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## Functions of Nuclear Radius as an Evaluation of Nuclear Charge Density

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### ABSTRACT

The goal of this paper is to show the shape of a nucleus depends mainly on its charge distribution. Both the radius of a nucleus and density distributions are important. The nuclear charge distribution of different values of nuclear radius  $R$  for both light and medium nucleus, scoped to computerize nuclear charge as function of nuclear radius. The nuclear charge density distribution is exponentially decreases with the increase in nuclear charge radius for both light and medium nucleus. Also for heavy nuclei indicate that the charge density is roughly constant out to a certain point and drops relatively slowly to zero.

**Keywords:** Electron scattering, Nuclear Radius, Nuclear Mass and Density

### INTRODUCTION

Nuclear structure encompasses a vast territory from the study of simple, few particle systems with close to 300 particles, from stable nuclei to the short-lived exotic nuclei, from ground state properties to excitations of such energy that the nucleus disintegrates into substructure and individual constituents, from the strong force that hold the atomic nucleus together to the effective interactions that describe the collective behavior observed in many heavy nuclei (Anderson, 2010). During the last decade, tremendous progress was obtained in the study of atomic transitions using high –precision laser spectroscopy and this allow the measurement of nuclear charge radii and also of nuclear moments for stable and even unstable isotopes (Anderson, 2010). The size of a nucleus grows slowly with atomic number in a way so as to keep the nuclear density essentially fixed. These observations again lend support to the fact that the nuclear force is short-ranged (Gernot, 2003). The nuclear density of stable nuclei with atomic weight  $A= 12-40$  can be described by Woods-Saxon expression, we extract the values of root mean square radii from the same experimental data in the frame of three different theoretical approaches: the optical approximation, the rigid target approximation and the exact expression of the Glauber theory (Mathieu, 2013).

### MATERIALS AND METHODS

#### Nuclear Mass and Density

The measurement of nuclear masses occupies an extremely important place in the development of nuclear physics. Mass spectrometer was the first technique of high precision available to the experiment and since the mass of nucleus increases in a regular way with the addition of one proton or neutron, measuring masses permitted the entire scheme of stable isotopes to be mapped (Kenneth, 1988). Nucleon distributions have studied by several nuclear reactions with the strong interaction probes. Among these proton elastic scattering provide the best information (Basdevant et al., 2005). Atomic electrons are sensitive to the properties of the nucleus they are bound to, such as nuclear mass, charge distribution, spin, magnetization distribution or even excited level scheme.



## Nuclear radius

An atom has a nucleus of charge  $Z$  and one electron, the nucleus has a radius  $R$ , inside which the charge proton is uniformly distributed (Gernot, 2003). The radius that we measure depends on the kind of experiment we are doing to measure the nuclear shape. In some experiments, such as high-energy electron scattering, muonic X-rays, optical and X-ray isotope shifts and energy differences of mirror nuclei, we measure the Coulomb interaction of a charged particle with the nucleus and these experiments would then determine the distribution of nuclear charge, primarily the distribution of protons but also involving somewhat the distribution of neutrons, because of their internal structure (Kenneth, 1988). The nucleus is often approximated by a homogeneous charge sphere, the radius  $R$  of this sphere is then quoted as the nuclear radius (Hussey, 2009).

## Electron Scattering

Electron can be used as probes for evaluation of the charge distribution in a nucleus because they undergo relative weak interactions with the nucleus, not strong interactions. Electron scattering by a nucleus is given by

$$F(q) = \frac{4\pi}{q} \int_0^{\infty} r \sin(qr) \rho(r) dr \quad 1$$

The mean value of  $r^2$ , taking over the charge distribution of a nucleus can be observed from experiment on the scattering of highly energetic electron by nuclei. It is possible to give an expression for  $\rho(r)$ , for the Fourier Transform yields

$$\langle r^2 \rangle = \frac{1}{2\pi r^2} \int_0^{\infty} F(q) \sin(qr) q dq \quad 2$$

The formula is of limited usefulness until experiments are carried out which give the form factor with considerable accuracy and over the sufficient range of  $q$ . One can prove that even for heavy nuclei, the low energy electron scattering depends to good approximation only on the root mean square radius of the charge distribution: for very small  $q$ -values only the first phase shift  $\delta_1$  contributes, which depends on the charge distribution  $\rho(r)$  through  $\sin(\delta_1 - \delta_1^c) \sim \gamma^{r+1}$  3

where  $\gamma = \sqrt{(1 - a^2 z^2)}$  at higher energies the form factor becomes sensitive to details of charge distribution, so that a model has to be specified for  $\rho(r)$ . For all but the lightest nuclei the electron scattering results can be remarkably well described by the so called Femi model (Ha 56)

$$\rho(r) = \rho_0 \frac{1 + \frac{wr^2}{c^2}}{1 + \exp\left\{\frac{r-c}{a}\right\}} \quad 4$$

For  $w=0$  the half density radius  $c$  is the distance at which the charge density has dropped to half its central value and skin thickness the distance over which the charge density decrease from 90% to 10% of its central value. The so called wine bottle parameter  $w$ , which modifies the central behavior of  $\rho(r)$ . One form of density dependence which has been studied extensively is

$$\rho(r) = \rho_0 \frac{1}{1 + \exp\left\{\frac{r-c}{a}\right\}} \quad 5$$



This distribution has been referred to as Femi density function. For  $R \gg a$ , is essentially  $\rho_0$  until  $r - R$  is a few times  $a$  and then it falls to negligible values in a distribution determined by  $a$  and independent of  $R$ .

## RESULTS

The charge density as function of radius for light, medium and heavy nuclei was computed using equation (5) and presented in tables and figures 1 to 3 respectively.

Table 1: Charge density as function of radius for light nuclei

Nucleus	Mass Number (A)	Radius (R)	$\frac{\rho(r)}{\rho_0}$
H	1	0.42	0.75
He	4	1.67	0.46
Li	7	2.92	0.2
Be	9	3.75	0.1
B	11	4.58	0.04
C	12	5	0.03
N	14	5.83	0.01
O	16	6.67	0.005
F	19	7.92	0.002
Ne	20	8.33	0.001
Na	23	9.58	0.0003
Mg	24	10	0.0002
Al	27	11.25	$5.92 \times 10^{-5}$
Si	28	11.67	$3.90 \times 10^{-5}$
P	31	12.92	$1.12 \times 10^{-5}$
S	32	13.33	$7.37 \times 10^{-6}$
Cl	35.5	14.79	$1.71 \times 10^{-6}$
Ar	40	16.67	$2.63 \times 10^{-7}$
K	39	16.25	$3.99 \times 10^{-7}$
Ca	40	16.67	$2.63 \times 10^{-7}$

Table 2: Charge density as function of radius for medium nuclei

Nucleus	Mass Number (A)	Radius (R)	$\frac{\rho(r)}{\rho_0}$
Sc	45	18.75	$3.27 \times 10^{-8}$
Ti	47.9	19.95	$9.78 \times 10^{-9}$
V	50.9	21.20	$2.80 \times 10^{-9}$
Cr	52	21.67	$1.77 \times 10^{-9}$
Mn	54.9	22.88	$5.29 \times 10^{-10}$



Fe	55.8	23.25	$3.64 \times 10^{-10}$
Co	58.9	24.54	$9.99 \times 10^{-11}$
Ni	58.7	24.46	$1.09 \times 10^{-10}$
Cu	63.5	26.46	$1.47 \times 10^{-11}$
Zn	65	27.08	$7.87 \times 10^{-12}$
Ga	70	29.17	$9.80 \times 10^{-13}$
Ge	73	30.42	$2.81 \times 10^{-13}$
As	75	31.25	$1.22 \times 10^{-13}$
Se	79	32.92	$2.31 \times 10^{-14}$
Br	80	33.33	$1.52 \times 10^{-14}$
Kr	84	35	$2.87 \times 10^{-15}$
Rb	85.5	35.63	$1.54 \times 10^{-15}$
Sr	88	36.67	$5.42 \times 10^{-16}$
Y	89	37.08	$3.57 \times 10^{-16}$
Zr	91	37.92	$1.55 \times 10^{-16}$

**Table 3: Charge density as function of radius for heavy nuclei**

Nucleus	Mass Number (A)	Radius (R)	$\frac{\rho(r)}{\rho_0}$
Nb	92.9	38.71	$6.88 \times 10^{-17}$
Mo	95.9	39.96	$1.97 \times 10^{-17}$
Tc	99	41.25	$5.41 \times 10^{-18}$
Ru	101.1	42.13	$2.26 \times 10^{-18}$
Rh	102.9	42.88	$1.07 \times 10^{-18}$
Pd	106.4	44.33	$2.48 \times 10^{-19}$
Ag	107.9	44.96	$1.33 \times 10^{-19}$
Cd	112.4	46.83	$2.04 \times 10^{-20}$
In	114.8	47.83	$7.49 \times 10^{-21}$
Sn	118.7	49.46	$1.48 \times 10^{-21}$
Sb	121.8	50.75	$4.06 \times 10^{-21}$
Te	127.6	53.17	$3.62 \times 10^{-23}$
I	126.9	52.88	$4.84 \times 10^{-23}$
Xe	131.3	54.71	$7.74 \times 10^{-24}$
Cs	132.9	55.38	$3.98 \times 10^{-24}$
Ba	137.3	57.21	$6.36 \times 10^{-25}$
La	138.9	57.88	$3.26 \times 10^{-25}$
Hf	178.5	74.38	$2.23 \times 10^{-32}$
Ta	181	75.42	$7.86 \times 10^{-33}$
W	183.9	76.63	$2.35 \times 10^{-33}$
Re	186.2	77.58	$9.00 \times 10^{-34}$

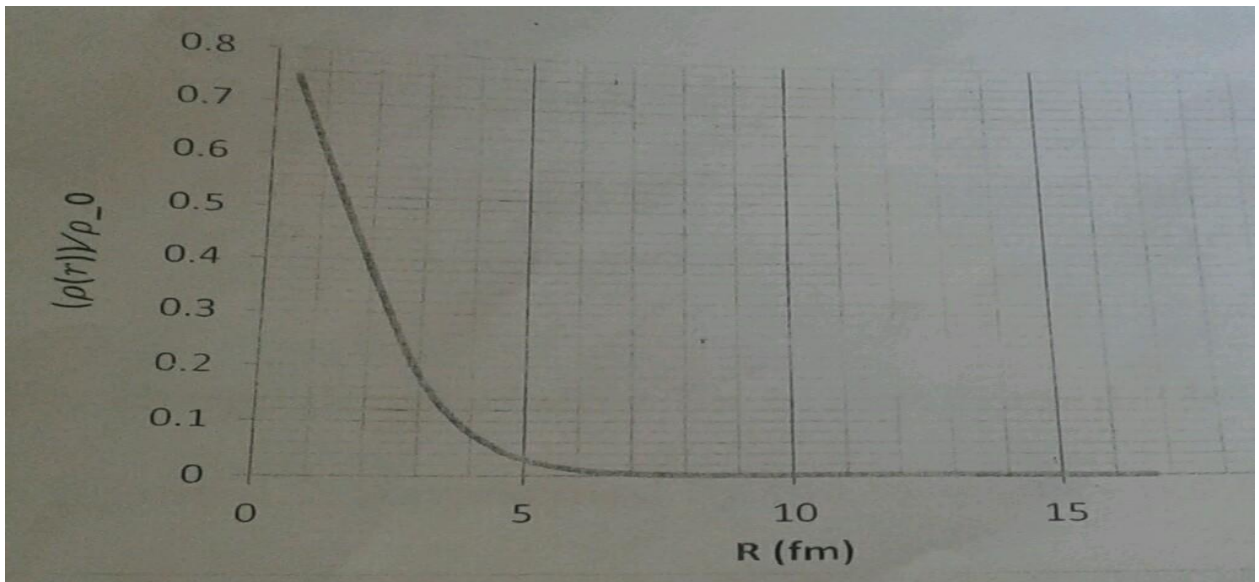


Figure 1: curve of a charge density distribution as a function of radius for light nuclei.

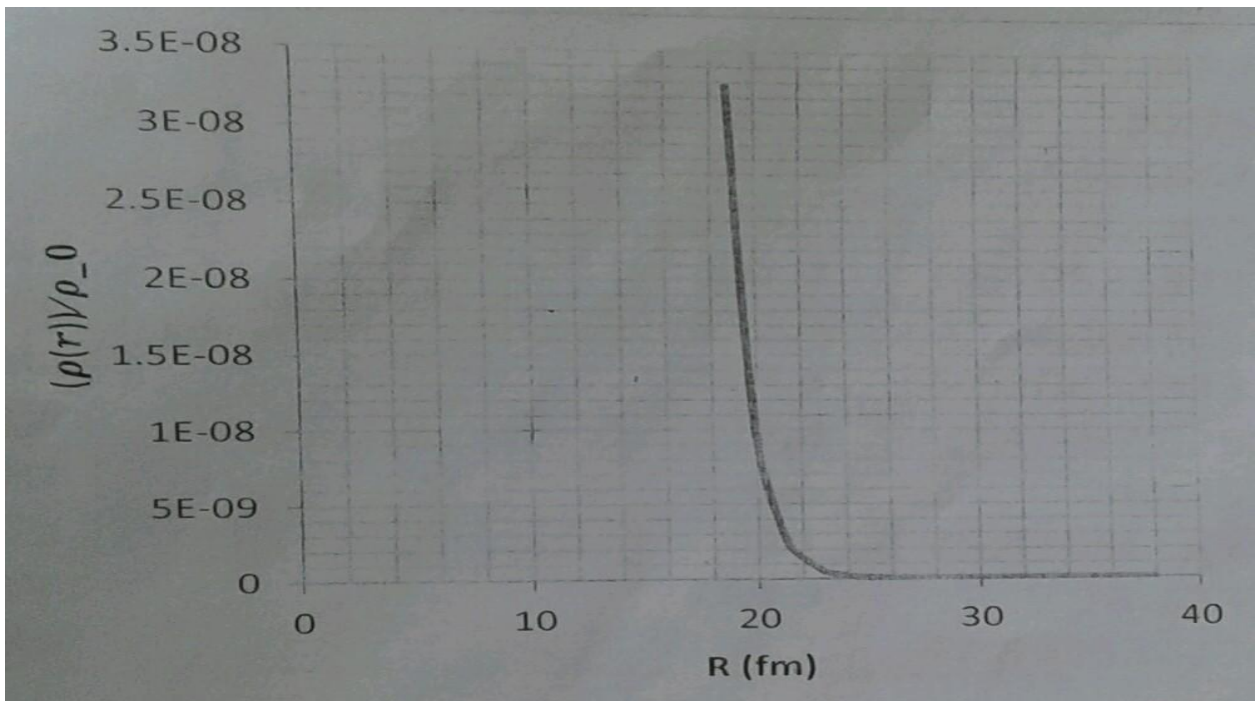


Figure 2: curve of a charge density distribution as a function of radius for medium nuclei.

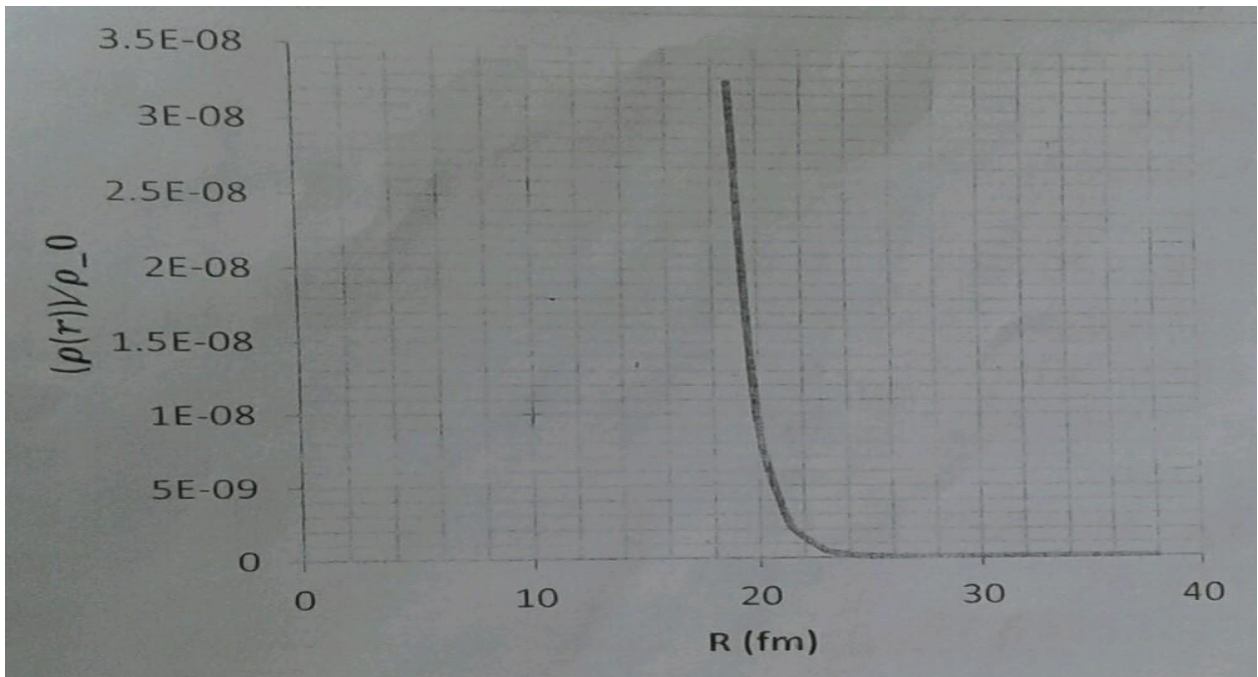


Figure 3: curve of a charge density distribution as a function of radius for heavy nuclei.

## DISCUSSIONS

It has been observed that Figure 1 have the charge density curve of a light nuclei decreases exponentially with the increase in radius  $R$ . The charge density distribution for light is nuclei is constant at a very large nuclear charge radius  $R$ , i.e. as the mass of the nuclei increases, the charge density distribution becoming independent on the nature of the nuclei. The charge density is roughly constant out to a certain point and the drops relatively slowly to zero. The distance over which this drop occurs is nearly independent of the size of the nucleus and is usually taken to be constant. Also figure 2 shows that the charge density curve of a medium nuclei decrease exponentially with the increase in radius  $R$ . The charge distribution for a medium nuclei decrease slowly with the increase in nuclear charge radius and becoming constant at a very large nuclear charge radius  $R$ , which is the mass of the nuclei increase, the charge density distribution becoming independent on the nature of the nuclei. This show little change in nuclear charge distribution for medium nuclei at a very small nuclear charge radius which depend on the mass of the nuclei and shows constant charge distribution as the mass of the nuclei increases. The charge density is roughly constant out to a certain point and then drops relatively slowly to zero. The distance over which this drop occurs is nearly independent of the size of the nucleus and is usually taken to be constant.

Figure 3 indicate that the charge density curve of a heavy nuclei decreases exponentially with the increase in radius  $R$ . The charge density distribution for a heavy nuclei decreases slowly with the increase in nuclear charge radius and becoming constant at a very large nuclear charge radius  $R$ , which is as the mass of the nuclei increases, the charge density distribution becoming independent on the nature of the nuclei. This shows a slight change in nuclear charge distribution for heavy nuclei at a very small nuclear charge radius which



depend on the mass of the nuclei and shows constant charge distribution as the mass of the nuclei increases. The charge density is roughly constant out to a certain point and then drops relatively slowly to zero. The distance over which this drop occurs is nearly independent of the size of the nucleus and is usually taken to be constant.

## CONCLUSION

From the results analyzed it can be concluded that, the charge density is roughly constant out to a certain point and the drops relatively slowly to zero. The distance over which this drop occurs is nearly independent of the size of the nucleus and is usually taken to be constant.

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