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## Modeling of Production Function with Application to Crude Oil Producing Sector in Nigeria

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### ABSTRACT

This study focuses on modeling of production function with application to crude oil producing sector in Nigeria. The Cobb-Douglas Production function with multiplicative error term is considered in this study. The objectives of the study are to estimate parameters of Cobb-Douglas production functions and examine measures of Returns to Scale of the Cobb-Douglas production function in the crude oil producing sector in Nigeria. The data for the study are crude oil production in Nigeria from 1998-2017, and are collected from the World Bank (World Development Indicator). The methods of ordinary least square (OLS) are used in the estimation of the production function. The study finds that coefficients for capital (K) and Labour (L) are - 0.0573 and 0.4656 respectively for the entire crude oil producing sector and the coefficients for capital is relatively low. The adjusted  $R^2$  implies that the model is able to explain only 29.2 percent of the total variation in the crude oil production. The findings also reveal that the sector exhibits decreasing returns to scale. This study suggests that increases in capital and labour employment will boost the production of crude oil in Nigeria.

**Keywords:** Cobb-Douglas Production function, Return to Scale, Crude oil, capital, labour.

### INTRODUCTION

Considering the ambitious growth targets of the vision 20: 2020 of Nigeria, it is very important to give serious attention to the crude oil producing sector for employment expansion, productivity enhancement and increasing per capita income oriented to reduction of poverty in the country. Crude oil production contributes significantly to the Nigeria economy, for the past years, it has been a major source of revenue, energy and exchange for the Nigerian economy. In spite of the important role played by crude oil producing sector or industry in Nigeria, it faced with problems of estimation of its parameters in the production function and finding the measures of Returns to scale of the production function in the crude oil producing sector. Production function is a quantitative link between production inputs and outputs. Johnson (2012) affirms that production function establishes the functional relationship between the quantity of a specific product that can be produced within a time and a set of inputs used, given the existing technology in a socio-cultural environment. According to Shaiara and Md (2016) Production function provides quantitative link between inputs and outputs. A production function is a function that summarizes the conversion of inputs into outputs. It can be applied to a single firm, an industry or an entire nation for policy making.



The traditional theory of production function of a firm expresses output as a function of two inputs, capital (K) and Labour (L) in the form of Cobb-Douglas Production function. According to Isaac (2011), the production function may be wrongly specified and the form of mis-specification at micro level may not be exactly as it is at the macro level. The aim of the study is modeling of production function with application to crude oil producing sector in Nigeria.

The Objectives of the study are as follows:

- Estimate Cobb-Douglas Production function
- Examine measures of Returns to Scale of the production function in the crude oil producing sector.

### Cobb-Douglas Production Function

The simplest and most popular production function used frequently in economics is the Cobb-Douglas production function. It was developed by Charles W. Cobb and Paul H. Douglas in 1928. The Cobb-Douglas production function omitting the error term is of the form.

$$Y = \theta_1 K^{\theta_2} L^{\theta_3} \quad (1)$$

Where Y is the output

K is the capital input

L is the Labour Input

$\theta_1, \theta_2$  and  $\theta_3$  are positive constants parameters.

### Literature Review

A research carried out by Hossain and Islam (2013); employed the nonlinear Cobb-Douglas production function to assess returns to scale and to estimate the level of productivity and allocative efficiency in manufacturing firms in the south west region of Bangladesh. The study found that fertilizer and seafood manufacturing firms have increasing returns to scale, while textile, jute, and cement manufacturing firms have decreasing returns to scale. It was concluded that the average capital productivity is less than the capital productivity of all firms in the study. Halid. (2015) examined the relationship between crude oil production and the Nigeria economic performance by using the Cobb-Douglas (CD) production model and the model was fitted to Nigeria's data of crude oil production, capital input, labour input, oil domestic consumption and oil exports. The study revealed that crude oil output has an essential impact on the Nigerian economy because there was a significant relationship between the variables in the model. MD et al (2013), applied non-linear Cobb-Douglas (CD) production function with Autocorrelation problem to selected manufacturing industries in Bangladesh, and identified that Cobb-Douglas production function with additive errors was more suitable for some selected manufacturing industries. The main aim of the study was to detect the autocorrelation problem of Cobb-Douglas production model with additive errors, finally, the results showed that autocorrelation is present in some of the



manufacturing industries. The autocorrelation problem was removed and the parameters of the Cobb-Douglas production function with additive errors was re-estimated.

Dana and Jaromir (2007); affirmed that the practical application of the production function method requires making certain assumptions on the functional form of the production technology, returns to scale, and characteristics of the technological progress, as well as of the functioning of the markets. They argued that the functional form however, includes the assumption of a constant share of labour in output, which may be for a converging country. In this study, they tested whether this fact renders the application of Cobb-Douglas production unreliable for the Czech economy. They applied a more general form of production function and allowed labour share to develop according to the empirical data. Hossain and Bhatti (2004); examined some major manufacturing industries in Bangladesh, using nonlinear Cobb-Douglas production model. In this study, the estimated results clearly showed that the manufacturing industries of Bangladesh generally seem to underline the case of increasing returns to scale, that is a unit increase in either capital or labour input cases of output to grow at a more than one percent rate. The underlying assumptions to be satisfied for the production model. The sum of the elasticities  $\theta_1 + \theta_2$  provides that returns to scale in question. This implies that if  $\theta_1 + \theta_2 = 1$ , the production operates under constant returns to scale, if  $\theta_1 + \theta_2 > 1$  or  $\theta_1 + \theta_2 < 1$ , the production operates under increasing or decreasing returns to scale respectively, where  $\theta_1$  and  $\theta_2$  are the estimated parameters.

According to Damoder (2013), if a model is nonlinear regression model whether the variables of such a model are linear or not and some models look nonlinear in the parameters but are intrinsically linear because with suitable transformation they can be made linear in the parameter regression models. But if such, models cannot be linearized in the parameters, they are called intrinsically nonlinear models, and that there are several methods or algorithms to estimate nonlinear regression models. Afzal and Manni (2013); investigated the nature and extent of productivity changes in the Cobb-Douglas production model components and the growth of the knowledge economy of selected countries, namely, Thailand, Malaysia, Singapore Philippines and south Korea which were analysed over the period 2005 to 2010. In this study, the total factor productivity (TFP) index, individual country's efficiency and productivity changes which took place within this period were estimated. The result obtained indicated that the Philippines and Singapore have highest increase in total factor productivity within the periods, and this growth in productivity was derived from both technical efficiency gain and technological progress. There was a remarkable growth in total factor productivity for Thailand and Philippines for the knowledge.

Many studies have been done so far on Cobb-Douglas production model. For example, the study by Chowdhury and Islam (2015); also applied the nonlinear Cobb-Douglas (C-D) production functions in garments industries in Bangladesh which affirmed that both labour and capital contributed significantly to the total output in garment industries. The statistical Analytical system (SAS) was applied to estimate the Cobb-



Douglas production function econometrically in this study. Prajneshu (2008); fitted Cobb-Douglas production functions to the area of agricultural production by first linearizing the models through logarithmic transformation and then using method of least squares. This procedure is valid only when the underlying assumption of multiplicative error terms is justified. The results obtained are not satisfactory, and nonlinear estimation methods generally gave parameter estimates exhibiting extremely high correlation, indicating that the parameters are not estimated independently. Expected-value parameters was used and highlighted in this study, the developed methodology has been illustrated by using it to the wheat yield time-series data of Punjab. Halid (2012);proposed a nonlinear Cobb-Douglas production model for the analysis of the Mexican Economy by estimating its parameters; a production function data of Mexican economy between 1955- 1974 was fitted to the model and the estimated Cobb- Douglas function was found to be

$$GDP = 0.5292 \text{ Labour}_t^{0.1810} \text{ capital}^{0.8827}$$

Norhidayu, et al. (2016); said crude palm oil millers should improve their productivity by introducing policies to induce technological innovation in order to shift up the production frontier. It was also highly recommended that known technology should be applied which include improvement in learning by doing processes and improved managerial practices. It was concluded that the mill should inject more capital into upgrading their processing capacity and Machinery. Ashfaq and Muhammad (2015); estimated the nonlinear Cobb-Douglas production function to investigate the relationship between the production of cement and inputs labour and capital. The results of the estimates showed that there is a constant return to scale in the cement industry, moreover, the empirical results also showed that the capital contributes relatively less than the labour during the production process. From this study, it was concluded that there is a strong relationship between the input and output variables.

### Material and Method

The Data for this study is the crude oil production in Nigeria from 1998 – 2017 obtained from World Bank (World Bank Development Indicator).

The Cobb-Douglas production function with Multiplication error term is of the form.

$$Y = \theta_1 K^{\theta_2} L^{\theta_3} e^u \quad (2)$$

Where  $Y$  = output (crude oil production level)

$K$  = Capital investment

$L$  = Labour (workers) inputs

$\theta_1$ ,  $\theta_2$  and  $\theta_3$  are positive parameters to be estimated.  $U$  = error term or stochastic disturbance term,  $e$  = Base of natural logarithm.

From Equation (2) the relationship between the output and the two inputs is nonlinear. In order to use the ordinary least square estimation method (OLS), the Equation (2) must be linearized. Thus, we take the natural logarithms of Equation (2), we have

$$\ln Y = \ln \theta_1 + \theta_2 \ln K + \theta_3 \ln L + u \quad (3)$$

$$= A + \theta_2 \ln K + \theta_3 \ln L + u \quad (4)$$

Where  $A = \ln \theta_1$



Equation (4) is linear in the parameters  $A$ ,  $\theta_2$  and  $\theta_3$  and is therefore a linear regression model. It is nonlinear in the variables  $Y$ ,  $K$  and  $L$  but linear in the logs of these variables. Model (4) is log linear model.

The sum of the parameters of capital and labour i.e.  $(\theta_1 + \theta_2)$  is the returns to scale.

Let  $\lambda = \theta_1 + \theta_2$

When  $\lambda = 1$ , means constant returns to scale.

When  $\lambda > 1$ , means increase returns to scale.

When  $\lambda < 1$ , means decrease returns to scale

## Results and Discussion

The estimated results are tabulated in table 1.

**Table 1: The Ordinary Least Square Estimation Result**

Dependent Variable: LOG(Y)

Method: Least Squares (OLS)

Date: 09/21/18 Time: 15:48

Sample: 1998 – 2017

Included observations: 20

$\text{LOG}(Y) = C(1) + C(2) * \text{LOG}(K) + C(3) * \text{LOG}(L)$

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	6.567924	0.409206	16.05041	0.0000
C(2)	-0.057283	0.063124	-0.907468	0.3768
C(3)	0.465619	0.255172	1.824725	0.0857
R-squared	0.366640	Mean dependent var	7.746186	
Adjusted R-squared	0.292127	S.D. dependent var	0.065364	
S.E. of regression	0.054994	Akaike info criterion	-2.825694	
Sum squared resid	0.051414	Schwarz criterion	-2.676334	
Log likelihood	31.25694	Hannan-Quinn criter.	-2.796538	
F-statistic	4.920484	Durbin-Watson stat	1.147841	
Prob(F-statistic)	0.020608			

From the estimated result, we obtain the model below,

$$\ln y = 1.8822 - 0.0573 \ln K + 0.4656 \ln L$$

The coefficients for  $K$  and  $L$  imply that productivity of inputs that is Labour is more productive than capital. The findings indicate that capital should be employed more in the crude oil producing sector, in order to increase production as well as economic development. The findings also show that sector exhibits decreasing returns to scale. The analysis shows that coefficient for capital is -0.0572 and coefficient for Labour is 0.4656 for the entire crude oil producing sector. By implication, the result implies that, a unit



increase in the capital will cause 0.0572 percent decrease in the crude oil production holding other explanatory variables constant. That is, any additional capital investment in the crude oil production will lead to 0.0572 percent reduction in the crude oil production at the location under study. Similarly, a unit increase in the labour input will cause 0.4656 percent increase in the crude oil production holding capital investment and the intercept as constant. The adjusted  $R^2$  implies that the model is able to explain only 29.2 percent of the total variation in the crude oil production. Thus, it implies that the two explanatory variables are responsible for just 29.2 percent of the causes of variation in the crude oil production. This means that, there are other variables that are responsible for fluctuation in the crude oil production that the model didn't capture. The F-statistic value (4.920) shows that the overall model is statistically significant at 1% and 5% significance levels, because it is greater than the critical values of 2.57 and 3.79 at 1% and 5% respectively. The Durbin Watson Statistic is a measure of autocorrelation, it shows that the model is free from autocorrelation as its value (1.148) at 1% level of significance.

## CONCLUSION

From the findings of the study, we can conclude that Cobb-Douglas Production function is useful and powerful tool for the analysis and evaluation of the governmental structural policies in the context of crude oil producing sector of Nigeria. It also reveals that this sector exhibits decreasing returns to scale. This means that increase in production is less than increase in inputs (capital, labour). This study suggests that increase in capital and labour employment will boost the production of crude oil in Nigeria.

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