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FOURIER TIME SERIES ANALYSIS OF NIGERIA GROSS DOMESTIC PRODUCT

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ABSTRACT
The study Fourier time series analysis of Nigeria gross domestic product from 2005 to 2015 of quarterly data is an important part of modern time series analysis. The study employed Fourier periodic time series analysis and adequately model gross domestic product over time. The detailed background of Nigeria gross domestic product to date is highlighted in the study. The parameters of the model yield an adequate model for the study. A computational algorithm for the model estimation gives better understanding of the procedures for estimation. The descriptive property of the gross domestic product was obtained on the transformed data. The test of unit root was achieved by Augmented Dickey-Fuller of order 1, with ACF, PACF and Residuals plots. The test of significance of the general model was achieved and the P-P Plot indicates that the model fits the data. The research work provides an autoregressive model of order 2, Fourier series model, time series decomposition plot, component analysis, trend analysis plot for linear model of gross domestic product. The research work provides forecast values of five years from 2016 to 2020.
Keyword: Fourier time series analysis, Nigeria GDP, ACF, PACF

INTRODUCTION

Statistical Fourier analysis is an important part of modern time-series analysis yet it frequency poses an impediment that prevents a full understanding of temporal stochastic processes and of the manipulations to which their data are amenable. A periodic time series fluctuates in some but maintains steady values and is not obviously steadily increasing or decreasing. This means that the series repeats itself after intervals (Priestly, 1981). Any well-behaved periodic function can be expressed as (possibly infinite) sum of sine and cosine functions. Hence a periodic function can be expressed as a sum of cosine and sine terms over a discrete set of frequencies. Gross Domestic Product (GDP) is the total value of goods and services produced within the country during a year. Therefore, the most comprehensive measure of the total output of performance of an economy is the Gross Domestic product. Although, GDP is the most widely used measure of national output of an economy, two other concepts are frequently cited, Net Domestic Product and Gross National Product (GNP) the relationship among these three concepts (GDP, GNP and Net Domestic Product) is that they measure an economy's output. GDP includes both the nationals and non-nationals of an economy and GDP must be equal to the value of only the final products.

Kumar, (2010) in his work titled" Macroeconomics theory, analysis and policy', said that use of this word "gross" along with "domestic product" indicates that we are calculating domestic product inclusive of the depreciation allowance or consumption fixed capital. Etuk (2012) presented time series analysis of Nigeria monthly inflation rates and fitted a multiplicative seasonal autoregressive integrated moving average model. He showed that the model is adequate and forecasts agreed closely with observations. Raza *et al.*, (2013) discussed the impact of inflation on economic growth of Pakistan and estimated short run and long run relationship between inflation and economic growth. They suggested that government should maintain inflation in single digit which is favorable for economic growth.

Ekpenyong, *et al.,* (2014) proposed the application of periodogram analysis and fourier analysis to model inflation rates in Nigeria. The main objective of the study was to identify cycles and fit the appropriate model to forecast future values. Konarasinghe *et al.,* (2015) suggested a study based on fourier transformation on model fitting for Sri Lankan share market. They also analyzed the monthly returns by ARIMA (Auto Regressive Integrated Moving Average) and concluded that fourier transformation along with multiple regression is suitable.

Ekpenyong, *et al.,* (2014) consider the application of Periodogram and Fourier Series Analysis to model all-items monthly inflation rates in Nigeria from 2003 to 2011. The main objectives are to identify inflation cycles, fit a suitable model to the data and make forecasts of future values. To achieve these objectives, monthly all-items inflation rates for the period were obtained from the Central Bank of Nigeria (CBN) website. Periodogram and Fourier series methods of analysis are used to analyze the data. Based on the analysis, it was found out that inflation cycle within the period was fifty one (51) months, which relates to the two government administrations within the period. Further, appropriate significant Fourier series model comprising the trend, seasonal and error components is fitted to the data and this model is further used to make a forecast of the inflation rates for thirteen months and this forecast compares favourably with the actual values for the thirteen months. The econometric models have been used to model inflation rates, but they are restrictive in their theoretical formulations and often lack to incorporate the dynamic structure of the data and have tendencies to inflict improper restrictions and specifications on the structural variables Saz, (2011). Odusanya and Atanda (2010) determined the dynamic and simultaneous interrelationship between inflation and its determinants –growth rate of Gross Domestic Product (GDP), growth rate of money supply (M2), fiscal deficit, exchange rate (U.S dollar to Naira), importance and interest rates, using econometric time series model. Ayinde *et al.,* (2010) examined the factors affecting inflation in Nigeria using cointegration and descriptive statistics. They observed that there were variations in the trend pattern of inflation rates and some variables considered were significant in determining inflation in Nigeria. These variables include annual total import, annual consumer price index for food, annual agricultural output, interest rate, annual government expenditure, exchange rate and annual crude oil export. Stockton and Glassman (1987) conducted a comparative study on three different inflation processes namely rational expectations model, monetarist model and the expectation augmented Philips curve that are based on economic theory relationships that explain and form inflation.

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METHOD OF ANALYSIS

Model Building

One approach to achieve this objective is to adopt the iterated modeling procedure of Box and Jenkins (1976) which consists of the following steps:

1. Model specification,
2. Estimation,
3. Model checking (residual analysis).

If a fitted model is judged to be inadequate via model checking statistics, the procedure is iterated to refine the model. A model that passed rigorous model checking can then be used to make inference, e.g. forecasting or policy simulation.

Fourier Time Series Model

$$X_t = T_t + \sum_{i=1}^{N} \{ai \text{ coswit} + \text{biSinWit}\} + Z_t \quad (1)$$

Where

T_t	=	The trend equation
X_t	=	The inflation rate at time t.
W_t	=	The angular frequency measured in radians
Z_t	=	The error term
a,b,	=	The coefficients

In short, the series consists of the trend, seasonal and error components. The series is first detrended and the original least squares estimates of the parameters obtained on the detrended series as;

$$\hat{a} = \frac{2}{N}\sum_{t=1}^{N} \Delta X_t Cos\ Wit \quad (2)$$

$$\hat{b} = \frac{2}{N}\sum_{t=1}^{N} \Delta X_t Sios\ Wit \quad (3)$$

Since, $$\sum_{t=1}^{N} Coswit\ Cos\ wjt = \begin{cases} o, i \neq j \\ N/2, i = j \end{cases} \quad (4)$$

$$\sum_{t=1}^{N} Sinwit\ Sin\ wjt = \begin{cases} N/2, i \neq j \\ n, i \neq j \end{cases} \quad (5)$$

$$\sum_{t=1}^{N} Sinwit\ Cos\ wjt \quad (6)$$

$$\sum_{t=1}^{N} Coswit = \text{o for } 1 < i < N/2, \text{ where } wi = 2\bar{n}f_i \quad (7)$$

The above results are obtained from complex variables as orthogonality and independent properties of fourier series or sinusoidal models. In case of time series with ever number of observations N=2q, q = $^N/_2$, the same definition are applicable except for

$$\hat{a}q = \frac{1}{N}\sum_{i=o}^{N}(-1)\Delta x_t \qquad (8)$$

$$\hat{b}q = 0 \qquad (9)$$

Data Analysis

Table 1: Descriptive Statistics of the Gross Domestic Product

Statistic	GDP
Mean	5.105478
Median	5.089446
Maximum	5.507147
Minimum	4.787899
Std. Dev.	0.171384
Skewness	0.324718
Kurtosis	2.502090
Probability	0.541249
Sum	224.6410
Sum Sq. Dev.	1.263012
Observations	44

Figure 1: Time Plot of the gross domestic product against index

Table 1: Augmented Dickey- Fuller Test for Stationarity.

Null Hypothesis: GDP has a unit root
Exogenous: Constant

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.718911	0.0080
Test critical values:	1% level	-3.610453	
	5% level	-2.938987	
	10% level	-2.607932	

*MacKinnon (1996) one-sided p-values.

Table 3: Component Series with Detrend and Forecast

Time	GDP	Trend	Seasonal	Detrend	Deseason	Predict	Error
1	4.78790	5.04121	0.97598	0.94975	4.90572	4.92013	-0.132236
2	4.85792	5.04422	0.98229	0.96307	4.94552	4.95487	-0.096956
3	5.03652	5.04723	1.01759	0.99788	4.94945	5.13602	-0.099499
4	5.07012	5.05023	1.02414	1.00394	4.95062	5.17214	-0.102018
5	4.85655	5.05324	0.97598	0.96108	4.97606	4.93187	-0.075324
6	4.90852	5.05625	0.98229	0.97078	4.99703	4.96669	-0.058167
7	5.09067	5.05926	1.01759	1.00621	5.00266	5.14826	-0.057590
8	5.13170	5.06226	1.02414	1.01372	5.01075	5.18446	-0.052758
9	4.91100	5.06527	0.97598	0.96954	5.03185	4.94361	-0.032616
10	4.96119	5.06828	0.98229	0.97887	5.05065	4.97850	-0.017313
11	5.15368	5.07128	1.01759	1.01625	5.06458	5.16050	-0.006819
12	5.20740	5.07429	1.02414	1.02623	5.08466	5.19678	0.010623
13	4.95633	5.07730	0.97598	0.97617	5.07830	4.95535	0.000977
14	5.01637	5.08030	0.98229	0.98741	5.10682	4.99032	0.026050
15	5.21319	5.08331	1.01759	1.02555	5.12306	5.17274	0.040449
16	5.27602	5.08632	1.02414	1.03730	5.15167	5.20909	0.066927
17	5.00523	5.08933	0.97598	0.98348	5.12840	4.96709	0.038137
18	5.08822	5.09233	0.98229	0.99919	5.17997	5.00213	0.086089
19	5.28363	5.09534	1.01759	1.03695	5.19229	5.18498	0.098652
20	5.34996	5.09835	1.02414	1.04935	5.22387	5.22141	0.128549
21	5.07590	5.10135	0.97598	0.99501	5.20082	4.97883	0.097073
22	5.16327	5.10436	0.98229	1.01154	5.25637	5.01395	0.149318
23	5.36022	5.10737	1.01759	1.04951	5.26755	5.19722	0.163000
24	5.43245	5.11038	1.02414	1.06302	5.30441	5.23373	0.198722
25	5.14322	5.11338	0.97598	1.00583	5.26978	4.99057	0.152645
26	5.23555	5.11639	0.98229	1.02329	5.32996	5.02576	0.209791
27	5.43134	5.11940	1.01759	1.06093	5.33744	5.20946	0.221878
28	5.50715	5.12240	1.02414	1.07511	5.37735	5.24605	0.261097
29	5.07012	5.12541	0.97598	0.98921	5.19489	5.00231	0.067809

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30	4.85655	5.12842	0.98229	0.94699	4.94413	5.03758	-0.181028
31	4.90852	5.13143	1.01759	0.95656	4.82366	5.22170	-0.313181
32	5.09067	5.13443	1.02414	0.99148	4.97069	5.25837	-0.167699
33	5.13170	5.13744	0.97598	0.99888	5.25798	5.01405	0.117648
34	4.91100	5.14045	0.98229	0.95536	4.99956	5.04939	-0.138395
35	4.96119	5.14345	1.01759	0.96456	4.87542	5.23394	-0.272751
36	5.15368	5.14646	1.02414	1.00140	5.03221	5.27069	-0.117006
37	5.20740	5.14947	0.97598	1.01125	5.33555	5.02579	0.181608
38	4.95633	5.15248	0.98229	0.96193	5.04571	5.06121	-0.104879
39	5.01637	5.15548	1.01759	0.97302	4.92964	5.24618	-0.229813
40	5.21319	5.15849	1.02414	1.01060	5.09032	5.28301	-0.069817
41	5.27602	5.16150	0.97598	1.02219	5.40586	5.03753	0.238492
42	5.00523	5.16450	0.98229	0.96916	5.09549	5.07302	-0.067794
43	5.08822	5.16751	1.01759	0.98466	5.00025	5.25842	-0.170199
44	5.28363	5.17052	1.02414	1.02188	5.15910	5.29532	-0.011693

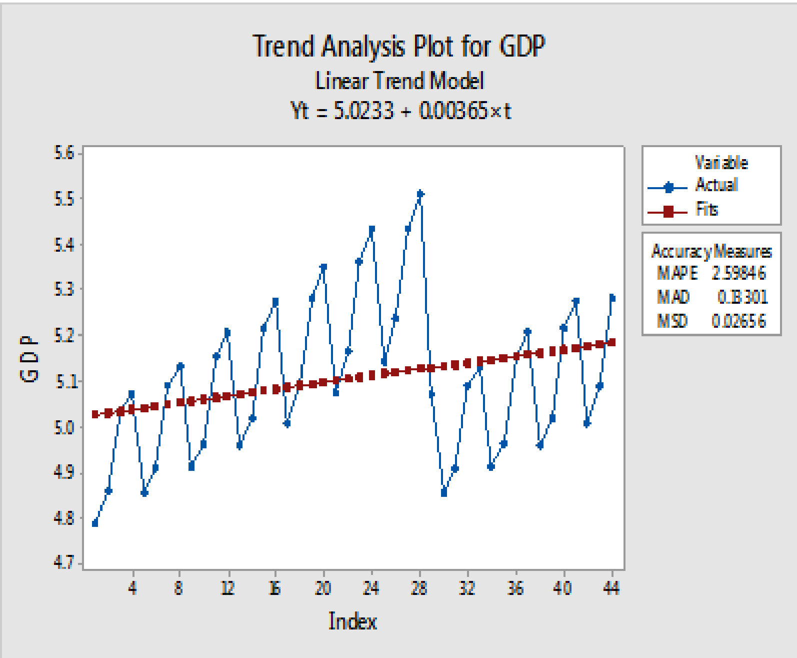

Figure 2: Graphical representative of the Trend Analysis Plot for the Estimated Gross Domestic Product

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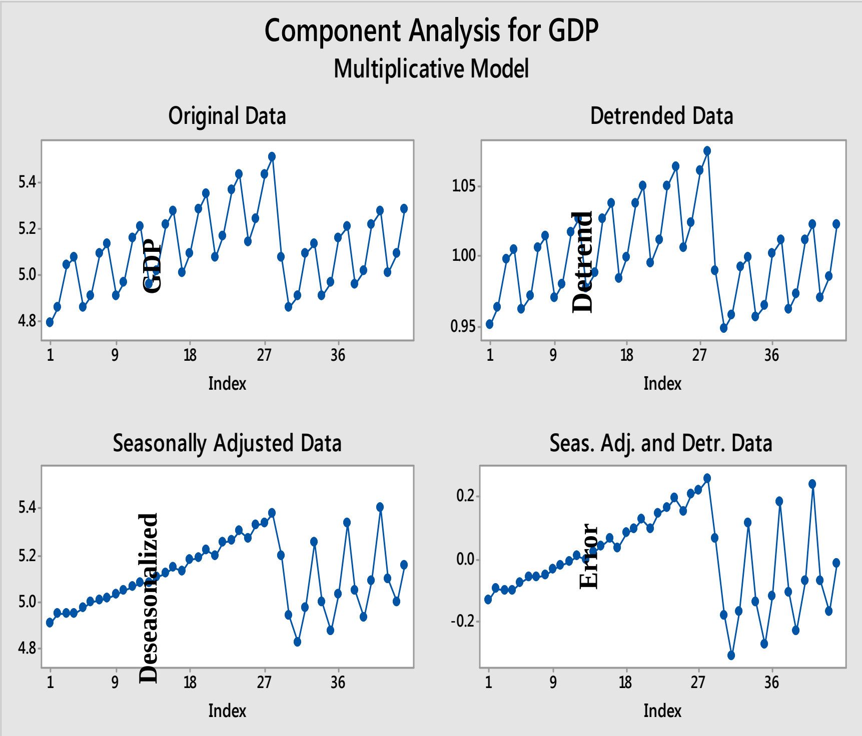

Figure 3: Graphical representative of the Component Analysis of Gross Domestic Product Multiplicative Model

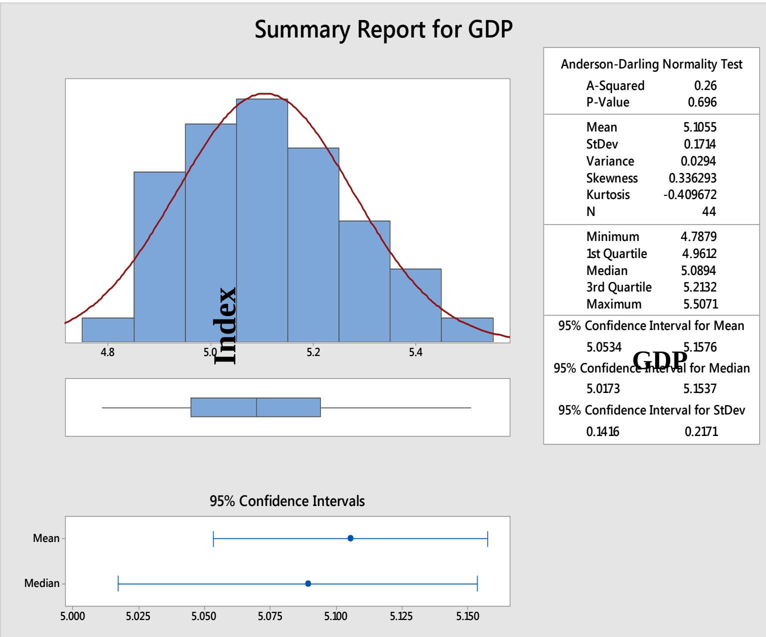

Figure 4: Graphical representative of the Summary Statistics of Gross Domestic Product

Table 4: Parameter Estimates and Significance Test of the Seasonal Component

Predictor	coefficient	StDev	T	P-Value
Coswt	-0.4567	0.0034	-0.64	0.002
Sinwt	0.3123	0.0034	0.19	0.003
Cos2wt	-0.3893	0.0034	-0.32	0.000
Sin2wt	-0.4522	0.0034	0.87	0.112
Cos3wt	0.0634	0.0034	-0.11	0.541
Sin3wt	0.0489	0.0034	0.27	0.001
Cos4wt	0.0495	0.0034	-0.082	0.340
Sin4wt	-0.0929	0.0034	-0.39	0.161
Cos5wt	0.0673	0.0034	-0.21	0.000
Sin5wt	-0.0349	0.0034	0.37	0.634
Cos6wt	0.0210	0.0034	-0.43	0.411
Sin6wt	-0.0281	0.0034	-0.67	0.621
Cos7wt	-0.0591	0.0034	0.04	0.306
Sin7wt	0.0030	0.0034	0.07	0.703

Table 5: Parameter Estimates of the Trend Model of GDP

Predictor	coefficient	StDev	T	P-Value
Constant	5.0233	0.1714	11	0.001
T	-0.0072	0.0092	-2.43	0.030
S= 0.8764	R-Sq=3.01	R-Sq(adj)=2.6		

Table 6: Significance Test of the Parameter Estimates of the Error Component

Type	coefficient	StDev	T	P-Value
AR(1)	0.7340	0.003	18.79	0.003
AR(2)	0.9636	0.014	21.36	0.001
Observations	44			

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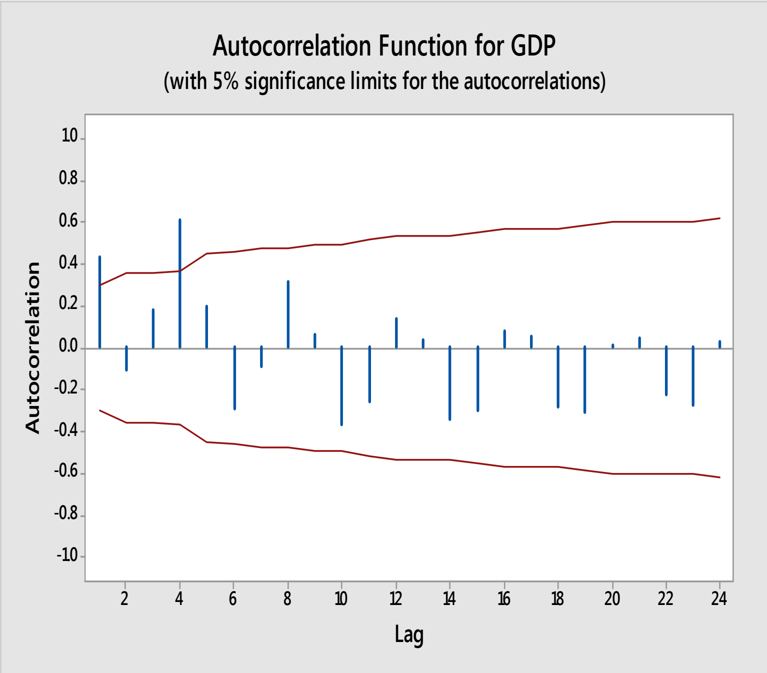

Figure 5: Graphical representative of Autocorrelation Function of the Gross Domestic Product

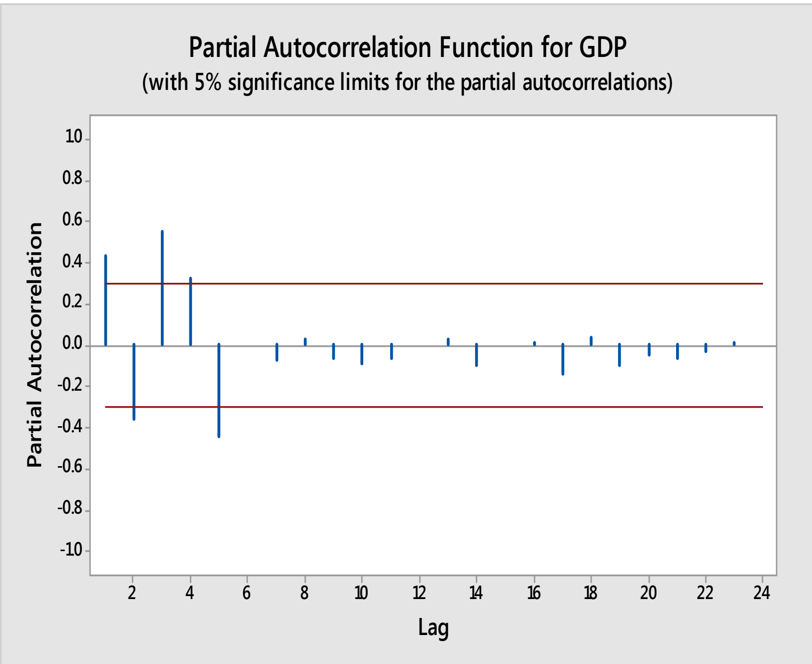

Figure 6: Graphical representative of Partial Autocorrelation Function of the Gross Domestic Product.

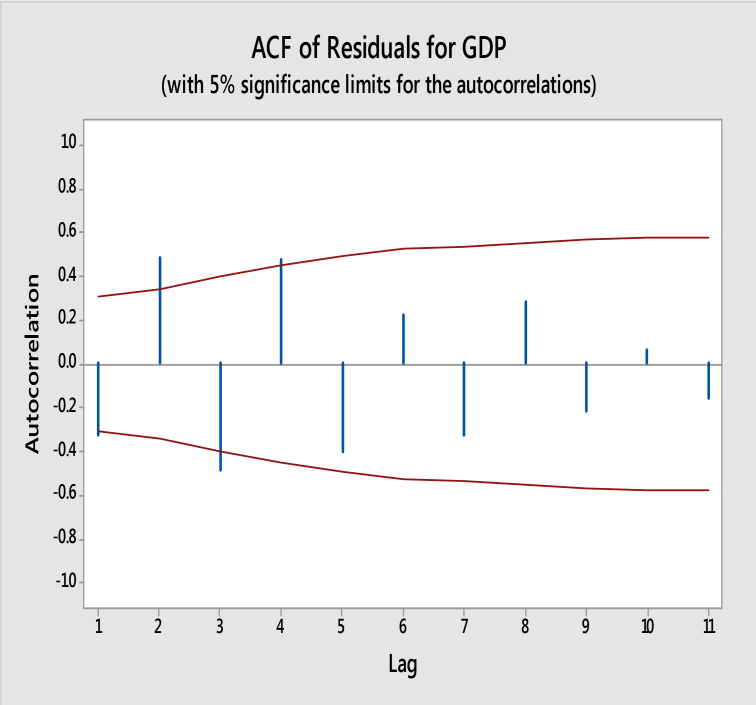

Figure 7: Graph of ACF of Residuals for Gross Domestic Product

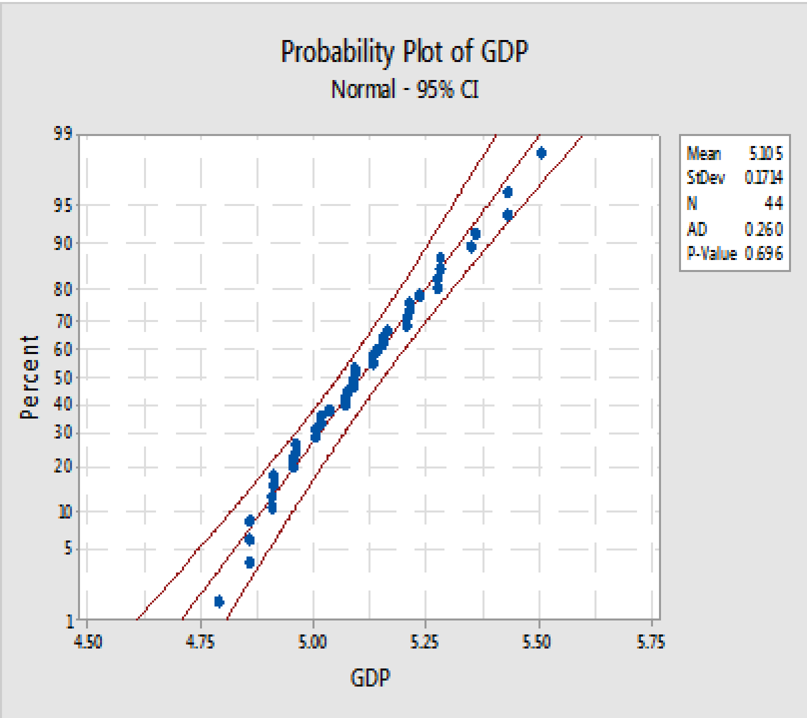

Figure 8: Graphical representative of P-P Plot for testing Goodness of fit of Gross Domestic Product at 95%.

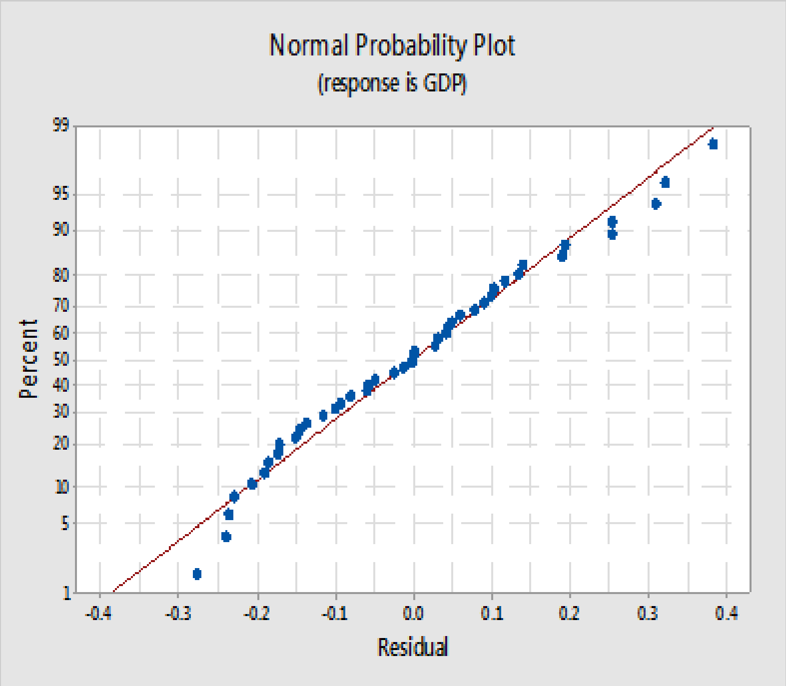

Figure 9: Graphical representative of Normal P-P Plot for Testing Goodness of Fit of Gross Domestic Product.

Table 7: Forecast for 2016 to 2020 of Quarterly Gross Domestic Product

Period	Forecast
45	5.08484
46	5.27066
47	5.30764
48	5.06101
49	5.09665
50	5.28290
51	5.31996
52	5.07275
53	5.10847
54	5.29514
55	5.33228
56	5.08449
57	5.12029
58	5.30738
59	5.34460
60	5.09623
61	5.13210
62	5.31962
63	5.35692
64	5.10797

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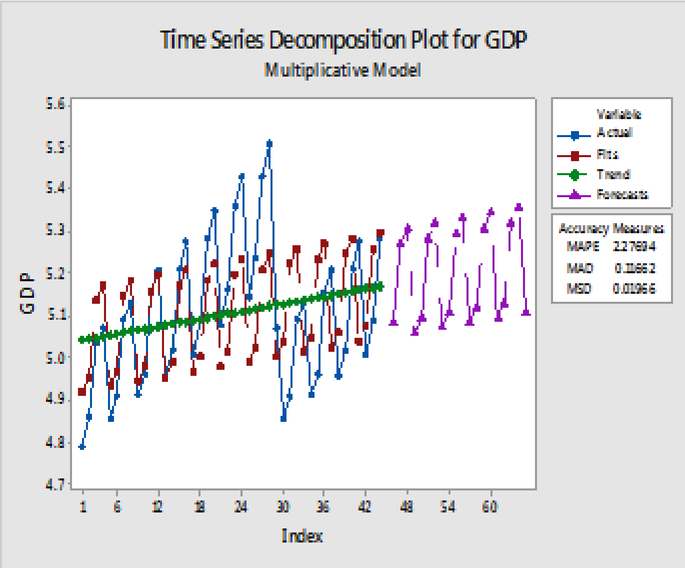

Figure 10: Graphical Representative of Time Series Decomposition Plot of Gross Domestic Product

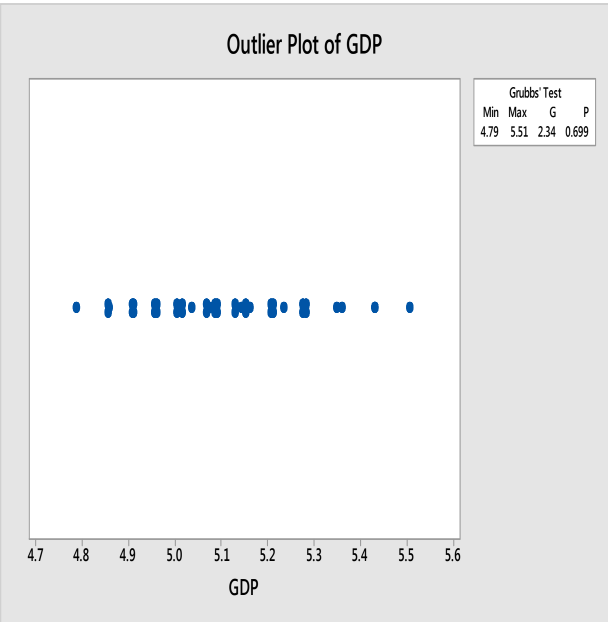

Figure 11: Outliers Plot of Gross Domestic Product.

DISCUSSIONS

The present study has sought to model gross domestic product in Nigeria via Fourier time series modeling. The study employed computational algorithm in the study to aid the estimation process (see fig 1). The empirical findings of the study in the preceding chapter are summarized below. There exist variations in the raw data because the gross domestic product has different units of measurement. In order to bring stability and uniformity, the data was transformed or transformation was done in natural logarithm (ln). It was evident that the transformation brought consistency in the series used for the analysis. The time plot of the gross domestic product indicates that the series has trend and hence

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non-stationarity. The unit root results formally confirmed the earlier conclusion that the series are non-stationary at their level, using the ADF unit root test (Table 2). However, the series is stationary after first difference. It is also relevant to stress that achieving stationarity is a precondition for the estimation or modeling of time series variable.

Typically, a fitted model is said to be adequate if all fitted parameters are significant (at a specified level) see table 4, 5 and 6, for univariate model fit estimation, respectively. To develop a Fourier time series model for the gross domestic product, the time of exposure in terms of uniform time step (quarterly) were considered. Although it is obvious that the mean of the series is not stationary, the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the non-differenced data are estimated using Minitab procedure, with the analytical procedures see fig 1, fig 2 and fig 3. The estimated autocorrelations decay slowly by lag 11 for the series, therefore check of this is to estimate an autoregressive model of the order 1. The study reviews that trend analysis, component analysis of multiplication model of seasonal data, summary statistics of gross domestic product, time series decomposition of the actual, the fit, trend and forecasts was duly established see fig 6, 7, 8 and 9 respectively. From the study the gross domestic product provides the linear trend of $y_t = 5.0233 + 0.00365t$ see fig 6 on trend analysis. The estimation procedures provides an output in table 4 and fig 6 on trend analysis, hence the study yields the Fourier series of gross domestic product:

$$F_{gdp} = y_t = 5.0233 - 0.4567cos\omega t + 0.3123sin\omega t - 0.3893cos2\omega + 0.0489sin3\omega t + 0.0673cos5\omega t + 0.7340z_{t-1} + 0.9636z_{t-2} \quad (1)$$

Further analysis from the study observed that the behavior of the autocorrelation and partial autocorrelation function suggest an autoregressive model of order 2 see (table 6) which yields the model:

$$y_t = \mu + \varphi_{t-1} + \varphi_{t-2} + e \quad (2)$$

and numerically

$$y_t = 5.0233 + 0.7340_{t-1} + 0.9636_{t-2} \quad (3)$$

Testing the overall goodness-of-fit, the fig 4 the p-p plot indicates that the model fits the data well or adequate enough for the forecast value to be estimated. It is adequate that every good model is fit for forecast which the study achieved in table 7 of quarterly data for the period of five years, 2016 (46, 47, 48, 49), 2017(50, 51, 52, 53), 2018(54, 55, 56, 57), 2019(58, 59, 60, 61), 2020(62, 63, 64, 65), also see fig 9. It of interest that the outlier plot was evident that the population was from same population, see fig 11.

CONCLUSION

The study Fourier time series analysis of Nigeria gross domestic product from 2005 to 2015 of quarterly data is important part of modern time series which frequently pose full understanding of temporal stochastic process and manipulation of the data. In order to get information about the data, the assumptions about its unobserved component and that the data was from same population was established. The classical time series analysis assumes that the systematic components, i.e. trend, business cycle and seasonal cycle are not influenced by stochastic disturbances and can thus be represented by deterministic functions of time. So stochastic impact is restricted to the residuals, which on the other hand, do not contain any systematic movements. Recently, the idea of decomposing a time series has been taken up again, particularly for the modeling of seasonal variation which the study achieved by modeling gross domestic product via Fourier time series method which assumed that all components of the time series can be represented by a simple stochastic models.

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APPENDIX

STATISTICAL ANALYSIS OF THE NIGERIAN GROSS DOMESTIC PRODUCT (FROM Q1 2005 – Q4 2015)

TIME	GDP
Q1 2005	4.8
Q2 2005	4.9
Q3 2005	5.0
Q4 2005	5.1
Q1 2006	4.9
Q2 2006	4.9
Q3 2006	5.1
Q4 2006	5.1
Q1 2007	4.9
Q2 2007	5.0
Q3 2007	5.2
Q4 2007	5.2
Q1 2008	5.0
Q2 2008	5.0
Q3 2008	5.2
Q4 2008	5.3
Q1 2009	5.0
Q2 2009	5.1
Q3 2009	5.3
Q4 2009	5.3
Q1 2010	5.1
Q2 2010	5.2
Q3 2010	5.4
Q4 2010	5.4
Q1 2011	5.1
Q2 2011	5.2
Q3 2011	5.4
Q4 2011	5.5
Q1 2012	5.1
Q2 2012	4.9
Q3 2012	4.9
Q4 2012	5.1
Q1 2013	5.1
Q2 2013	4.9
Q3 2013	5.0
Q4 2003	5.2
Q1 2014	5.2
Q2 2014	5.0

Q3 2014	5.0
Q4 2014	5.2
Q1 2015	5.3
Q2 2015	5.0
Q3 2015	5.1
Q4 2015	5.3