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COMPUTATION OF ERRORS IN COMMON PHYSICS EXPERIMENTS

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ABSTRACT

In conducting experiments in physics, the uncertainty in a measurement is first decided before the measurement is made and the measurements repeated and then the individual measured values are combined into an average final value or the measured value is combined mathematically with other measured values, either via combining equations, curve fitting and graphical analysis, to find a final measured value. The process of evaluating uncertainty associated with the measurement is often called error analysis. The uncertainty of a single measurement is limited by the precision and accuracy of the measurement. Every uncertainty in result of experiment from the expected one has important significance. Even on undergraduate level of our universities, error analysis and interpretation of the result should be done instead of writing just the points of precautions. This approach will make the students somewhat familiar about the error analysis on advanced level. This fact necessitated this work the computation of error in undergraduate common physics experiments. This shall enhance the standardization of our laboratory reporting.

Keywords: *Computation, Errors analysis, Uncertainty and Measurement.*

INTRODUCTION

The interpretation of some physical phenomena is obtained by conducting experiment and making measurements. It is important to understand how to express such data and how to analyze and draw meaningful conclusions from it. In doing this, it is crucial to understand that all measurements of physical quantities are subject to uncertainties. It is never possible to measure anything exactly without an error no matter how small therefore to draw valid conclusion the error must be indicated and dealt with properly. All measurements have some degree of uncertainty that may come from a variety of sources. The process of evaluating this uncertainty associated with a measurement result is often called uncertainty analysis or error analysis. Error analysis may seem tedious; however, without proper error analysis, no valid scientific conclusions can be drawn. In fact, bad things can happen if error analysis is ignored. The failure to specify the error for a given measurement can have serious consequences in science and in real life. Since there is no way to avoid error analysis, it is best to learn how to do it right. Error in a scientific measurement usually does not mean a mistake or blunder. Instead, the terms error and uncertainty both refer to unavoidable imprecision in measurements (Taylor, 1982). The uncertainty of a single measurement is limited by the precision and accuracy of the measuring instrument, along with any other factors that might affect the ability of the experimenter to make the measurement. In order to determine the accuracy of a particular measurement, we have to know the true value. Sometimes we have a textbook measured value which is known precisely, and we assume that this is our ideal value, and use it to estimate the accuracy of our result. In some cases, we know a theoretical value which is calculated from basic principles, and this also may be taken as an ideal value. The main purpose of error analysis is to check whether the result of experiment agrees with a theoretical prediction or results from other experiments or not. Generally speaking, a measured result agrees with a theoretical prediction if the prediction lies within the range of experimental uncertainty.

Definitions of Terms

Error

An error is the difference between measured value and true value of a quantity

Accuracy

The accuracy of a measurement is limited by the least count of the measuring instrument

Types of Errors

1. Constant Error: The error due to constant causes due to faulty calibration of a measuring instrument. Such faulty instruments should be discarded as soon as possible.
2. Systematic Error: The error which is governed by some systematic rule Example; zero error on screw gauge, index error of optical bench, etc. Such errors can be reduced by associated systematic rules.
3. Random Error: The error occurring due to unknown reasons, example, error due to change in condition of experiment, incorrect judgment of an observer in making measurement etc. Such errors can be minimized by repeating experiment many times and taking arithmetic mean.

Common sources of error in physics laboratory experiments

Incomplete definition **(may be systematic or random):** carefully consider and specify the conditions that could affect the measurement.
Failure to account for a factor **(usually systematic):** Care/account about all possible implicit and explicit factors should be done before beginning the

experiment so that arrangements can be made to account for the confounding factors before taking data.

Environmental factors (systematic or random): Be aware of errors introduced by your immediate working environment.

Instrument resolution (random) - All instruments have finite precision that limits the ability to resolve small measurement differences.

Failure to calibrate or check zero of instrument (systematic) - Whenever possible, the calibration of an instrument should be checked before taking data.

Physical variations (random) - It is always wise to obtain multiple measurements over the entire range being investigated.

Parallax (systematic or random) - If the observer's eye is not squarely aligned with the pointer and scale, the reading may be too high or low.

Instrument drift (systematic) - Most electronic instruments have readings that drift over time. Occasionally this source of error can be significant and should be considered.

Lag time and hysteresis (systematic) - Some measuring devices require time to reach equilibrium, and taking a measurement before the instrument is stable will result in a measurement that is generally too low. The most common example is taking temperature readings with a thermometer that has not reached thermal equilibrium with its environment. A similar effect is hysteresis where the instrument readings lag behind and appear to have a memory effect as data are taken sequentially moving up or down through a range of values. Hysteresis is most commonly associated with materials that become magnetized when a changing magnetic field is applied.

Personal errors: Gross personal errors, sometimes called mistakes or blunders, should be avoided and corrected if discovered (Bork *et al.,* 1993).

COMPUTATION OF ERRORS

Absolute and Relative errors

Let x_1, x_2, x_3 …xn, be the n measurements of a physical quantity.

True value $\bar{x} = \frac{\sum X}{n}$

Error:

$$\Delta x = \text{measured value} - \text{true value} = x_1 - x$$

Relative error $= \frac{\Delta x}{\bar{x}}$

Percentage error

$$\%\ \text{error} = \frac{\Delta x}{\bar{x}} \times 100\%$$

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Combination (propagation) of Errors:

The errors are communicated in different mathematical operations (Norton and Salagean, 2002, Sen, 2001) .

i. Errors in sum or Difference:

a. In Sum: Let X = (a + b) be the quantity. If Δa is error in measurement of 'a' and Δb be that in 'b', then

Permissible error in $\Delta x = \Delta a + \Delta b$

Relative error $\frac{\Delta x}{x} = \frac{\Delta a + \Delta b}{a+b}$

% error $=\frac{\Delta x}{x} \times 100\% = \left(\frac{\Delta a + \Delta b}{a+b}\right) \times 100\%$

b. In Difference: Let X = (a— b) be the quantity. If Δa is error in measurement of 'a', Δb be that in 'b', then

Permissible error in $X = \Delta a + \Delta b$ [Since, error is always additive]

Relative error $= \frac{\Delta x}{x} = \frac{\Delta a + \Delta b}{a-b}$

% error $=\frac{\Delta x}{x} \times 100\% = \left(\frac{\Delta a + \Delta b}{a-b}\right) \times 100\%$

ii. Errors in Quotient and Product :

a. In quotient

Let $X = \frac{a}{b}$ be the quantity. If Δa is error in measurement of a, Δb be that in 'b', then Taking log:

$$\log X = \log a - \log b$$

Differentiating we get

$$\frac{\Delta x}{x} = \frac{\Delta a}{a} - \frac{\Delta b}{b}$$

Permissible error in $X = \Delta x = X \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) = ab \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$ [Since, error is additive]

Relative error $=\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$

Percentage error $= \frac{\Delta x}{x} \times 100\% = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100\%$

b. In Product

Let X = a b be the quantity. If Δa is error in measurement of 'a', Δb be that in 'b', then Taking log:

$$\log X = \log a + \log b$$

Differentiating we get

$$;\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

Permissible error in X= $\Delta x = X\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) = ab\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Relative error $= \frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$

% error $= \frac{\Delta x}{x} \times 100\% = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100\%$

iii. Errors due to raised power:

Let $X = K\frac{a^n b^m}{C^p}$ be the quantity, where K is a constant. If Δa is error in measurement of 'a', Δb be that in 'b' and Δc that in 'c' then

Taking log of both sides we have :

Log X = Log K + nlog a + mLog b – pLog c

Differentiating we get

$$\frac{\Delta x}{x} = n\frac{\Delta a}{a} + m\frac{\Delta b}{b} - p\frac{\Delta c}{c}$$

Permissible error in X $= \Delta x$

$$= X\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b} - p\frac{\Delta c}{c}\right)$$

$$= ab\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b} + p\frac{\Delta c}{c}\right)$$

Relative error $= \left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b} + p\frac{\Delta c}{c}\right)$

% error $= \frac{\Delta x}{x} \times 100\% = \left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b} + p\frac{\Delta c}{c}\right) \times 100\%$

Significant Figures

In the measured value of a physical quantity, the digits about the correctness of which we are sure plus the last digit which is doubtful are called significant figures.

Rules for counting significant figures

i. All the non-zero digits are significant. Example, 98576 have five significant digits.
ii. All zeros occurring in between the two non-zero digits are significant. Example, 90008 have five significant digits.
iii. In a number without decimal, zeros on the right of non-zero digit are not significant. Example, 7890 has only three significant digits (789).
iv. In a number with a decimal point, zeros on the right of the last non zero digit are significant. Example, 9.78000 have six significant digits.
v. In a value less than one, zeros occurring between the decimal point and non-zero digit on the right are not significant. Example 0.00987 has only three significant digits.

Note that front zeros are always not significant, middle zeros are always significant and last zeros are significant only if there is decimal point.

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Remarks

i. The change in units of measurement of a quantity does not affect the number of significant figures.

ii. When one value is expressed in an exponential form, the exponential term does not affect the number of significant figures.

Example $98.7 = 0.0987 \times 10^3 = 98700 \times 10^{-3}$ has only three significant figures. [The powers of 10 are not counted as significant figures.]

iii. The error in measurement is equal to the least count (L.C.) of the measuring instrument.

iv. If least count of measuring instrument is not given, then error is ± 1 in the last digit of measurement.

v. Greater the number of significant figures in a measurement, smaller is the percentage error.

vi. Never round off intermediate results when performing a chain of calculations. The associated round-off errors can quickly propagate and cause the final result to be unnecessarily inaccurate. So, keep as many digits as is possible to avoid rounding off errors. Take two more significant figures than required and at the end, round off the final result to a reasonable number of significant figures.

Rounding Off:

The process of cutting off superfluous digit and retaining as many desired is called rounding off. In rounding off process if the digit next to the one to be rounded off is more than 5, the digit next to one to be rounded off is increased by 1; if it is less than 5, digit to be rounded is left unchanged. But if the next digit is exactly 5, then the digit to be rounded is increased by 1 if it is odd and keep unchanged if it is even. This can be remembered by the concept that we physicists always prefer even parity or symmetry. Example let's round off the following numbers on second decimal position.

$$5.478 \rightarrow 5.48$$
$$5.473 \rightarrow 5.47$$
$$5.475 \rightarrow 5.48$$
$$5.465 \rightarrow 5.47$$

Mathematical Operations and Significant Figures

The following mathematical operation are commonly applied in common physics experiments (Ashlock, 2010)

a. Addition and subtraction:

Example: $34.321 + 5.4 = 39.721 \rightarrow 39.72$

b. Multiplication

$34.321 \times 5.4 = 185.3334 \rightarrow 185.33$

c Division

Example $\frac{34.321}{5.4} = 6.3557407 \rightarrow 6.4$

Note: While expressing final result of a measurement/experiment, the measurement must be expressed up to proper significant figures, depending upon the least count of your measuring instrument and order of expected result.]

Examples of Error Analysis in Experiments:

Example 1. Fresnel's Bi-prism

Object of the Experiment: *To determine the wavelength of the light wave emitted by the given light source.*

Formula Used: $\lambda = \frac{\beta d}{D}$

where, β = fringe width, d = slit separation and D = distance between slit & source.

Calculation:

$$\lambda_i = \left[\frac{\beta}{D} d\right]_i$$

$\Rightarrow$ The mean $\bar{\lambda} = \frac{\Sigma \lambda_i}{n}$ for different D

Error Analysis

$$\Delta\lambda = \lambda\sqrt{\left[\left(\frac{\Delta\beta}{\beta}\right)^2 + \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta D}{D}\right)^2\right]}$$

{Recall section on errors in Product and Quotient}

$$(\Delta\lambda)^2 = \lambda^2\left[\left(\frac{\Delta\beta}{\beta}\right)^2 + \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta D}{D}\right)^2\right]$$

{Since, $\lambda = \lambda(\beta, d, D)$ and correlation coefficient for independent variables is zero i.e. product of average mean deviations of independent variables is zero}

Or, $\Delta\lambda = \lambda\sqrt{\left[\left(\frac{\Delta\beta}{\beta}\right)^2 + \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta D}{D}\right)^2\right]}$

Where,

a. $\beta = \frac{b-a}{n}$ where $\beta = \beta(b, a, n)$

Or, $\Delta\beta = \beta\sqrt{\left[\left(\frac{\Delta b}{b}\right)^2 + \left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta n}{n}\right)^2\right]}$

[On calculation, take $\frac{\Delta n}{n} = 0$ because for distinct Eigen states $\Delta n = 0$]

b. $d = \sqrt{d_1 d_2}$

Taking logarithm and differentiate we get,

$(\Delta d)^2 = \frac{1}{4} d^2 [\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta d_2}{d}\right)^2$

Methods to choose values of $\Delta\beta$, Δd, for the calculation of $\Delta\lambda$:

We take more than three observations for any observable like … etc. Therefore, we adopt different methods to choose values of $\Delta\beta$, Δd for the calculation of $\Delta\lambda$

A. **First Method:** (Method of maximum probable error)

$\Delta\beta$ = maximum $(\Delta\beta_i)$

Δd = maximum (Δd_i)

ΔD = Least Count.

B. **Second Method:** (Method of Standard deviation)

$\Delta\beta = \frac{\sigma\beta}{\sqrt{n}}$ where $\sigma_x = \frac{1}{n}\sum f_i(x_i - \bar{x}]^2$

$\Delta d = \frac{\sigma d}{\sqrt{n}}$

ΔD = Least count

Third Method: *(Method of least square fitting):* (For the equations which can be transformed in the form $y = mx + c$

Numerical examples to further explain the application of error analysis in common physics experiments are hereby presented (Scarborough, 1993)

Example 2: Radioactive sample

Aim of the Experiment: *To determine the half-life of a given radioactive sample.*

Formula used: $N = N_o e^{-\lambda t}$

Where, N_o = numbers of atom present at time $t = 0$.

Calculation:

Taking logarithm on both sides, we get,

$\ln N = \ln N_o - \lambda t$

Now, this equation is in the form of straight line. So, comparing with

$y = mx + c$, we get,

$y = \ln N$, $x = t$, $m = \lambda$ and $c = \ln N_o$
Now, taking time of counts on x- axis, logarithm of number of decaying atoms per second on y-axis, we can determine λ and N_o by finding slope and y-intercept of straight line on plotting graph of lnN versus t.
At the same time we also calculate m and c by the method of least square fitting using the normal equations: $\Sigma y = m\Sigma x + nc\Sigma y = m\Sigma x + n$
$\Sigma xy = m\Sigma x^2 + c\Sigma x\Sigma xy = m\Sigma x^2 + c\Sigma x$.

.
Solving these two equations we can obtain the values of m and c.
Hence, using, $T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$ we can determine the half life time of the given radioactive substance.

Discussion of the Result of the Experiment

After computation of the value of the physical quantity you must interpret your result. This section is included as discussion section on writing the experiment you did in your laboratory. There, you should state how precise and how accurate your result is. You should discuss the sources responsible for error on your result. Not only this, you must discuss about the significance of your result. Every deviation in result from the expected one has important significance.
For example: On sugar solution experiment for determination of refractive index at different concentrations, if we start with sugar solution of concentration above 40%, then relation between refractive index and concentration of sugar solution comes out to be non-linear at those high concentrations with respect to low concentration. But at low concentrations below 20% the relation is linear. This is because at high concentration sugar is dominating and at low concentration water is dominating. That means at high concentration about 80% the ref. index of the solution is near to that of sugar and at very low concentration the ref. index of solution is near to that of water. That means, at high concentrations about 80% and above you are measuring the refractive index of sugar rather than that of solution. Similarly at very low concentration 2.5% or near to it you are measuring refractive index of water. So, we make sugar solution of concentration equal and below 20% in our lab.

Our trend for general physics practical filing

In our traditional method of submission of practical reports, a general physics practical report consists:
1. Object of your experiment.
2. Apparatus required.
3. Theory.
4. Observations and calculations.
5. Error analysis.
6. Result of your experiment.

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7. Discussion.

What should a standard physics laboratory report include?

Actually, a standard physics lab report includes following requirements.

A. Experimental Description

1. Provide a statement of the physical theory or principle observed during the experiment.
2. Briefly describe the experiment you performed.
3. Discuss the relevance of the experiment to the theory.

B. Discussion of Graphs

1. For each graph, discuss the proximity of the data points to the line of best fit.
2. For each graph, state the theoretical values for slope and y-intercept.
3. For each graph, perform a percent difference calculation between the theoretical slope and y-intercept, and the measured values of slope and y-intercept.

C. Uncertainty Analysis Discussion

1. Discuss the method used to determine the measured uncertainty value for each measured quantity.
2. Consider each sub-experiment and determine the dominant source of uncertainty.
3. Determine the percent difference, percent uncertainty and percent variation. Then,
   (i) Determine if each percent difference value is larger than or smaller than the dominant percent uncertainty for the experiment.
   (ii) Draw conclusions based on your findings; for example determine the factors contributing to percent difference values greater than the percent uncertainty value.
   (iii) Determine experimental factors which could have contributed to variation of measurements, i.e. how you would account for measured values that are not constant.

4. What did you think of the experiment and what would you do to improve the experiment?

D. Inquiry based laboratory conclusions/ discussion:

Finally, predict the conclusions of your lab report. In particular, a paragraph should be written for each experiment. Each paragraph should contain the following information:

1. <u>Prediction</u>: What was the prediction that was made about the system?
2. <u>Reasoning</u>: What was the reasoning behind the prediction (i.e. why did you make the prediction that you made)?
3. <u>Experimental Result</u>: What was the result of the experiment?

4. Analysis:
(i) If the experiment agreed with you prediction, write a brief statement indicating this agreement.
(ii) If the experiment did not agree with the prediction, discuss what was wrong with the reasoning that led to the prediction that you made.

Example1: Determination of value of 'g' by using simple pendulum.
Suppose: l = 75cm by using meter scale. Then, Δl = 1mm. Suppose the time for 20 oscillations is 35 seconds measured with a wrist watch of 1second resolution. Then, g = $4\pi^2 \frac{1}{T^2}$= 9.668m/s^2 and taking log, we have,

$$\log g = \log 4\pi^2 + \log l - 2\log T$$

Differentiating we get ;$\frac{\Delta g}{g} = \frac{\Delta l}{l} - 2\frac{\Delta T}{T}$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} - 2\frac{\Delta T}{T}$$

Permissible error in g is Δg = $g(\frac{\Delta l}{l} + 2\frac{\Delta T}{T})$

Relative error in g: $\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$

$$\frac{\Delta g}{g} = (\frac{0.001}{0.75} + 2\frac{1}{35}) = 0.058$$

Result: value of g = (9.668 ±0. 058)m/s^2.

Example .2: To determine the surface tension of given liquid at room temperature. Suppose the quantities density 'd', acceleration due to gravity 'g' and angle of contact θ are taken from the table of constants, while height of liquid raised inside capillary tube and diameter of the capillary tube are measured as h = 3.00 ± 0.01cm and D = 0.250 ± 0.001mm

Then, T = $\frac{Dhdg}{4cos\theta}$ gives observed value for T where h= 3.00cm and D= 0.250cm.

Permissible error in T = ΔT = $\left(\frac{\Delta D}{D} + \frac{\Delta h}{h} + \frac{\Delta d}{d} + \frac{\Delta h}{h}\right)$

(d, g and Θ are constants} [Here, ΔD = 0.001cm and Δh = 0.01cm]

Relative error in T =$\frac{\Delta T}{T} = \left(\frac{\Delta D}{D} + \frac{\Delta h}{h} + \frac{\Delta d}{d}\right)$

% error on T =$\frac{\Delta T}{T} \times 100\% = \left(\frac{\Delta D}{D} + \frac{\Delta h}{h}\right) \times 100\%$

Result: value of g = T ± ΔT =$\frac{Dhdg}{4cos\theta} \pm \frac{Dhdg}{4cos\theta}\left(\frac{\Delta D}{D} + \frac{\Delta h}{h}\right)$.

CONCLUSION

The concept of error needs to be well understood. What is and what is not meant by "error". A measurement may be made of a quantity which has an accepted value which can be looked up in a handbook (e.g. the density of brass). The difference between the measurement and the accepted value is not what is meant by error. Such accepted values are not "right" answers. They are just measurements made by other people which have errors associated with them as well. Nor does error mean "blunder" reading a scale backwards, misunderstanding what you are doing or elbowing your laboratory partner's measuring apparatus are blunders which can be caught and should simply be disregarded. , it cannot be determined exactly how far off a measurement is; if this could be done, it would be possible to just give a more accurate, corrected value. Error then has to do with uncertainty in measurements that nothing can be done about. If a measurement is repeated, the values obtained will differ and none of the results can be possible to do anything about such error, it can be characterized. For instance, the repeated measurements may cluster tightly together or they may spread widely. This pattern can be analyzed systematically.

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