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Solving first Order Linear Differential Equation Using Operational Amplifier Approach

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ABSTRACT

Operational Amplifier (Op-Amp) as it is popularly called is used to solve differential linear equations. A differential linear equation of first degree was considered for the design using cascades of inverter; summer and integrator configuration of operational amplifier. A general purpose op-amp, μA 741 of the 8 pin mini DIP type was chosen for the design. Its unique features and prime advantages was considered and hence its choice. The performance of the designed circuit was evaluated for the input cases of a unit step inputs. The output graphs obtained in each case compared to the analytically derived function showed the circuit worked as desired. Though, the little ripples seen in the output graphs are as a result of the thermal noise of the components used in building the circuits.

Keywords: Op-Amp, differential equation, summer; inverter and integrator

INTRODUCTION

An operational amplifier ("op amp") is a differential-input, high gain voltage amplifier, usually packaged in the form of a small integrated circuit. The term "operational" dates back to the early days of analog computers when these devices were employed in circuits that performed mathematical operations such as addition, subtraction, integration, and the solution of differential equations. Today's op amps are used in a much wider variety of circuits and operate at considerably lower voltages and powers; however, the name remains. The modern operational amplifier is a very useful and versatile building block for thousands of circuits in applications as diverse as audio, video, communications, process control and instrumentation.
An operational amplifier (or an op-amp) is an integrated circuit (IC) that operates as a voltage amplifier. An op-amp has a differential input. That is, it has two inputs of opposite polarity. An op-amp has

a single output and a very high gain, which means that the output signal is much higher than input signal. An op-amp is often represented in a circuit diagram with the following symbol:

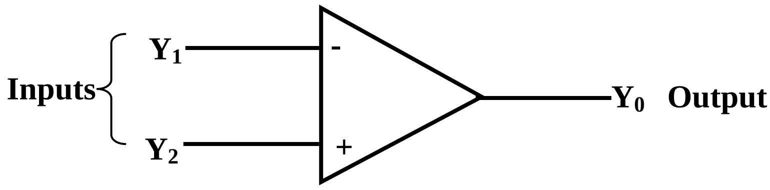

Figure 1: Simple representation of an Op-Amp

These amplifiers are called "operation" amplifiers because they were initially designed as an effective device for performing arithmetic operations in an analog circuit. The op-amp has many other applications in signal processing, measurement, and instrumentation. Operational amplifiers are widely used in electronic and measurement systems. They are among the active components of electronic circuits. Its applications in electronic circuits have received much attention due to their potential advantages in terms of high frequency performance, ease of design and miniaturization (Shinde, 2002). They can be configured to execute most mathematical operations such as integration, differentiation, simultaneous linear and quadratic equations among others.

Method (circuit design and implementation)

This paper is intended to build an Operational Amplifier circuit that would have the capacity to solve any first degree differential equation with any input. Consider the differential equation of the form

$a_n \Delta y_n + b_n y_n + c_n = 0 , n \geq 1$ .... (1)

Where, Δ is the forward differential operator ($\frac{\delta y}{\delta t}$).

(i.e.) equation (1) is of the form

$a_n \frac{\delta y}{\delta t} + b_n y + c_n = 0$ .... (2)

a_n ; $b_n and$ c_n are real constant integers.(Edward, 2000).

Rewriting equation (2) in terms of a solution for differential part, we would have

$a_n \frac{\delta y}{\delta t} = -(b_n y + c_n)$ .... (3)

Dividing equation (3) through by a_n, we would have

$$\frac{\delta y}{\delta t} = -(\frac{b_n}{a_n} y + \frac{c_n}{a_n}) \quad \ldots\ldots (4)$$

From equation (4), it shows that we have a first order differential whose solution is a summation of two quantities.
Consider the Operational Amplifier circuit shown in figure 2. RC (Resistor-Capacitor) Circuits in its simplest form, an R-C circuit contains a resistance, R, a capacitor, C, and an electromotive force, e.m.f (usually a battery). A circuit diagram of an R-C circuit looks like that shown in figure 3a

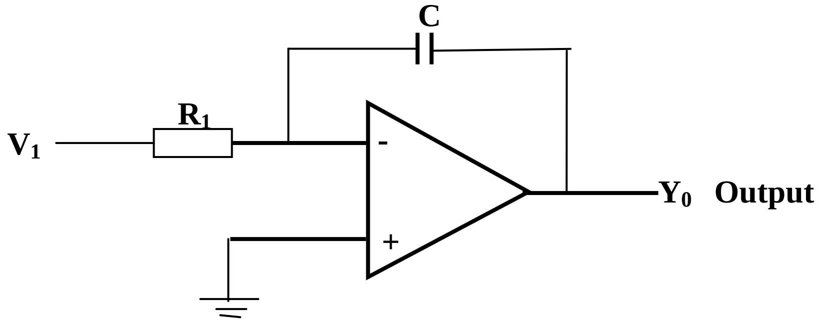

Figure 2: Representation of an Op-Amp with R-C as biasing component

Though by convention, figure 2a is an operational amplifier as an integrator whose input output relation is governed by

$$Y_o = -RC \int V_1 (t) \delta t \quad \ldots\ldots (5)$$

This was adopted for a simple fact that the integral of the higher-order derivative is the derivative that's one order lower.

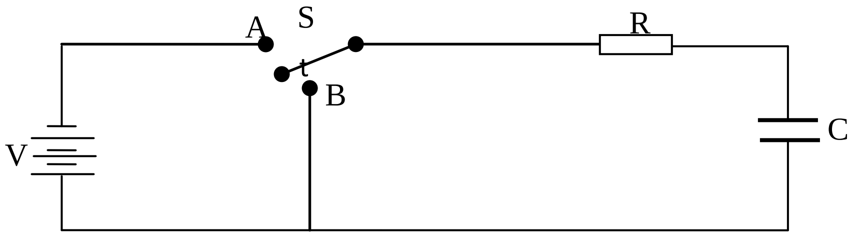

Figure 3a: Simplified R-C circuit of An op-amp

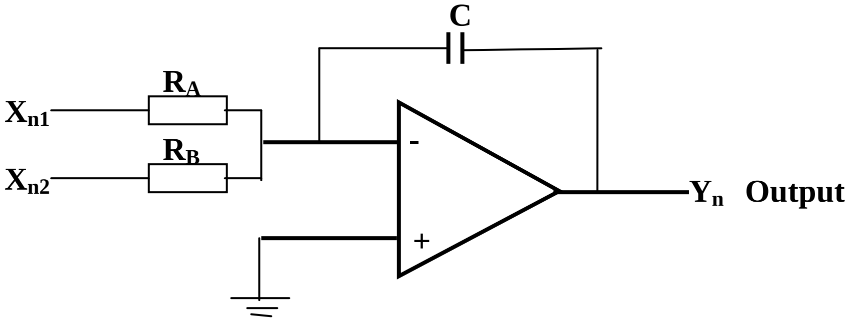

Figure 3b: Representation of an Op-Amp with R-C as biasing component

In figure 3b, the output (Y_n) is the input arriving at the negative terminal of the operational amplifier. The negative terminal of this output is located or indicated at or by the negative of the input terminal (Boylestad, 2002)

From figure 3a, the simplified RC circuit with two states of either A or B whose real operational amplifier circuit is shown in figure 3b would have an input –output relationship of the form:

$$Y_n = -A\frac{1}{RC}\sum_1^{\infty} X_{n1} - B\frac{1}{RC}\sum_1^{\infty} X_{n2} \quad \ldots\ldots (6)$$

Setting the RC values which represent the gain of the operational amplifier to be 1, equation 4 becomes:

$$Y_n = -A\sum_1^{\infty} X_{n1} - B\sum_1^{\infty} X_{n2} \quad \ldots\ldots (7)$$

In designing the circuit that would implementing equation 5, to obtain the constant coefficient of A and B that are linear in nature (El-Ali, *et-al, 2007*), an inverter circuit of figure 4 was considered

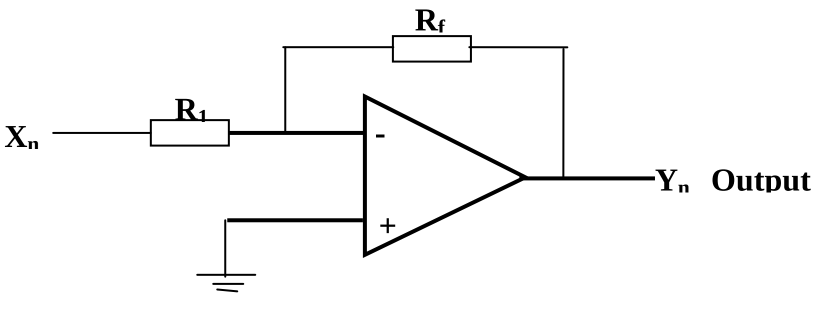

Figure 4: Representation of an Op-Amp with R-C as biasing component

The input – output relationship of an inverter shown in figure 4 is represented by:

$Y_n = \frac{R_f}{R_1} X_n$ .... (8)

To obtain a unity gain for the inverter, R_1 must set to equal to R_f, by so doing we have unity gain inverter.

Combining the circuits of figure 3b and 4, we obtain a circuit in figure 5, that would implement or solve a first order differential equation of the form:

$\Delta Y_n + A_n Y_n = B_n X_n$ .... (9).

The (Y_n) solution of equation (7) is $(-2)^n$ with other variables as (A_n = 1; $B_n \geq 0$ and $X_n \geq 0$)

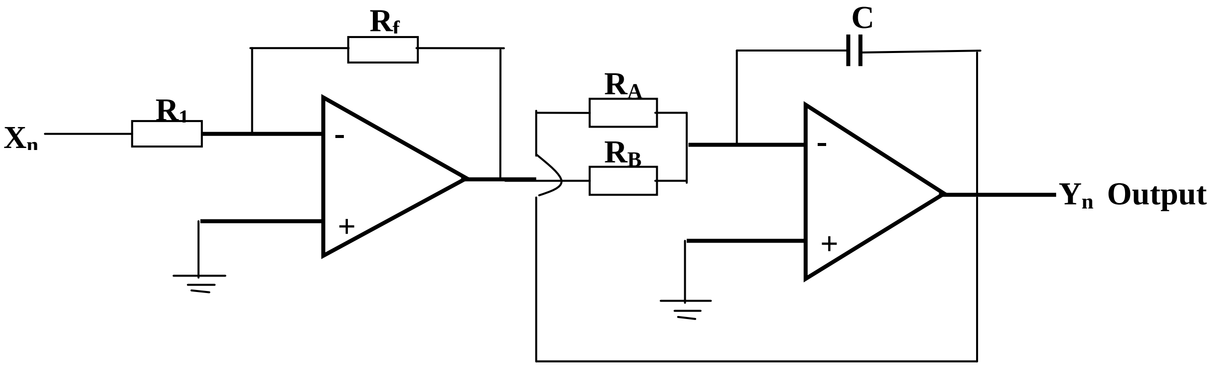

Figure 5: Simplified circuit to implement equation 9 above.

In building a circuit that would solve the first order differential equation of equation (1), the approach to differential equation of equation (9) was adopted taken into cognisance sets of equations (4). In doing so, different values of inputs functions were considered.

In all cases, different values of **a_n, b_n and c_n** in equation (1) were considered in order to verify the workability of the circuit built, setting initial condition to $V_o(t)$ = K y(t). K as the integrating factor (in all cases to 1V/m

Case 1:

If **a_n = 1, b_n = 5 and c_n = 2,** equation (1) becomes

$\Delta y_n + 5y_n + 2 = 0$ .... (10)

Rewriting equation (10), in comparison with equation (4) we have

$\frac{\delta y}{\delta t} = -(\frac{5}{1}y + \frac{2}{1})$ .... (11).

Equation (11) shows that the circuit to implement the system requires a summer of one arm having a gain of -5 and a constant of -2 as a solution of the first degree differential equation.

The circuit to implement this system is shown in figure 6 with all values of the discrete component that supported the accomplishment of the design. While, a switch (t) is introduced to initiate the initial value at time (t) with D.C. voltage source as a exciter or input source across (V_1).

Component values: R_1 = 1.0MΩ; R_2 = 0.2MΩ; R_3 = 1.0MΩ; R_4 = 0.2MΩ; Rf_1 = 1.0MΩ;
Rf_2 = 0.2MΩ and C_1 = 1.0μf.

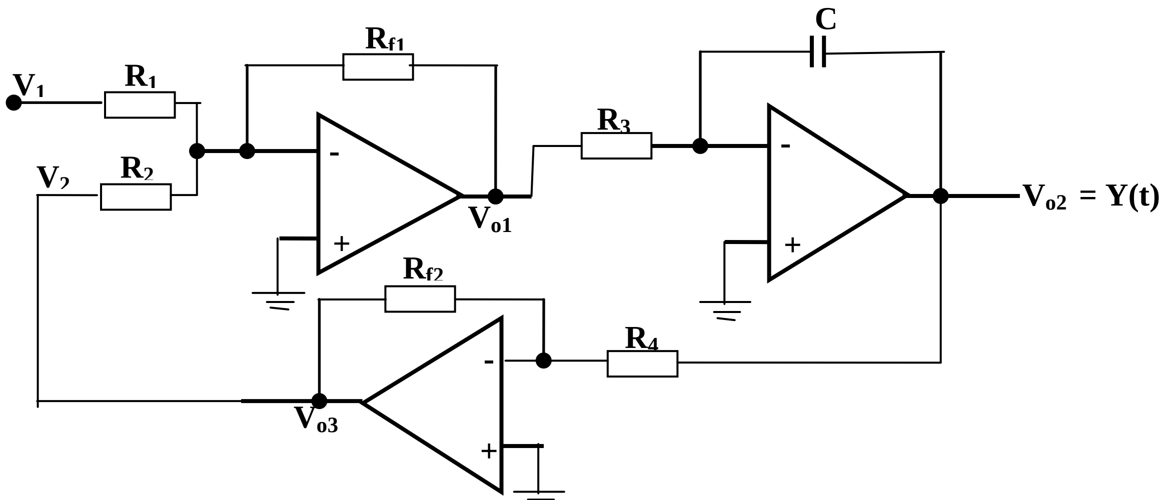

Figure 6: Simplified circuit to implement equation 9 above.

$V_{01} = -\frac{\delta y}{\delta t}$; $V_{03} = V_{02} - y(t)$; $V_{02} = y(t)$; $\frac{R_{f1}}{R_1} = \frac{1.0}{1.0} = 1$; $\frac{R_{f1}}{R_2} = \frac{1.0}{0,2} = 5$; $\frac{R_{f2}}{R_{41}} = \frac{1.0}{1.0} = 1$

Case 2:

If **a_n = 1, b_n = 1 and c_n = 1**, equation (1) becomes

$\frac{\delta y}{\delta t} = -(y + 1)$ .... (12)

The equation (12) is the same as equation (11) with the difference of gain and the constant been Unity (1). The circuit to implement this system is same as that shown in figure 6. The only changes are from the values of discrete component (resistors) used.

Generalising the circuit, to solve a general first order differential equation with any coefficients defined, a precision variable resistors could replaced the R(s) to be able to vary the gains in compliance with desired target.

CONCLUSION

By comparing what was obtained from the analytical result to the output displayed on CRO, showed that the designed circuit worked as desired. The first order differential equation was solved using the designated inputs with the outputs as a function of the inputs. Solving any first order differential equations with any arbitrary coefficients requires huge sets of discrete resistive values. Though, the amplifier saturation is of great concern and thus must be taken into cognisance since the constant coefficients of a_n, b_n **and** c_n are translatable to the ratio of resistor values.

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