AN EVALUATION OF RIGID ROTOR BEHAVIOR OF NUCLEAR DEFORMATION OF MASS

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ABSTRACT
The goal of this paper is to obtain rigid rotor behavior of the nuclear deformation of mass. Empirical formulae have been achieved. The nuclear kinematic and dynamic moments of inertia have been calculated. In this paper, movement of inertia is one of the most significant quantities to characterize the nuclear rotational band. Theoretical equations were put forward for the possible clarification which has been achieved.
Key word: Rigid, $^{152}Dy\ (SD-1),^{144}Eu(SD-2),\ ^{144}Eu(SD-3)\ and\ ^{192}Hg\ (SD-1)$

INTRODUCTION
In nuclear physics, hyper deformation is theoretically predicted states of an atomic nucleus with extremely elongated shape and very high angular momentum (Adamian et al., 2007). Less elongated states, super deformation, have been well observed, but the experimental evidence for hyper deformation is more limited. Hyper deformed states correspond to an axis ratio of 3:1 (Schunck et al., 2007). They would be caused by a third minimum in the potential energy surface, the second causing super deformation and the first minimum being normal deformation (Abusara et al., 2009). The first super deformed states to be observed were the fission isomers, low-spin states of elements in the actinide and lanthanide series. The strong force decays much faster than the Coulomb force, which becomes stronger when nucleons are greater than 2.5 femtometers apart (Schunck et al., 2007). For this reason, these elements undergo spontaneous fission. In the late 1980s, high-spin super deformed rotational bands were observed in other regions of the periodic table. Specific elements include ruthenium, rhodium, palladium, silver, osmium, iridium, platinum, gold, and mercury. The existence of super deformed states occurs because of a combination of macroscopic and microscopic factors, which together lower their energies, and make them stable minima of energy as a function of deformation. Macroscopically, the nucleus can be described by the liquid drop model (Adamian et al., 2007). The liquid drop's energy as a function of deformation is at a minimum for zero deformation, due to the surface tension term. However, the curve may become soft with respect to high deformations because of the Coulomb repulsion (especially for the fission isomers, which have high Z) and also, in the case of high-spin states, because of the increased moment of inertia (Abusara et al., 2009). Modulating this macroscopic behavior, the microscopic shell correction creates certain super deformed magic numbers that are analogous to the spherical magic numbers. For nuclei near these magic numbers, the shell correction creates a second minimum in the energy as a function of deformation.

The rigid rotor is a mechanical model of rotating systems. An arbitrary rigid rotor is a 3-dimensional rigid object, such as a top. To orient such an object in space requires three angles, known as Euler angles. A special rigid rotor is the linear rotor requiring only two

angles to describe, for example of a diatomic molecule. More general molecules are 3-dimensional, such as water (asymmetric rotor), ammonia (symmetric rotor), or methane (spherical rotor) (Podolsky, 1928). Rotational transitions of a molecule occur when the molecule absorbs a photon [a particle of a quantized electromagnetic (em) field]. Depending on the energy of the photon (i.e., the wavelength of the em field) this transition may be seen as a sideband of a vibrational and/or electronic transition. Pure rotational transitions, in which the vibronic (vibrational plus electronic) wave function does not change, occur in the microwave region of the electromagnetic spectrum. Typically, rotational transitions can only be observed when the angular momentum quantum number changes by 1 (Goldstein et al., 2001). This selection rule arises from a first-order perturbation theory approximation of the time-dependent Schrödinger equation. According to this treatment, rotational transitions can only be observed when one or more components of the dipole operator have a non-vanishing transition moment.

MATERIALS AND METHODS
Linear rotor
The linear rigid rotor model consists of two point masses located at fixed distances from their center of mass. The fixed distance between the two masses and the values of the masses are the only characteristics of the rigid model. However, for many actual diatomics this model is too restrictive since distances are usually not completely fixed (Kronig et al., 1927). Corrections on the rigid model can be made to compensate for small variations in the distance. Even in such a case the rigid rotor model is a useful point of departure (zeroth-order model).

Coordinates of the rigid rotor
Different branches of physics and engineering use different coordinates for the description of the kinematics of a rigid rotor. In molecular physics Euler angles are used almost exclusively. In quantum mechanical applications it is advantageous to use Euler angles in a convention that is a simple extension of the physical convention of spherical polar coordinates. The first step is the attachment of a right- handed orthonormal frame (3-dimensional system of orthogonal axes) to the rotor (a body-fixed frame) (Gimzewski et al., 1998). This frame can be attached arbitrarily to the body, but often one uses the principal axes frame—the normalized eigenvectors of the inertia tensor, which always can be chosen orthonormal, since the tensor is symmetric. When the rotor possesses a symmetry-axis, it usually coincides with one of the principal axes. It is convenient to choose as body-fixed z - axis the highest-order symmetry axis. One starts by aligning the body-fixed frame with a space-fixed frame (laboratory axes), so that the body-fixed x , y, and z axes coincide with the space-fixed X , Y , and Z axis. Secondly, the body and its frame are rotated actively over a positive angle around the z -axis (by the right-hand rule), which moves the - to the - axis. Thirdly, one rotates the body and its frame over a positive angle around the -axis. The z -axis of the body- fixed frame has after these two rotations the longitudinal angle (commonly designated by) and the colatitudes angle (commonly designated by), both with respect to the space-fixed frame. If the rotor were cylindrical symmetric around its z -axis, like the linear rigid rotor, its orientation in space

would be unambiguously specified at this point (Dennison, 1931). If the body lacks cylinder (axial) symmetry, a last rotation around its z-axis which has polar coordinates and is necessary to specify its orientation completely.

There are general two kinds of moment of inertia for illustrating the high spin phenomena, the dynamic moment of inertia

$$\xi^2 = \hbar \frac{dI_x}{d\omega} = \hbar^2 \left[\frac{d^2 E}{dI_x^2} \right] \qquad\qquad 1$$

and the kinematic moment of inertia,

$$\xi^1 = \frac{\hbar I_x}{\omega} = \hbar^2 I_x \left[\frac{dE}{dI_x} \right] \qquad\qquad 2$$

Usually, ξ^1 and ξ^2 are obtained from the intra band γ transition energies by using the formulae

$$\xi^1(J-1) = \frac{(2J-1)}{E_\gamma(J)} \qquad\qquad 3$$

$$\xi^2(J) = \frac{\hbar^2}{E_\gamma(J+2) - E_\gamma(J)} \qquad\qquad 4$$

Where $E_\gamma(J) \equiv E_\gamma(J \to J-2)$

The expression for the rotation frequency $(\hbar\omega(I))$ of the nuclei

$$\hbar\omega(I) = E_\gamma(J+1 \to J-1) \qquad\qquad 5$$

The well known expression for the ground state band energy levels for rotational spectra

$$E = \frac{\hbar^2}{2\xi(J)} J(J+1) \qquad\qquad 6$$

The three parametric simplest energy formula for the deformed nuclei is the Bohr-Mottelson energy expansion in terms of the power of $J(J+1)$ (James, 1971),

$$E = AJ(J+1) + BJ(J+1)^2 + CJ(J+1)^3 \qquad\qquad 7$$

The moment of inertia of deformed nuclei ascends linearly with level energy i.e

$$\xi(J) = bE + a \qquad\qquad 8$$

Substitution of equation (8) in equation (6), they obtained the two-parameter ab formula

$$E = a\left[\sqrt{1 + b\,J(J+1)} - 1 \right] \qquad\qquad 9$$

Equating equation (8) and equation (5)

$$\xi = \frac{1}{2ab}\left(1 + \sqrt{1 + \frac{2}{a}E} \right) \qquad\qquad 10$$

Using equation (8) and equation (6) a new expression for the energy called pq formula:

$$E = a\left(\left\{ p^2 + [p^4 + q^3]^{\frac{1}{2}} \right\}^{\frac{1}{3}} + \left\{ p^2 - [p^4 + q^3]^{\frac{1}{2}} \right\}^{\frac{1}{3}} \right) \qquad\qquad 11$$

where $= \frac{b\,J(J+1)}{2}$ and $q = \frac{b\,J(J+1)}{3}$ (James, 1971). Further observed that ξ depends upon the spin (J) and energy (E) by relation

$$\xi = \xi_0(1 + aJ + bE) \qquad\qquad 12$$

The energy term bE in equation (10) and replacing it in equation (5) derived a new formula containing two parameter called the soft rotor formula

$$E = \frac{1}{\xi_0(1+\alpha J)} J(J+1) \qquad\qquad 13$$

The concept of arithmetic mean of two terms used in the Bohr-Mottelson expression by the geometric mean to introduce a single term energy formula for ground band level energies of soft rotor and called it power law

$$E = aJ^b \qquad\qquad 14$$

The index b can be determined from the ratio

$$R_J = \frac{E}{E(2)} = \left(\frac{J}{2}\right)^b \qquad\qquad 15$$

For any spin(J).

RESULTS AND DISCUSSIONS

Calculating the staggering quantity

$$\Delta^4 E_\gamma(J) = \frac{1}{16}\left[6E_\gamma(J) - 4E_\gamma(J-2) - 4E_\gamma(J+2) + E_\gamma(J-4) + E_\gamma(J+4)\right] \qquad 16$$

Table 1: Energy of Transition for the $^{192}Hg\,(SD-1),^{194}Hg\,(SD-1)(SD-2)(SD-3)$ and $^{194}Pb\,(SD-1)$

S/N	$^{192}Hg\,(-1)$		$^{194}Hg\,(-1)$		$^{194}Hg\,(-2)$		$^{194}Hg\,(-3)$		$^{194}Pb\,(-1)$	
	$\hbar\omega$	$\Delta^4 E_\gamma(K$	$\hbar\omega$	$\Delta^4 E_\gamma(K$	$\hbar\omega$	$\Delta^4 E_\gamma(K$	$\hbar\omega$	$\Delta^4 E_\gamma(K$	$\hbar\omega$	$\Delta^4 E_\gamma(K$
1	0.11	0.00	0.13	0.60	0.08	0.13	0.10	0.13	0.06	-0.18
2	0.14	-0.19	0.15	0.60	0.16	-0.13	0.13	0.00	0.09	0.13
3	0.16	0.20	0.18	-0.19	0.18	0.04	0.15	0.06	0.12	-0.18
4	0.18	-0.07	0.20	0.19	0.20	0.04	0.18	-0.03	0.14	0.25
5	0.20	0.00	0.22	-0.60	0.22	-0.19	0.19	0.06	0.16	-0.25
6	0.22	0.00	0.23	0.00	0.23	0.25	0.21	0.00	0.18	0.25
7	0.24	0.07	0.25	0.00	0.25	-0.19	0.23	0.06	0.20	-0.18
8	0.26	-0.19	0.28	0.00	0.27	0.04	0.24	-0.03	0.22	-0.01
9	0.28	0.23	0.30	0.00	0.29	0.00	0.26	0.06	0.24	0.18
10	0.29	-0.19	0.33	0.00	0.30	0.04	0.28	0.00	0.25	-0.14
11	0.31	0.10	0.34	0.60	0.33	-0.19	0.30	0.06	0.27	-0.08
12	0.33	-0.19	0.35	-0.13	0.35	0.25	0.32	-0.03	0.29	0.13
13	0.34	0.30	-	-	-	-	-	-	0.31	-0.08
14	0.36	-0.31	-	-	-	-	-	-	-	-
15	0.38	0.19	-	-	-	-	-	-	-	-

Table 2: Energy of Transition for the $^{152}Dy\,(SD-1)$, $^{151}Dy\,(SD-4)$, $^{153}Dy\,(SD-2)(SD-3)$ and $^{151}Tb\,(SD-2)$

S/N	$^{152}Dy\,(-1)$		$^{151}Dy\,(-4)$		$^{153}Dy\,(-2)$		$^{153}Dy\,(-3)$		$^{151}Tb\,(-2)$	
	$\hbar\omega$	$\Delta^4 E_\gamma(K$	$\hbar\omega$	$\Delta^4 E_\gamma(K$	$\hbar\omega$	$\Delta^4 E_\gamma(K$	$\hbar\omega$	$\Delta^4 E_\gamma(K$	$\hbar\omega$	$\Delta^4 E_\gamma(K$
1	0.31	0.00	0.36	-0.20	0.33	0.00	0.36	0.07	0.38	0.32
2	0.33	-0.13	0.39	0.14	0.36	0.07	0.38	-0.13	0.39	-0.18
3	0.35	0.06	0.42	-0.20	0.39	-0.07	0.40	0.07	0.42	0.06
4	0.38	0.06	0.44	0.30	0.42	-0.22	0.43	0.07	0.43	0.06
5	0.40	-0.13	0.46	-0.39	0.45	0.50	0.45	-0.20	0.46	-0.18
6	0.43	0.06	0.49	0.20	0.46	-0.60	0.48	0.25	0.48	0.32
7	0.44	0.06	0.51	0.10	0.50	0.50	0.50	-0.13	0.50	-0.38

8	0.46	-0.13	0.53	-0.13	0.52	-0.22	0.53	-0.20	0.54	0.32
9	0.50	0.06	0.55	-0.13	0.54	-0.15	0.55	0.38	0.55	-0.18
10	0.51	0.00	0.58	0.10	0.56	0.16	0.58	-0.25	0.58	0.06
11	0.53	0.00	0.60	0.20	0.58	-0.08	0.60	0.07	0.60	0.00
12	0.55	0.00	0.63	0.52	0.60	-0.06	0.63	0.07	0.63	0.00
13	0.58	0.00	-	-	0.63	0.01	0.65	-0.25	0.65	0.06
14	0.61	0.00	-	-	0.65	-0.28	-	-	0.68	-0.18
15	0.64	0.06	-	-	-	-	-	-	0.70	0.18
16	0.67	-0.19	-	-	-	-	-	-	0.73	0.13
17	0.69	0.19	-	-	-	-	-	-	-	-

Table 3: Energy of Transition for the $^{142}Eu(SD)$, $^{143}Eu(SD)$, $^{144}Eu(SD-2)$ and $^{144}Eu(SD-3)$

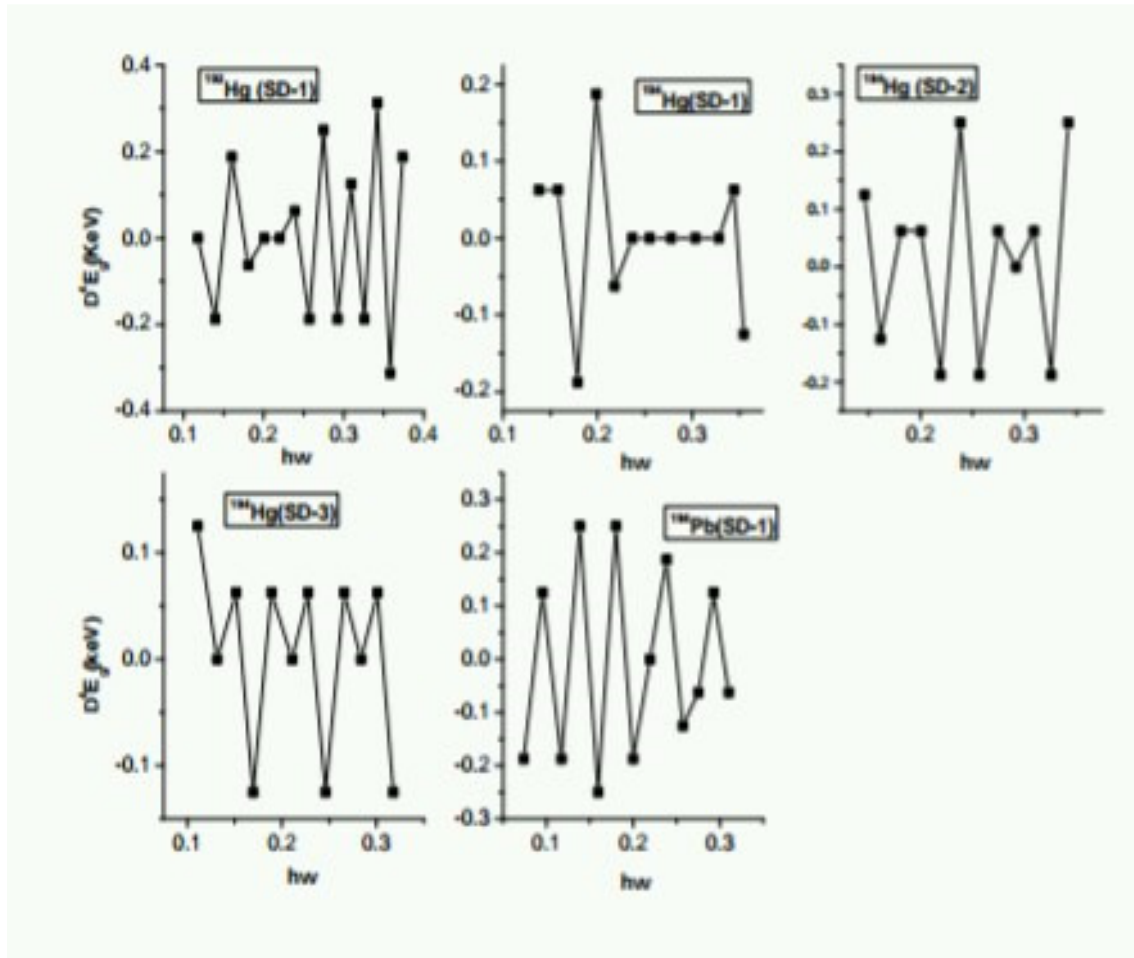| S/N	$^{142}Eu (SD)$		$^{143}Eu (SD)$		$^{144}Eu (SD-2)$		$^{144}Eu (SD-3)$	
	$\hbar\omega$	$\Delta^4 E_\gamma (KeV)$	$\hbar\omega$	$\Delta^4 E_\gamma (KeV)$	$\hbar\omega$	$\Delta^4 E_\gamma (KeV)$	$\hbar\omega$	$\Delta^4 E_\gamma (KeV)$
1	1.00	-0.32	0.50	-0.13	0.50	0.10	0.60	0.02
2	1.50	0.20	1.00	0.18	0.90	0.00	1.10	-0.20
3	1.80	-0.03	1.50	-0.12	1.30	-0.11	1.50	0.23
4	2.30	-0.06	1.80	0.00	1.50	0.11	1.95	-0.13
5	2.80	-0.09	2.00	0.09	2.00	-0.10	2.45	-0.01
6	3.30	0.30	2.30	-0.08	2.50	-0.10	2.90	-0.00
7	4.00	-0.26	3.00	0.04	3.00	0.11	3.50	0.04
8	4.50	0.05	3.50	-0.07	3.50	-0.15	4.20	-0.30
9	5.10	-0.03	4.00	0.13	4.20	0.01	4.50	0.40
10	5.60	-0.12	4.50	-0.11	4.80	-0.40	5.30	-0.28
11	-	-	5.00	0.06	5.50	-052	6.00	0.35
12	-	-	5.50	-0.02	6.00	0.39	6.80	-0.31
13	-	-	6.50	-0.01	6.50	1.10	7.50	-0.01
14	-	-	7.00	-0.03	7.50	-0.39	-	-
15	-	-	7.50	0.06	-	-	-	-
16	-	-	8.50	-0.13	-	-	-	-
17	-	-	9.00	0.15	-	-	-	-
18	-	-	10.00	-0.22	-	-	-	-

Fig. 1: $\Delta^4 E_\gamma$ versus $\hbar\omega$ for the $^{192}Hg\ (SD-1),^{194}Hg\ (SD-1)(SD-2)(SD-3)\ and\ ^{194}Pb\ (SD-1)$
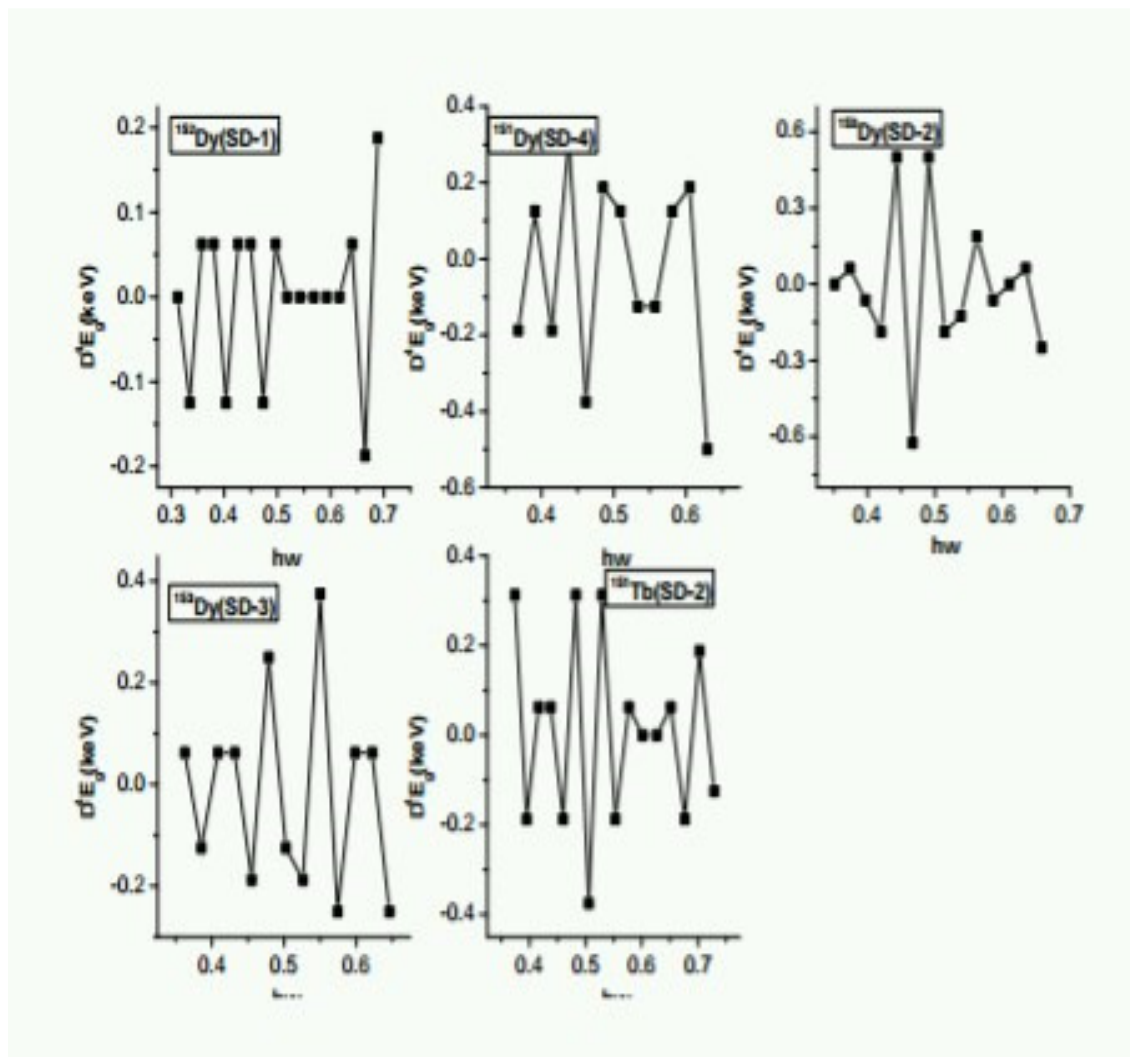
Fig. 2: $\Delta^4 E_\gamma$ versus $\hbar\omega$ for the $^{152}Dy\ (SD-1)$, $^{151}Dy\ (SD-4)$, $^{153}Dy\ (SD-2)(SD-3)\ and\ ^{151}Tb\ (SD-2)$
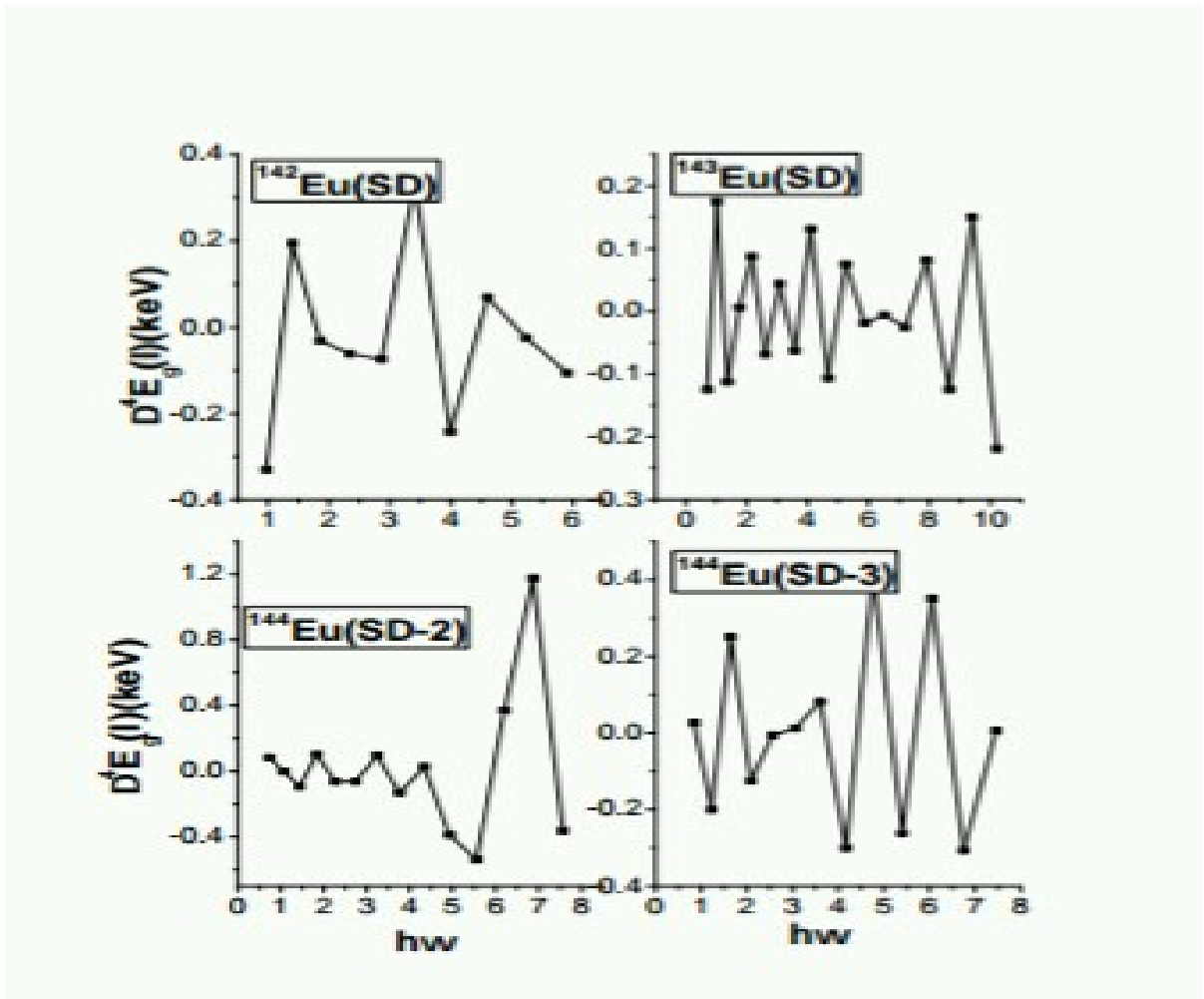
Fig. 3: $\Delta^4 E_\gamma$ versus $\hbar\omega$ for the $^{142}Eu(SD)$, $^{143}Eu(SD)$, $^{144}Eu(SD-2)$ and $^{144}Eu(SD-3)$

Fig. 1 indicate the staggering pattern for $^{192}Hg\ (SD-1)$, $^{194}Hg\ (SD-1)(SD-2)(SD-3)$ and $^{194}Pb\ (SD-1)$. For $^{192}Hg\ (SD-1)$, value lies from -0.3 to 0.3 and with increasing neutron number staggering also gets increased. Fig. 2 shows the occurrence of $\Delta 1 = 2$ staggering effect in the γ-ray transitions energies of $^{152}Dy\ (SD-1)$, $^{151}Dy\ (SD-4)$, $^{153}Dy\ (SD-2)(SD-3)$ and $^{151}Tb\ (SD-2)$ and the values lies between -0.6 to 0.6. Fig.3 prove the variation of staggering index with $\hbar\omega$ for $^{142}Eu(SD)$, $^{143}Eu(SD)$, $^{144}Eu(SD-2)$ and $^{144}Eu(SD-3)$. The SD nuclei $^{142}Eu(SD)$, $^{143}Eu(SD)$ and $^{144}Eu(SD-3)$ indicate large amplitude while $^{144}Eu(SD-2)$ indicate low amplitude of staggering index at low value of $\hbar\omega$.

CONCLUSION
The systematic behavior has been analysis which the band movement of inertia turn out nearly similar but the same could not be said for identical SD bands. The analysis shows the implication of identical bands.

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