatrix}1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\\1.0000\end{pmatrix}$	$\begin{pmatrix}-0.0623\\0.3330\\-0.5098\\0.4937\\-0.3629\\0.2183\\-0.1111\\0.0480\\-0.0171\\0.0043\end{pmatrix} * 1.0e - 05$	9

CONCLUSION

The Jacobi and the Gauss-Seidel iterations are applied to the four problems with varying tolerance parameters. The effect of the tolerance parameters is seen from the number of iterations, the solutions as well as the errors involved. Thus, as the number of iterations increases, the errors are drastically reduced and the iterative solutions get closer to the exact solutions as the value of r increase for the tolerance parameter $tol = 10^{-r}$. Hence the tolerance parameter significantly affects the degree of accuracy of the solution. Also the Gauss-Seidel iteration converged twice as fast as the Jacobi method in the examples used. Therefore the more relaxed the parameter, the smaller the error involved and the closer the iterates approach the exact solution.

REFERENCES

1. Barrett, R et al. (Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods). Society for Industrial and Applied Mathematics, Philadelphia 1st Ed. 1994,
2. R. O. Bagnara, (Unified Proof for the Convergence of Jacobi and Gauss-Seidel Methods). Society for Industrial and Applied Mathematics. 1995, 37(1): 93-97.
3. O. M. Bamigbola, A. A. Ibrahim. (On Algebraic Structure of Improved Gauss-Seidel iteration), International Journal of Mathematical, Computational, Physical and Quantum Engineering, 2014, 8(10), 1181-1186.
4. A. S. Householder. Theory of Matrices in Numerical Analysis. Johnson: Blaisdell Publishing Company, 1964.
5. A. A. Ibrahim. Characterization and Application of Stationary Iterative Methods. University of Ilorin, Nigeria: Unpublished Ph.D. Thesis, 2014.
6. C. T. Kelly. Iterative Methods for Linear and Nonlinear Equations. Philadelphia: SIAM, 1995.
7. A. Kharab, R. B. Guenther. An Introduction to Numerical Methods: A Matlab Approach. Chapman and Hall/CRC, 2002.
8. E. Kreyszig. Advanced Engineering Mathematics. John Wiley and Sons, New York, 1999
9. C. D. Meyer. Matrix Analysis and Applied Linear Algebra. Philadelphia, PA: Society for Industrial and Applied Mathematics, 2000.
10. Y. Saad. Iterative Methods for Sparse Linear Systems, 2nd edition. New York: PWS Publishing, 2000.
11. Scheid, F. Theory and Problems of Numerical Analysis. McGraw-Hill Book Company, New York, 1968.